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Some Structures on Neutrosophic Topological Spaces

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Abstract

In this paper, we define boundary of neutrosophic soft set, neutrosophic soft dense set, neutrosophic soft basis and neutrosophic soft subspace topology on neutrosophic soft topological spaces. Furthermore, some important theorems are proved and interesting examples are given.

Keywords: neutrosophic soft set, neutrosophic soft topological spaces, boundary of neutrosophic soft set, neutrosophic soft dense set, neutrosophic soft basis, neutrosophic soft subspace topology

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1 Introduction

The theory of fuzzy set was introduced by Zadeh in 1965 [19]. Fuzzy sets have been applied in many real life problems to handle uncertainty. After Zadeh, Smarandache introduced the theory of neutrosophic set [17]. This theory is the generalization of many theories such as; fuzzy set [19], intuitionistic fuzzy set [7]. In recent years, there have been many academic studies on the theory of neutrosophic set [3, 4, 8, 9, 13, 14, 16]. Many classical methods were not enough to solve problems related to uncertainties. Therefore Molodtsov introduced the soft set theory in 1999 [12]. The soft set theory is completely a new approach for dealing with uncertainties and vagueness. After Molodtsov, many different studies have been done on soft set theory. Also, many authors studied on different combination of fuzzy set, soft set, intuitionistic set, neutrosophic set, etc. [1–6, 8, 11, 15, 16, 18]. One of these combinations, neutrosophic soft set theory was first introduced by Maji [10]. Later, this theory was modified by Deli and Broumi [8]. Also, Bera presented neutrosophic soft topological spaces [4]. Recently, researchers have shown great interest in this theory. Operations on the neutrosophic soft set theory were re-defined as different from [4, 8] by Ozturk T. Y. et. al [13]. They also studied some separation axioms on neutrosophic soft topological spaces [9].

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In this paper, considering these newly defined operations, unlike [13], boundary of neutrosophic soft set, neutrosophic soft basis, neutrosophic soft dense set, neutrosophic soft subspaces on neutrosophic soft topological spaces are defined. In addition, some important theorems together with proofs are given and study is supported by many different examples.

2 Preliminary

Definition 1. [8] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X . Then, a neutrosophic soft set $(\tilde{F}, E)$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F} : E \rightarrow P(X)$ where $\tilde{F}$ is called approximate function of the neutrosophic soft set $(\tilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,

$$(\tilde{F}, E) = \left\{ \left(e, \langle x, T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where $T_{\tilde{F}(e)}(x)$, $I_{\tilde{F}(e)}(x)$, $F_{\tilde{F}(e)}(x) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$. Since supremum of each T , I , F is 1 so the inequality $0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2. [4] Let $(\tilde{F}, E)$ be neutrosophic soft set over the universe set X . The complement of $(\tilde{F}, E)$ is denoted by $(\tilde{F}, E)^c$ and is defined by:

$$(\tilde{F}, E)^c = \left\{ \left(e, \langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x) \rangle : x \in X \right) : e \in E \right\}.$$

Obvious that, $((\tilde{F}, E)^c)^c = (\tilde{F}, E)$.

Definition 3. [10] Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two neutrosophic soft sets over the universe set X . $(\tilde{F}, E)$ is said to be neutrosophic soft subset of $(\tilde{G}, E)$ if $T_{\tilde{F}(e)}(x) \leq T_{\tilde{G}(e)}(x)$, $I_{\tilde{F}(e)}(x) \leq I_{\tilde{G}(e)}(x)$, $F_{\tilde{F}(e)}(x) \geq F_{\tilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$. It is denoted by $(\tilde{F}, E) \subseteq (\tilde{G}, E)$.

$(\tilde{F}, E)$ is said to be neutrosophic soft equal to $(\tilde{G}, E)$ if $(\tilde{F}, E)$ is neutrosophic soft subset of $(\tilde{G}, E)$ and $(\tilde{G}, E)$ is neutrosophic soft subset of $(\tilde{F}, E)$. It is denoted by $(\tilde{F}, E) = (\tilde{G}, E)$.

Definition 4. [13] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X . Then their union is denoted by $(\tilde{F}_1, E) \cup (\tilde{F}_2, E) = (\tilde{F}_3, E)$ and is defined by:

$$(\tilde{F}_3, E) = \left\{ \left(e, \langle x, T_{\tilde{F}_3(e)}(x), I_{\tilde{F}_3(e)}(x), F_{\tilde{F}_3(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where

$$\begin{aligned} T_{\tilde{F}_3(e)}(x) &= \max \{ T_{\tilde{F}_1(e)}(x), T_{\tilde{F}_2(e)}(x) \}, \\ I_{\tilde{F}_3(e)}(x) &= \max \{ I_{\tilde{F}_1(e)}(x), I_{\tilde{F}_2(e)}(x) \}, \\ F_{\tilde{F}_3(e)}(x) &= \min \{ F_{\tilde{F}_1(e)}(x), F_{\tilde{F}_2(e)}(x) \}. \end{aligned}$$

Definition 5. [13] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X . Then their intersection is denoted by $(\tilde{F}_1, E) \cap (\tilde{F}_2, E) = (\tilde{F}_3, E)$ and is defined by:

$$(\tilde{F}_3, E) = \left\{ \left(e, \langle x, T_{\tilde{F}_3(e)}(x), I_{\tilde{F}_3(e)}(x), F_{\tilde{F}_3(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where

$$\begin{aligned} T_{\tilde{F}_3(e)}(x) &= \min \left\{ T_{\tilde{F}_1(e)}(x), T_{\tilde{F}_2(e)}(x) \right\}, \\ I_{\tilde{F}_3(e)}(x) &= \min \left\{ I_{\tilde{F}_1(e)}(x), I_{\tilde{F}_2(e)}(x) \right\}, \\ F_{\tilde{F}_3(e)}(x) &= \max \left\{ F_{\tilde{F}_1(e)}(x), F_{\tilde{F}_2(e)}(x) \right\}. \end{aligned}$$

