Applied Mathematics and Nonlinear Sciences

<https://www.sciendo.com>Exact solutions of $(2 + 1)$ -Ablowitz-Kaup-Newell-Segur equationHülya Durur^{1†}, Asif Yokuş²¹Department of Computer Engineering, Faculty of Engineering, Ardahan University, Ardahan, 75000, Turkey²Department of Actuary, Faculty of Science, Firat University, Elazığ, 23100, Turkey

Submission Info

Communicated by Hacı Mehmet Baskonus

Received April 8th 2020

Accepted June 20th 2020

Available online December 31st 2020

Abstract

The aim of the present study is to obtain different types of hyperbolic type solutions of the $(2+1)$ -Ablowitz-Kaup-Newell-Segur (AKNS) equation. In order to construction exact solutions of AKNS equation, $(1/G')$ -expansion method is successfully applied. At the end of this application, singular soliton wave with considerable importance for the shock wave structure and asymptotic behavior employees have emerged. By giving arbitrary values to the constants in the solutions obtained, 3D, 2D and contour graphics are presented. The method used in this article can be used in other nonlinear differential equations (NPDEs) as it is reliable, easy and effective. Ready package programs are used to solve complex and difficult processes in this study.

Keywords: $(1/G')$ -expansion method, hyperbolic traveling wave solution, singular soliton wave, analytical solution.**AMS 2010 codes:** 35-XX, 35Qxx.

1 Introduction

Different studies can be seen on NPDEs in various disciplines such as physical events, applied sciences, natural events and engineering. Scientists have used many methods to find analytical solutions of NPDEs. Some of these methods are new sub equation method [1, 2], $(1/G')$ -expansion method [3, 4], Homotopy analysis and Homotopy-Pade methods [5], (G'/G) -expansion method [6, 7], Variational Iteration Algorithm-I [8], decomposition method [9–11], sumudu transform method [12], sub equation method [13, 14], collocation method [15], the auto-Bäcklund transformation method [16], the Clarkson-Kruskal (CK) direct method [17], first integral method [18], homogeneous balance method [19], SGEEM [20], residual power series method [21], Modified Kudryashov method [22], sine-Gordon expansion method (SGEM) [23, 24] and so on [25–46].

Many analytical methods have been added to the literature to find the exact solution of the NPDEs. We know that each method produces different types of solutions. For example, sub equation method produces dark

[†]Corresponding author.Email address: hulyadurur@ardahan.edu.tr

solitons, Hirota bilinear form lump solitons. In this study, we aim to produce hyperbolic type wave solutions using the $(1/G')$ -expansion method. For this, we have dealt with the AKNS equation with strong nonlinearity. So let's take the AKNS equation [47],

$$4u_{xt} + u_{xxxz} + 8u_{xz}u_x + 4u_zu_{xx} = 0. \quad (1)$$

Scientists related to the equation of AKNS equation have produced several studies in the literature. Some of these are obtained traveling wave reductions of AKNS equation applying classical Lie symmetries [48], new soliton like solutions of AKNS equation obtained using the extended auxiliary equation method [49], the truncated Painlevé analysis of AKNS equation obtained with their symmetries [50], complex combined dark-bright soliton solutions of AKNS equation attained using SGEM [51], general solutions of the AKNS system have been obtained via standard truncated Painlevé expansions [52].

2 Description of the Method

Consider form of NPDEs,

$$\dagger \left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}, \frac{\partial^2 u}{\partial x^2}, \dots \right) = 0. \quad (2)$$

Let $\xi = x + z - ct$, $u(x, z, t) = u(\xi) = u$, where c is a constant and the speed of the wave. We can convert it into the following nODE

$$\theta(u, -cu', u'', u''', \dots) = 0. \quad (3)$$

The solution that makes the Eq. (3) correct is assumed to have the form

$$u(\xi) = a_0 + \sum_{i=1}^m a_i \left(\frac{1}{G'} \right)^i, \quad (4)$$

where a_i , ($i = 0, 1, \dots, m$) are constants, $G = G(\xi)$ provides the following second order IODE

$$G'' + \lambda G' + \mu = 0, \quad (5)$$

where λ and μ are constants to be determined after,

$$\frac{1}{G'(\xi)} = \frac{1}{-\frac{\mu}{\lambda} + B \cosh(\xi \lambda) - B \sinh(\xi \lambda)}, \quad (6)$$

where B is integral constant. If the desired derivatives of the Eq. (4) are calculated and substituting in the Eq. (3), a polynomial with the argument $(1/G')$ is attained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. The equation are solved using package program and put into place in the default Eq. (3) solution function. Lastly, the solutions of Eq. (2) are found.