Definition 6. [13] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X . Then " $(\tilde{F}_1, E)$ difference $(\tilde{F}_2, E)$ " operation on them is denoted by $(\tilde{F}_1, E) \setminus (\tilde{F}_2, E) = (\tilde{F}_3, E)$ and is defined by $(\tilde{F}_3, E) = (\tilde{F}_1, E) \cap (\tilde{F}_2, E)^c$ as follows:

$$(\tilde{F}_3, E) = \left\{ \left(e, \langle x, T_{\tilde{F}_3(e)}(x), I_{\tilde{F}_3(e)}(x), F_{\tilde{F}_3(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where

$$\begin{aligned} T_{\tilde{F}_3(e)}(x) &= \min \left\{ T_{\tilde{F}_1(e)}(x), F_{\tilde{F}_2(e)}(x) \right\}, \\ I_{\tilde{F}_3(e)}(x) &= \min \left\{ I_{\tilde{F}_1(e)}(x), 1 - I_{\tilde{F}_2(e)}(x) \right\}, \\ F_{\tilde{F}_3(e)}(x) &= \max \left\{ F_{\tilde{F}_1(e)}(x), T_{\tilde{F}_2(e)}(x) \right\}. \end{aligned}$$

Definition 7. [13] 1. A neutrosophic soft set $(\tilde{F}, E)$ over the universe set X is said to be null neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 0, I_{\tilde{F}(e)}(x) = 0, F_{\tilde{F}(e)}(x) = 1; \forall e \in E, \forall x \in X$. It is denoted by $0_{(X,E)}$.

2. A neutrosophic soft set $(\tilde{F}, E)$ over the universe set X is said to be absolute neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 1, I_{\tilde{F}(e)}(x) = 1, F_{\tilde{F}(e)}(x) = 0; \forall e \in E, \forall x \in X$. It is denoted by $1_{(X,E)}$.

Clearly, $0_{(X,E)}^c = 1_{(X,E)}$ and $1_{(X,E)}^c = 0_{(X,E)}$.

Proposition 1. [13] Let $(\tilde{F}_1, E), (\tilde{F}_2, E)$ and $(\tilde{F}_3, E)$ be neutrosophic soft sets over the universe set X . Then,

1. $(\tilde{F}_1, E) \cup [(\tilde{F}_2, E) \cup (\tilde{F}_3, E)] = [(\tilde{F}_1, E) \cup (\tilde{F}_2, E)] \cup (\tilde{F}_3, E)$ and $(\tilde{F}_1, E) \cap [(\tilde{F}_2, E) \cap (\tilde{F}_3, E)] = [(\tilde{F}_1, E) \cap (\tilde{F}_2, E)] \cap (\tilde{F}_3, E);$
2. $(\tilde{F}_1, E) \cup [(\tilde{F}_2, E) \cap (\tilde{F}_3, E)] = [(\tilde{F}_1, E) \cup (\tilde{F}_2, E)] \cap [(\tilde{F}_1, E) \cup (\tilde{F}_3, E)]$ and $(\tilde{F}_1, E) \cap [(\tilde{F}_2, E) \cup (\tilde{F}_3, E)] = [(\tilde{F}_1, E) \cap (\tilde{F}_2, E)] \cup [(\tilde{F}_1, E) \cap (\tilde{F}_3, E)];$
3. $(\tilde{F}_1, E) \cup 0_{(X,E)} = (\tilde{F}_1, E)$ and $(\tilde{F}_1, E) \cap 0_{(X,E)} = 0_{(X,E)};$
4. $(\tilde{F}_1, E) \cup 1_{(X,E)} = 1_{(X,E)}$ and $(\tilde{F}_1, E) \cap 1_{(X,E)} = (\tilde{F}_1, E).$

Proposition 2. [13] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X . Then,

1. $\left[(\tilde{F}_1, E) \cup (\tilde{F}_2, E) \right]^c = (\tilde{F}_1, E)^c \cap (\tilde{F}_2, E)^c$;
2. $\left[(\tilde{F}_1, E) \cap (\tilde{F}_2, E) \right]^c = (\tilde{F}_1, E)^c \cup (\tilde{F}_2, E)^c$.

Definition 8. [9] Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X . Then neutrosophic soft set $x_{(\alpha, \beta, \gamma)}^e$ is called a neutrosophic soft point, for every $x \in X, 0 < \alpha, \beta, \gamma \leq 1, e \in E$, and defined as follows:

$$x_{(\alpha, \beta, \gamma)}^e(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } e' = e \text{ and } y = x, \\ (0, 0, 1) & \text{if } e' \neq e \text{ or } y \neq x. \end{cases}$$

Definition 9. [9] Let $(\tilde{F}, E)$ be a neutrosophic soft set over the universe set X . We say that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)$ read as belongs to the neutrosophic soft set $(\tilde{F}, E)$, whenever $\alpha \leq F_{\tilde{F}(e)}(x), \beta \leq I_{\tilde{F}(e)}(x)$ and $\gamma \geq T_{\tilde{F}(e)}(x)$.

Definition 10. [9] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X . A neutrosophic soft set $(\tilde{F}, E)$ in $(X, \overset{NSS}{\tau}, E)$ is called a neutrosophic soft neighborhood of the neutrosophic soft point $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)$, if there exists a neutrosophic soft open set $(\tilde{G}, E)$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{G}, E) \subset (\tilde{F}, E)$.

Definition 11. [9] Let $x_{(\alpha, \beta, \gamma)}^e$ and $y_{(\alpha', \beta', \gamma')}^{e'}$ be two neutrosophic soft points. For the neutrosophic soft points $x_{(\alpha, \beta, \gamma)}^e$ and $y_{(\alpha', \beta', \gamma')}^{e'}$ over a common universe X , we say that the neutrosophic soft points are distinct points if $x_{(\alpha, \beta, \gamma)}^e \cap y_{(\alpha', \beta', \gamma')}^{e'} = 0_{(X, E)}$.

It is clear that $x_{(\alpha, \beta, \gamma)}^e$ and $y_{(\alpha_1, \beta_1, \gamma_1)}^{e'}$ are distinct neutrosophic soft points if and only if $x \neq y$ or $e' \neq e$.

Definition 12. [13] Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\overset{NSS}{\tau} \subset NSS(X, E)$. Then $\overset{NSS}{\tau}$ is said to be a neutrosophic soft topology on X if

1. $0_{(X, E)}$ and $1_{(X, E)}$ belongs to $\overset{NSS}{\tau}$
2. the union of any number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$
3. the intersection of finite number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$.

Then $(X, \overset{NSS}{\tau}, E)$ is said to be a neutrosophic soft topological space over X . Each members of $\overset{NSS}{\tau}$ is said to be neutrosophic soft open set.

Definition 13. [13] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E)$ be a neutrosophic soft set over X . Then $(\tilde{F}, E)$ is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 14. [13] Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$ be a neutrosophic soft set. Then, the neutrosophic soft interior of $(\tilde{F}, E)$, denoted $(\tilde{F}, E)^\circ$, is defined as the neutrosophic soft union of all neutrosophic soft open subsets of $(\tilde{F}, E)$.

Clearly, $(\tilde{F}, E)^\circ$ is the biggest neutrosophic soft open set that is contained by $(\tilde{F}, E)$.

Definition 15. [13] Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$ be a neutrosophic soft set. Then, the neutrosophic soft closure of $(\tilde{F}, E)$, denoted $\overline{(\tilde{F}, E)}$, is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of $(\tilde{F}, E)$.

Clearly, $\overline{(\tilde{F}, E)}$ is the smallest neutrosophic soft closed set that containing $(\tilde{F}, E)$.

3 Some Structures on Neutrosophic Soft Topological Spaces

Definition 16. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$ be a neutrosophic soft set over X . If $Fr(\tilde{F}, E) = \overline{(\tilde{F}, E)} \cap ((\tilde{F}, E)^c)$, then $Fr(\tilde{F}, E)$ is said to be boundary of the neutrosophic soft set $(\tilde{F}, E)$.

Example 1. Let $X = \{x_1, x_2, x_3\}$ be an initial universe set, $E = \{e_1, e_2\}$ be a set of parameters and

$$\tau^{NSS} = \left\{ 0_{(X, E)}, 1_{(X, E)}, (\tilde{F}_1, E), (\tilde{F}_2, E), (\tilde{F}_3, E), (\tilde{F}_4, E) \right\}$$

be a neutrosophic soft topology over X . Here, the neutrosophic soft sets $(\tilde{F}_1, E)$, $(\tilde{F}_2, E)$, $(\tilde{F}_3, E)$ and $(\tilde{F}_4, E)$ over X are defined as following;

$$\begin{aligned} (\tilde{F}_1, E) &= \left\{ e_1, \{ \langle x_1, 0.6, 0.4, 0.7 \rangle, \langle x_2, 0.3, 0.5, 0.2 \rangle, \langle x_3, 0.4, 0.6, 0.9 \rangle \} \right\}, \\ (\tilde{F}_2, E) &= \left\{ e_1, \{ \langle x_1, 0.2, 0.7, 0.6 \rangle, \langle x_2, 0.3, 0.7, 0.9 \rangle, \langle x_3, 0.5, 0.8, 0.2 \rangle \} \right\}, \\ (\tilde{F}_3, E) &= \left\{ e_1, \{ \langle x_1, 0.2, 0.4, 0.7 \rangle, \langle x_2, 0.3, 0.5, 0.9 \rangle, \langle x_3, 0.4, 0.6, 0.9 \rangle \} \right\}, \\ (\tilde{F}_4, E) &= \left\{ e_1, \{ \langle x_1, 0.6, 0.7, 0.6 \rangle, \langle x_2, 0.3, 0.7, 0.2 \rangle, \langle x_3, 0.5, 0.8, 0.2 \rangle \} \right\}. \end{aligned}$$

Suppose that the neutrosophic soft set $(\tilde{F}, E)$ over X is defined as;

$$(\tilde{F}, E) = \left\{ e_1, \{ \langle x_1, 0.7, 0.6, 0.3 \rangle, \langle x_2, 0.5, 0.7, 0.1 \rangle, \langle x_3, 0.8, 0.8, 0.6 \rangle \} \right\}.$$

Then, let us find the boundary of the neutrosophic soft set $(\tilde{F}, E)$:

$$\overline{(\tilde{F}, E)} = 1_{(X, E)} \text{ and } (\tilde{F}, E)^c = (\tilde{F}_1, E)^c \cap (\tilde{F}_3, E)^c$$

Therefore,

$$\begin{aligned} Fr(\tilde{F}, E) &= \overline{(\tilde{F}, E)} \cap ((\tilde{F}, E)^c) = 1_{(X, E)} \cap (\tilde{F}_1, E)^c \cap (\tilde{F}_3, E)^c \\ &= \left\{ e_1, \{ \langle x_1, 0.7, 0.7, 0.6 \rangle, \langle x_2, 0.2, 0.5, 0.3 \rangle, \langle x_3, 0.9, 0.9, 0.4 \rangle \} \right\} \\ &= \left\{ e_2, \{ \langle x_1, 0.4, 0.3, 0.3 \rangle, \langle x_2, 0.5, 0.8, 0.1 \rangle, \langle x_3, 0.9, 0.2, 0.7 \rangle \} \right\}. \end{aligned}$$

Theorem 1. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}_1, E), (\tilde{F}_2, E) \in NSS(X, E)$. Then,

1. $(\tilde{F}_1, E)^o = (\tilde{F}_1, E) \setminus Fr(\tilde{F}_1, E)$,
2. $\overline{(\tilde{F}_1, E)} = (\tilde{F}_1, E) \cup Fr(\tilde{F}_1, E)$,
3. $Fr((\tilde{F}_1, E) \cup (\tilde{F}_2, E)) \subseteq Fr(\tilde{F}_1, E) \cup Fr(\tilde{F}_2, E)$,
4. $Fr((\tilde{F}_1, E)^c) = Fr(\tilde{F}_1, E)$,
5. $1_{(X, E)} = (\tilde{F}_1, E)^o \cup Fr(\tilde{F}_1, E) \cup (1_{(X, E)} \setminus (\tilde{F}_1, E))^o$,
6. $Fr(\overline{(\tilde{F}_1, E)}) \subseteq Fr(\tilde{F}_1, E)$,
7. $Fr((\tilde{F}_1, E)^o) \subseteq Fr(\tilde{F}_1, E)$,
8. $(\tilde{F}_1, E)$ is a neutrosophic soft open set $\Leftrightarrow Fr(\tilde{F}_1, E) = \overline{(\tilde{F}_1, E)} \setminus (\tilde{F}_1, E)$,
9. $(\tilde{F}_1, E)$ is a neutrosophic soft closed set $\Leftrightarrow Fr(\tilde{F}_1, E) = (\tilde{F}_1, E) \setminus (\tilde{F}_1, E)^o$.

Proof. 1. $(\tilde{F}_1, E) \setminus Fr(\tilde{F}_1, E) = (\tilde{F}_1, E) \setminus \left(\overline{(\tilde{F}_1, E)} \cap \overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right)$

$$= (\tilde{F}_1, E) \cap \left(\overline{(\tilde{F}_1, E)} \cap \overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right)^c$$

$$= \left((\tilde{F}_1, E) \cap \left(\overline{(\tilde{F}_1, E)} \right)^c \right) \cup \left((\tilde{F}_1, E) \cap \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right)^c \right)$$

$$= \left((\tilde{F}_1, E) \setminus \overline{(\tilde{F}_1, E)} \right) \cup \left((\tilde{F}_1, E) \setminus \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right) \right)$$

$$= (\tilde{F}_1, E) \setminus \overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} = (\tilde{F}_1, E) \cap \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right)^c$$

$$= (\tilde{F}_1, E) \cap (\tilde{F}_1, E)^o = (\tilde{F}_1, E)^o.$$

2. It is clear.

$$3. Fr((\tilde{F}_1, E) \cup (\tilde{F}_2, E)) = \overline{(\tilde{F}_1, E) \cup (\tilde{F}_2, E)} \cap \overline{(1_{(X, E)} \setminus ((\tilde{F}_1, E) \cup (\tilde{F}_2, E)))}$$

$$= \left(\overline{(\tilde{F}_1, E)} \cup \overline{(\tilde{F}_2, E)} \right) \cap \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \cap \overline{(1_{(X, E)} \setminus (\tilde{F}_2, E))} \right)$$

$$\subseteq \left(\overline{(\tilde{F}_1, E)} \cup \overline{(\tilde{F}_2, E)} \right) \cap \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right) \cap \left(\overline{(1_{(X, E)} \setminus (\tilde{F}_2, E))} \right)$$

$$\subseteq \left(\overline{(\tilde{F}_1, E)} \cap \overline{(1_{(X, E)} \setminus (\tilde{F}_1, E))} \right) \cup \left(\overline{(\tilde{F}_2, E)} \cap \overline{(1_{(X, E)} \setminus (\tilde{F}_2, E))} \right)$$

$$= Fr(\tilde{F}_1, E) \cup Fr(\tilde{F}_2, E)$$

$$4. Fr((\tilde{F}_1, E)^c) = \overline{(\tilde{F}_1, E)^c} \cap \overline{((\tilde{F}_1, E)^c)^c} = \overline{(\tilde{F}_1, E)^c} \cap \overline{(\tilde{F}_1, E)}$$

$$= Fr(\tilde{F}_1, E)$$

5. It is clear.

$$6. Fr(\overline{(\tilde{F}_1, E)}) = \overline{(\tilde{F}_1, E)} \cap \overline{((\tilde{F}_1, E))^c} \subseteq \overline{(\tilde{F}_1, E)} \cap \overline{(\tilde{F}_1, E)^c} = Fr(\tilde{F}_1, E)$$

7. It is clear.

8. Suppose that $(\tilde{F}_1, E)$ is a neutrosophic soft open set. Then $(\tilde{F}_1, E)^c$ is a neutrosophic soft closed set and $\overline{(\tilde{F}_1, E)^c} = (\tilde{F}_1, E)^c$. In here,

$$Fr(\tilde{F}_1, E) = \overline{(\tilde{F}_1, E)} \cap \overline{((\tilde{F}_1, E)^c)} = \overline{(\tilde{F}_1, E)} \cap (\tilde{F}_1, E)^c = \overline{(\tilde{F}_1, E)} \setminus (\tilde{F}_1, E).$$

From the condition-1,

$$\begin{aligned}(\tilde{F}_1, E)^o &= (\tilde{F}_1, E) \setminus Fr(\tilde{F}_1, E) = (\tilde{F}_1, E) \setminus \left((\tilde{F}_1, E) \setminus \left((\tilde{F}_1, E) \cap \left((\tilde{F}_1, E) \cap (\tilde{F}_1, E)^c \right)^c \right) \right) \\&= (\tilde{F}_1, E) \cap \left(\left((\tilde{F}_1, E) \right)^c \cup \left((\tilde{F}_1, E)^c \right)^c \right) \\&= \left((\tilde{F}_1, E) \cap \left((\tilde{F}_1, E) \right)^c \right) \cup \left((\tilde{F}_1, E) \cap (\tilde{F}_1, E) \right) \\&= \left((\tilde{F}_1, E) \cap \left((\tilde{F}_1, E) \right)^c \right) \cup (\tilde{F}_1, E) = (\tilde{F}_1, E)\end{aligned}$$

That is, $(\tilde{F}_1, E)$ is a neutrosophic soft open set. $\square$

Definition 17. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$.

- a) $(\tilde{F}, E)$ is said to be a neutrosophic soft dense set in $(X, \overset{NSS}{\tau}, E)$ if $\overline{(\tilde{F}, E)} = 1_{(X, E)}$,
- b) $(\tilde{F}, E)$ is said to be a neutrosophic soft co-dense set in $(X, \overset{NSS}{\tau}, E)$ if $\overline{(1_{(X, E)} \setminus (\tilde{F}, E))} = 1_{(X, E)}$,
- c) $(\tilde{F}, E)$ is said to be a neutrosophic soft not-dense set in any part of $(X, \overset{NSS}{\tau}, E)$ if $\overline{(\tilde{F}, E)}$ is a neutrosophic soft dense set over $(X, \overset{NSS}{\tau}, E)$.

Theorem 2. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$. Then,

- 1. $(\tilde{F}, E)$ is a neutrosophic soft dense set in $(X, \overset{NSS}{\tau}, E)$ iff $(\tilde{F}, E) \cap (\tilde{U}, E) \neq 0_{(X, E)}$ for each $0_{(X, E)} \neq (\tilde{U}, E) \in \overset{NSS}{\tau}$,
- 2. $(\tilde{F}, E)$ is a neutrosophic soft co-dense set in $(X, \overset{NSS}{\tau}, E)$ iff $\left(1_{(X, E)} \setminus (\tilde{F}, E) \right) \cap (\tilde{U}, E) \neq 0_{(X, E)}$ for each $0_{(X, E)} \neq (\tilde{U}, E) \in \overset{NSS}{\tau}$,
- 3. $(\tilde{F}, E)$ is a neutrosophic soft not-dense set in any part of $(X, \overset{NSS}{\tau}, E)$ iff there is a neutrosophic soft open set $(\tilde{V}, E) \in \overset{NSS}{\tau}$ such that $(\tilde{V}, E) \cap (\tilde{F}, E) = 0_{(X, E)}$ and $0_{(X, E)} \neq (\tilde{V}, E) \subseteq (\tilde{U}, E)$ for each $0_{(X, E)} \neq (\tilde{U}, E) \in \overset{NSS}{\tau}$.

Proof. Straightforward. $\square$

Definition 18. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $\overset{NSS}{B}$ be a sub-family of $\overset{NSS}{\tau}$. $\overset{NSS}{B}$ is said to be a neutrosophic soft basis for the neutrosophic soft topology $\overset{NSS}{\tau}$ if every element of $\overset{NSS}{\tau}$ can be written as the neutrosophic soft union of elements of $\overset{NSS}{B}$.

Theorem 3. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $\overset{NSS}{B}$ be a neutrosophic soft basis for $\overset{NSS}{\tau}$. Then, $\overset{NSS}{\tau}$ equals to the collection of all neutrosophic soft unions of elements of $\overset{NSS}{B}$.

Proof. This is easily seen from the definition of neutrosophic soft basis. $\square$

Example 2. Let us consider the neutrosophic soft topology $\overset{NSS}{\tau}$ given in Example-1. Then, the family

$$\overset{NSS}{B} = \left\{ 0_{(X, E)}, 1_{(X, E)}, (\tilde{F}_1, E), (\tilde{F}_2, E), (\tilde{F}_3, E) \right\}$$

is a neutrosophic soft basis for the neutrosophic soft topology $\overset{NSS}{\tau}$.

Theorem 4. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $\mathcal{B}^{NSS}$ be a sub-family of τ^{NSS} . Then,

1. The family $\mathcal{B}^{NSS}$ is a neutrosophic soft basis of the neutrosophic soft topology τ^{NSS} iff there exist a neutrosophic soft set $(\tilde{B}, E) \in \mathcal{B}^{NSS}$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}, E) \subseteq (\tilde{F}, E)$ for each $(\tilde{F}, E) \in \tau^{NSS}$ and $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)$.
2. If the family $\mathcal{B}^{NSS} = \{(\tilde{B}_i, E)\}_{i \in I}$ is a neutrosophic soft basis for τ^{NSS} , then there exist a neutrosophic soft set $(\tilde{B}_{i_3}, E) \in \mathcal{B}^{NSS}$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_{i_3}, E) \subseteq (\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E)$ for each $(\tilde{B}_{i_1}, E), (\tilde{B}_{i_2}, E) \in \mathcal{B}^{NSS}$ and each $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E)$.

Proof. 1. $\Rightarrow$ Suppose that $\mathcal{B}^{NSS}$ is a neutrosophic soft basis of the neutrosophic soft topology τ^{NSS} , $(\tilde{F}, E) \in \tau^{NSS}$ and $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)$. Then $(\tilde{F}, E) = \bigcup_{(\tilde{B}, E) \in \mathcal{B}^{NSS}} (\tilde{B}, E)$. Therefore $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}, E) \subseteq (\tilde{F}, E)$ from $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E) =$

$$\bigcup_{(\tilde{B}, E) \in \mathcal{B}^{NSS}} (\tilde{B}, E) \text{ for } x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}, E) \subseteq (\tilde{F}, E).$$

$\Leftarrow$ Suppose that the condition of theorem to be provided. Then,

$$\begin{aligned} (\tilde{F}, E) &= \bigcup_{x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)} \{x_{(\alpha, \beta, \gamma)}^e\} \subseteq \bigcup_{x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)} (\tilde{B}, E) \subseteq (\tilde{F}, E) \\ \Rightarrow (\tilde{F}, E) &= \bigcup_{x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)} (\tilde{B}, E). \end{aligned}$$

That is, $\mathcal{B}^{NSS}$ is a neutrosophic soft basis for τ^{NSS} .

2. Let $(\tilde{B}_{i_1}, E), (\tilde{B}_{i_2}, E) \in \mathcal{B}^{NSS}$ and $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E)$. Since $(\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E)$ is a neutrosophic soft open set and $\mathcal{B}^{NSS}$ is a neutrosophic soft basis for τ^{NSS} , then $(\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E) = \bigcup_j (\tilde{B}_j, E) \Rightarrow x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E) = \bigcup_j (\tilde{B}_j, E) \Rightarrow \exists (\tilde{B}_{i_3}, E), x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_{i_3}, E) \subseteq (\tilde{B}_{i_1}, E) \cap (\tilde{B}_{i_2}, E)$. $\square$

Theorem 5. Let τ_1^{NSS} and τ_2^{NSS} be two neutrosophic soft topologies over X generated by the neutrosophic soft bases $\mathcal{B}_1^{NSS}$ and $\mathcal{B}_2^{NSS}$, respectively. Then $\tau_1^{NSS} \subseteq \tau_2^{NSS}$ iff for each $x_{(\alpha, \beta, \gamma)}^e \in NSS(X, E)$ and for each $(\tilde{B}_1, E) \in \mathcal{B}_1^{NSS}$ containing $x_{(\alpha, \beta, \gamma)}^e$, there exists $(\tilde{B}_2, E) \in \mathcal{B}_2^{NSS}$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_2, E) \subseteq (\tilde{B}_1, E)$.

Proof. $\Rightarrow$ Suppose that $\tau_1^{NSS} \subseteq \tau_2^{NSS}$ and $x_{(\alpha, \beta, \gamma)}^e \in NSS(X, E)$, $(\tilde{B}_1, E) \in \mathcal{B}_1^{NSS}$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_1, E)$. Since $\mathcal{B}_1^{NSS}$ is a neutrosophic soft basis for neutrosophic soft topology τ_1^{NSS} over X , then $\mathcal{B}_1^{NSS} \subseteq \tau_1^{NSS} \Rightarrow x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_1, E) \in \mathcal{B}_1^{NSS} \subseteq \tau_1^{NSS} \subseteq \tau_2^{NSS}$ i.e., $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_1, E) \in \tau_2^{NSS}$. Since $\mathcal{B}_2^{NSS}$ is a neutrosophic soft basis for τ_2^{NSS} , so for $(\tilde{B}_2, E) \in \mathcal{B}_2^{NSS}$ we have $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_2, E) \subseteq (\tilde{B}_1, E)$.

$\Leftarrow$ Conversely, assume that the hypothesis holds. Let $(\tilde{F}, E) \in \tau_1^{NSS}$. Since $\mathcal{B}_1^{NSS}$ is a neutrosophic soft basis for neutrosophic soft topology τ_1^{NSS} , then for $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}, E)$ there exist $(\tilde{B}_1, E) \in \mathcal{B}_1^{NSS}$ such that $x_{(\alpha, \beta, \gamma)}^e \in (\tilde{B}_1, E) \subseteq$

$(\tilde{F}, E)$. No by hypothesis, there exist $(\tilde{B}_2, E) \in \tilde{B}_2^{NSS}$ such that $(\tilde{B}_2, E) \subseteq (\tilde{B}_1, E) \Rightarrow (\tilde{B}_2, E) \subseteq (\tilde{B}_1, E) \subseteq (\tilde{F}, E) \Rightarrow (\tilde{B}_2, E) \subseteq (\tilde{F}, E) \Rightarrow (\tilde{F}, E) \in \tau_2^{NSS}$. This show that $\tau_1^{NSS} \subseteq \tau_2^{NSS}$. $\square$

Theorem 6. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X; E)$. Then the collection

$$\tau_{(\tilde{F}, E)}^{NSS} = \left\{ (\tilde{F}, E) \cap (\tilde{F}_i, E) : (\tilde{F}_i, E) \in \tau^{NSS} \text{ for } i \in I \right\}$$

is a neutrosophic soft topology on $(\tilde{F}, E)$ and $\left(X_{(\tilde{F}, E)}, \tau_{(\tilde{F}, E)}^{NSS}, E \right)$ is a neutrosophic soft topological space.

Proof. Since $0_{(\tilde{X}, E)} \cap (\tilde{F}, E) = 0_{(\tilde{F}, E)}$ and $1_{(X, E)} \cap (\tilde{F}, E) = (\tilde{F}, E)$, then $0_{(\tilde{F}, E)}$ and $(\tilde{F}, E) \in \tau_{(\tilde{F}, E)}^{NSS}$. Moreover,

$$\bigcap_{i=1}^n ((\tilde{F}_i, E) \cap (\tilde{F}, E)) = \left(\bigcap_{i=1}^n (\tilde{F}_i, E) \right) \cap (\tilde{F}, E)$$

and

$$\bigcup_{i \in I} ((\tilde{F}_i, E) \cup (\tilde{F}, E)) = \left(\bigcup_{i \in I} (\tilde{F}_i, E) \right) \cup (\tilde{F}, E)$$

for $\tau^{NSS} = \{(\tilde{F}_i, E) : i \in I\}$. Therefore $\tau_{(\tilde{F}, E)}^{NSS}$ is a neutrosophic soft topology over $(\tilde{F}, E)$. $\square$

Definition 19. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and $(\tilde{F}, E) \in NSS(X, E)$. Then the collection

$$\tau_{(\tilde{F}, E)}^{NSS} = \left\{ (\tilde{F}, E) \cap (\tilde{F}_i, E) : (\tilde{F}_i, E) \in \tau^{NSS} \text{ for } i \in I \right\}$$

is called a neutrosophic soft subspace topology on $(\tilde{F}, E)$ and $\left(X_{(\tilde{F}, E)}, \tau_{(\tilde{F}, E)}^{NSS}, E \right)$ is called a neutrosophic soft topological subspace of (X, τ^{NSS}, E) .

Example 3. Let us consider the neutrosophic soft topology τ^{NSS} and the neutrosophic soft set $(\tilde{F}, E)$ given in Example-1. Then the collection

$$\tau_{(\tilde{F}, E)}^{NSS} = \left\{ 0_{(\tilde{X}, E)}, (\tilde{F}, E), (\tilde{F}_1, E)', (\tilde{F}_2, E)', (\tilde{F}_3, E)', (\tilde{F}_4, E)' \right\}$$

is a neutrosophic soft sub-topology on $(\tilde{F}, E)$ of the neutrosophic soft topology τ^{NSS} . Here, the neutrosophic soft sets $(\tilde{F}_1, E)', (\tilde{F}_2, E)', (\tilde{F}_3, E)'$ and $(\tilde{F}_4, E)'$ over $(\tilde{F}, E)$ are defined as following:

$$\begin{aligned} (\tilde{F}_1, E)' &= (\tilde{F}, E) \cap (\tilde{F}_1, E) = \left\{ e_1, \{ \langle x_1, 0.6, 0.4, 0.7 \rangle, \langle x_2, 0.3, 0.5, 0.2 \rangle, \langle x_3, 0.4, 0.6, 0.9 \rangle \} \right. \\ &\quad \left. e_2, \{ \langle x_1, 0.3, 0.7, 0.4 \rangle, \langle x_2, 0.1, 0.2, 0.5 \rangle, \langle x_3, 0.7, 0.8, 0.9 \rangle \} \right\}, \\ (\tilde{F}_2, E)' &= (\tilde{F}, E) \cap (\tilde{F}_2, E) = \left\{ e_1, \{ \langle x_1, 0.2, 0.6, 0.6 \rangle, \langle x_2, 0.3, 0.7, 0.9 \rangle, \langle x_3, 0.5, 0.8, 0.6 \rangle \} \right. \\ &\quad \left. e_2, \{ \langle x_1, 0.1, 0.9, 0.5 \rangle, \langle x_2, 0.3, 0.4, 0.8 \rangle, \langle x_3, 0.4, 0.3, 0.7 \rangle \} \right\}, \\ (\tilde{F}_3, E)' &= (\tilde{F}, E) \cap (\tilde{F}_3, E) = \left\{ e_1, \{ \langle x_1, 0.2, 0.4, 0.7 \rangle, \langle x_2, 0.3, 0.5, 0.9 \rangle, \langle x_3, 0.4, 0.6, 0.9 \rangle \} \right. \\ &\quad \left. e_2, \{ \langle x_1, 0.1, 0.7, 0.5 \rangle, \langle x_2, 0.1, 0.2, 0.8 \rangle, \langle x_3, 0.4, 0.3, 0.9 \rangle \} \right\}, \\ (\tilde{F}_4, E)' &= (\tilde{F}, E) \cap (\tilde{F}_4, E) = \left\{ e_1, \{ \langle x_1, 0.6, 0.7, 0.6 \rangle, \langle x_2, 0.2, 0.5, 0.3 \rangle, \langle x_3, 0.5, 0.8, 0.4 \rangle \} \right. \\ &\quad \left. e_2, \{ \langle x_1, 0.3, 0.3, 0.5 \rangle, \langle x_2, 0.4, 0.6, 0.5 \rangle, \langle x_3, 0.7, 0.2, 0.7 \rangle \} \right\}. \end{aligned}$$

In addition, $\left((\tilde{F}, E), \overset{NSS}{\tau}_{(\tilde{F}, E)}, E\right)$ is a neutrosophic soft topological subspace of $(X, \overset{NSS}{\tau}, E)$.

Theorem 7. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $(\tilde{F}, E), (\tilde{K}, E) \in NSS(X, E)$. Then,

1. If $\overset{NSS}{B}$ is a neutrosophic soft base for $\overset{NSS}{\tau}$, then $\overset{NSS}{B}_{(\tilde{F}, E)} = \left\{(\tilde{B}, E) \cap (\tilde{F}, E) : (\tilde{B}, E) \in \overset{NSS}{B}\right\}$ is a neutrosophic soft base for the neutrosophic soft sub-topology $\overset{NSS}{\tau}_{(\tilde{F}, E)}$,
2. If $(\tilde{G}, E)$ is a neutrosophic soft closed set in $\overset{NSS}{\tau}_{(\tilde{F}, E)}$ and $(\tilde{F}, E)$ is a neutrosophic soft closed set in $\overset{NSS}{\tau}_{(K, E)}$, then $(\tilde{G}, E)$ is a neutrosophic soft closed set in $\overset{NSS}{\tau}_{(K, E)}$,
3. Let $(\tilde{G}, E) \subseteq (\tilde{F}, E)$. If $\overline{(\tilde{G}, E)}$ is the neutrosophic soft closure in $(X, \overset{NSS}{\tau}, E)$, then $\overline{(\tilde{G}, E)} \cap (\tilde{F}, E)$ is the neutrosophic soft closure in $\left(X_{(\tilde{F}, E)}, \overset{NSS}{\tau}_{(\tilde{F}, E)}, E\right)$.

Proof. 1. Since $\overset{NSS}{B}$ is a neutrosophic soft base for $\overset{NSS}{\tau}$ so for arbitrary $(\tilde{U}, E) \in \overset{NSS}{\tau}$, we have $(\tilde{U}, E) = \bigcup_{(\tilde{B}, E) \in \overset{NSS}{B}} (\tilde{B}, E)$. In case,

$$(\tilde{U}, E) \cap (\tilde{F}, E) = \left(\bigcup_{(\tilde{B}, E) \in \overset{NSS}{B}} (\tilde{B}, E) \right) \cap (\tilde{F}, E) = \bigcup_{(\tilde{B}, E) \in \overset{NSS}{B}} \left((\tilde{B}, E) \cap (\tilde{F}, E) \right)$$

for $(\tilde{U}, E) \cap (\tilde{F}, E) \in \overset{NSS}{\tau}_{(\tilde{F}, E)}$. Since arbitrary member of $\overset{NSS}{\tau}_{(\tilde{F}, E)}$ can be expressed as the union of members of $\overset{NSS}{B}_{(\tilde{F}, E)}$, hence the theorem is completed.

2. We first show that if $(\tilde{G}, E)$ is a neutrosophic soft closed set in $\overset{NSS}{\tau}_{(\tilde{F}, E)}$ then there exist a closed set $(\tilde{V}, E) \subseteq (\tilde{K}, E)$ i.e., $(\tilde{V}, E) \notin \overset{NSS}{\tau}$ such that $(\tilde{G}, E) = (\tilde{V}, E) \cap (\tilde{F}, E)$.

Let $(\tilde{G}, E)$ be a closed in $\overset{NSS}{\tau}_{(\tilde{F}, E)}$. Then $(\tilde{G}_i, E)^c$ is a neutrosophic soft open set in $\overset{NSS}{\tau}_{(\tilde{F}, E)}$ i.e., $(\tilde{G}_i, E)^c$ can be put as $(\tilde{G}, E)^c = (\tilde{U}, E) \cap (\tilde{F}, E)$ for $(\tilde{U}, E) \in \overset{NSS}{\tau} \Rightarrow \left((\tilde{G}, E)^c\right)^c = (\tilde{F}, E) \cap \left((\tilde{U}, E) \cap (\tilde{F}, E)\right)^c = (\tilde{U}, E)^c \cap (\tilde{F}, E)$. Here $(\tilde{U}, E)^c \notin \overset{NSS}{\tau}$ i.e., $(\tilde{U}, E)^c$ is a closed in $\overset{NSS}{\tau}$. So here acts as $(\tilde{V}, E) \subseteq (\tilde{K}, E)$.

Conversely, suppose that $(\tilde{G}, E) = (\tilde{V}, E) \cap (\tilde{F}, E)$ where $(\tilde{F}, E) \subseteq (\tilde{K}, E)$ and $(\tilde{V}, E)$ is closed in $\overset{NSS}{\tau}_{(K, E)}$. Clearly, $(\tilde{V}, E)^c \in \overset{NSS}{\tau}$ so that $(\tilde{V}, E)^c \cap (\tilde{F}, E) \in \overset{NSS}{\tau}_{(\tilde{F}, E)}$. Now,

$(\tilde{V}, E)^c \cap (\tilde{F}, E) = \left((\tilde{K}, E) \setminus (\tilde{V}, E)\right) \cap (\tilde{F}, E) = \left((\tilde{K}, E) \cap (\tilde{F}, E)\right) \setminus \left((\tilde{V}, E) \cap (\tilde{F}, E)\right) = (\tilde{F}, E) \setminus (\tilde{G}, E)$. This implies $(\tilde{F}, E) \setminus (\tilde{G}, E)$ is a neutrosophic soft open set in $(\tilde{F}, E)$ i.e., $(\tilde{G}, E)$ is a neutrosophic soft closed set in $\overset{NSS}{\tau}_{(\tilde{F}, E)}$.

3. $\overline{(\tilde{G}, E)} = \bigcap \left\{(\tilde{G}_i, E) : (\tilde{G}_i, E) \text{ is closed and } (\tilde{G}_i, E) \supseteq (\tilde{G}, E)\right\}$ is the neutrosophic soft closure of $(\tilde{G}, E)$ and so $\overline{(\tilde{G}, E)}$ is a neutrosophic soft closed set. Now, $\overline{(\tilde{G}, E)} \cap (\tilde{F}, E) = \bigcap \left\{(\tilde{G}_i, E) : (\tilde{G}_i, E) \text{ is closed and } (\tilde{G}_i, E) \supseteq (\tilde{G}, E)\right\} \cap (\tilde{F}, E) = \bigcap \left((\tilde{G}_i, E) \cap (\tilde{F}, E)\right)$. Since each $(\tilde{G}_i, E)$

is closed, then each $(\tilde{G}_i, E) \cap (\tilde{F}, E)$ is closed in $\tau_{(\tilde{F}, E)}^{NSS}$ by Theorem-5. Now $(G, E) \subseteq (\tilde{G}_i, E)$ and $(G, E) \subseteq (\tilde{F}, E)$. So $((\tilde{G}, E) \cap (\tilde{F}, E)) \subseteq ((\tilde{G}_i, E) \cap (\tilde{F}, E)) \Rightarrow (\tilde{G}, E) \subseteq (\tilde{G}_i, E) \cap (\tilde{F}, E)$. Therefore,

$$\overline{(\tilde{G}, E) \cap (\tilde{F}, E)} = \bigcap \left\{ ((\tilde{G}_i, E) \cap (\tilde{F}, E)) : ((\tilde{G}_i, E) \cap (\tilde{F}, E)) \text{ is closed and } ((\tilde{G}_i, E) \cap (\tilde{F}, E)) \supseteq (\tilde{G}, E) \right\}$$

. Thus, $\overline{(\tilde{G}, E) \cap (\tilde{F}, E)}$ is a neutrosophic soft closure of $(\tilde{G}, E)$ in $\tau_{(\tilde{F}, E)}^{NSS}$. $\square$

Theorem 8. Let $(X_{(\tilde{F}, E)}, \tau_{(\tilde{F}, E)}^{NSS}, E)$ be a neutrosophic soft subspace of a neutrosophic soft topological space (X, τ^{NSS}, E) over X . If $(\tilde{F}, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) , then a neutrosophic soft set $(\tilde{F}_1, E) \subseteq (\tilde{F}, E)$ is neutrosophic soft open set in $(X_{(\tilde{F}, E)}, \tau_{(\tilde{F}, E)}^{NSS}, E)$ iff $(\tilde{F}_1, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) .

Proof. Suppose that $(\tilde{F}, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) such that a neutrosophic soft subset $(\tilde{F}_1, E)$ of $(\tilde{F}, E)$ is open set in $(X_{(\tilde{F}, E)}, \tau_{(\tilde{F}, E)}^{NSS}, E)$. Then $(\tilde{F}_1, E) \in \tau_{(\tilde{F}, E)}^{NSS}$ and so $(\tilde{F}_1, E) = (\tilde{U}, E) \cap (\tilde{F}, E)$ for $(\tilde{U}, E) \in \tau^{NSS}$. But $(\tilde{F}_1, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) as $(\tilde{U}, E)$ and $(\tilde{F}, E)$ both are neutrosophic soft open set in (X, τ^{NSS}, E) .

Conversely, assume that $(\tilde{F}_1, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) when $(\tilde{F}, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) and $(\tilde{F}_1, E) \subseteq (\tilde{F}, E)$. Then $(\tilde{F}_1, E) \in \tau^{NSS}$. But $(\tilde{F}_1, E) \cap (\tilde{F}, E) = (\tilde{F}_1, E)$ and so $(\tilde{F}_1, E)$ is a neutrosophic soft open set in (X, τ^{NSS}, E) . Therefore, the first part is proved. $\square$

Theorem 9. Let $(X_{(\tilde{K}, E)}, \tau_{(\tilde{K}, E)}^{NSS}, E)$ be a neutrosophic soft subspace of a neutrosophic soft topological space (X, τ^{NSS}, E) over X . If $(\tilde{K}, E)$ is a neutrosophic soft closed set in (X, τ^{NSS}, E) , then a neutrosophic soft set $(\tilde{K}_1, E) \subseteq (\tilde{K}, E)$ is a neutrosophic soft closed set in $(X_{(\tilde{K}, E)}, \tau_{(\tilde{K}, E)}^{NSS}, E)$ iff $(\tilde{K}_1, E)$ is a neutrosophic soft closed set in (X, τ^{NSS}, E) .

Proof. Suppose that $(\tilde{K}, E)$ is a neutrosophic soft closed set in (X, τ^{NSS}, E) such that a neutrosophic soft subset $(\tilde{K}_1, E)$ of $(\tilde{K}, E)$ is neutrosophic soft closed set in $(X_{(\tilde{K}, E)}, \tau_{(\tilde{K}, E)}^{NSS}, E)$. Since $(\tilde{K}_1, E)$ is closed in $(X_{(\tilde{K}, E)}, \tau_{(\tilde{K}, E)}^{NSS}, E)$ and so $(\tilde{K}_1, E) = (\tilde{V}, E) \cap (\tilde{K}, E)$ for $(\tilde{V}, E)$ being neutrosophic soft closed set in (X, τ^{NSS}, E) . But $(\tilde{K}_1, E)$ is a neutrosophic soft closed set in (X, τ^{NSS}, E) as $(\tilde{V}, E)$ and $(\tilde{K}, E)$ both are neutrosophic soft closed sets in (X, τ^{NSS}, E) .

Conversely, assume that $(\tilde{K}_1, E)$ is a neutrosophic soft closed set in (X, τ^{NSS}, E) when $(\tilde{K}, E)$ is neutrosophic soft closed set in (X, τ^{NSS}, E) and $(\tilde{K}_1, E) \subseteq (\tilde{K}, E)$. Then $(\tilde{K}_1, E) \cap (\tilde{K}, E) = (\tilde{K}_1, E)$ and so $(\tilde{K}_1, E)$ is a neutrosophic soft closed in $(X_{(\tilde{K}, E)}, \tau_{(\tilde{K}, E)}^{NSS}, E)$. Hence the first part is proved. $\square$

4 Conclusion

In this study, we investigate some notions of neutrosophic soft topological space such as; boundary of neutrosophic soft set, neutrosophic soft dense set, neutrosophic soft basis and neutrosophic soft subspace topology. Furthermore we give some important theorems and many interesting examples. We hope that results of this paper will contribute to the studies on neutrosophic soft topological spaces.

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