3 Solutions of The AKNS equation

Let's consider the Eq. (1). By applying the transmutation $\xi = x + z - ct$, allows us to convert Eq. (1) into an nonlinear ODE

$$-4cu' + u''' + 6(u')^2 = 0. \quad (7)$$

In Eq. (7), $m = 1$ is found from the definition of balancing term and written according to Eq. (4)

$$u(\xi) = a_0 + a_1 \left(\frac{1}{G'} \right), \quad a_1 \neq 0. \quad (8)$$

If Eq. (8) is replacing in Eq. (7) and the coefficients are equal to zero, the following system is attained.

$$\begin{aligned} \left(\frac{1}{G'[\xi]}\right)^1 &: -4c\lambda a_1 + \lambda^3 a_1 = 0, \\ \left(\frac{1}{G'[\xi]}\right)^2 &: -4c\mu a_1 + 7\lambda^2\mu a_1 + 6\lambda^2 a_1^2 = 0, \\ \left(\frac{1}{G'[\xi]}\right)^3 &: 12\lambda\mu^2 a_1 + 12\lambda\mu a_1^2 = 0, \\ \left(\frac{1}{G'[\xi]}\right)^4 &: 6\mu^3 a_1 + 6\mu^2 a_1^2 = 0. \end{aligned} \quad (9)$$

When we solve Eq. (9) system with the help of computer ready package program, we can present the following situations.

Case1.

$$a_1 = -\mu, \quad c = \frac{\lambda^2}{4}, \quad (10)$$

if the values in Eq. (10) are written in place of Eq. (8), the hyperbolic type solution below are obtain

$$u_1(x, z, t) = -\frac{\mu}{-\frac{\mu}{\lambda} + B\left(\cosh\left[\lambda\left(x+z-\frac{t\lambda^2}{4}\right)\right] - \sinh\left[\lambda\left(x+z-\frac{t\lambda^2}{4}\right)\right]\right)} + a_0. \quad (11)$$

Fig. 1 3D, 2D and contour graphs respectively for $B = 0.9$, $\mu = 0.1$, $\lambda = 0.2$, $a_0 = 5$, $z = 1$ values of Eq. (11).

As seen in figure 1, we can observe asymptotic behavior in the $0 < x < 5$ range. Different interpretations and studies can be made for this situation.

Case2.

$$a_1 = -\mu, \quad \lambda = 2\sqrt{c}, \quad (12)$$

if the values in Eq. (12) are written in place of Eq. (8), the hyperbolic type solution below are obtain

$$u_2(x, z, t) = -\frac{\mu}{-\frac{\mu}{2\sqrt{c}} + B\left(\cosh\left[(-ct + x + z)2\sqrt{c}\right] - \sinh\left[(-ct + x + z)2\sqrt{c}\right]\right)} + a_0. \quad (13)$$

Fig. 2 3D, 2D and contour graphs respectively for $B = 0.7$, $\mu = 2$, $z = 1$, $c = -1$, $a_0 = 3$ values of Eq. (13).

4 Conclusions and Discussion

In this study, analytical solutions of the AKNS equation were obtained via the $(1/G')$ -expansion method. Although the nonlinear terms of this equation are strong, traveling wave solutions are obtained. The solutions obtained are of the hyperbolic type. By giving special values to the parameters in these solutions, 3D, 2D and contour graphics are presented. We consider these wave solutions are especially important for scientists studying shock wave structure and asymptotic behavior. In addition, it will be much more valuable when the physical meaning is loaded on the constants in these solutions. Algebraic equation obtained by classical expansion methods is simpler. However, the disadvantage of the expansion method we use is that it only provides hyperbolic type solution. The advantage of this method is that it is different from the solutions obtained in other expansion methods. While obtaining solutions in many NPDEs, we encountered less complex operations while solving this equation. However, this study also offers solutions with two cases. It is seen that the method applied in the study is easy, reliable, useful and can be used in future studies. Ready-made package programs are used to facilitate the complexity of the procedure in this package.

References

- [1] Kurt, A., Tasbozan, O., & Durur, H., (2019), *Fundamental Journal of Mathematics and Applications*, 2(2), 173-179.
- [2] Tasbozan, O., Kurt, A., & Durur, H., (2019), *International Journal of Engineering Mathematics and Physics*.
- [3] Yokuş, A., & Durur, H., (2019), *Journal of Balikesir University Institute of Science and Technology*, 21(2), 590-599.
- [4] Durur, H., & Yokuş, A., (2019), *Afyon Kocatepe Universitesi Fen ve Muhendislik Bilimleri Dergisi*, 19(3), 615-619.
- [5] Kheiri, H., Alipour, N., & Dehghani, R., (2011), *Mathematical Sciences*, 5(1), 33-50.
- [6] Durur, H., (2020), *Modern Physics Letters B*, 34(03), 2050036.
- [7] Yokuş, A., & Kaya, D., (2015), *Istanbul Ticaret Universitesi Fen Bilimleri Dergisi*, 14(28).
- [8] Ahmad, H., Rafiq, M., Cesarano, C., & Durur, H., (2020), *Earthline Journal of Mathematical Sciences*, 3(2), 229-247.
- [9] Kaya, D., & Yokuş, A., (2002), *Mathematics and Computers in Simulation*, 60(6), 507-512.
- [10] Kaya, D., & Yokuş, A., (2005), *Applied Mathematics and Computation*, 164(3), 857-864.
- [11] Yavuz, M., & Ozdemir, N., (2018), *Konuralp Journal of Mathematics*, 6(1), 102-109.
- [12] Yavuz, M., & Ozdemir, N., (2018), *Mathematical Studies and Applications* 2018 4-6 October 2018, 442.
- [13] Durur, H., Tasbozan, O., Kurt, A., & Şenol, M., (2019), *Erzincan University Journal of the Institute of Science and Technology*, 12(2), 807-815.
- [14] Durur, H., Kurt, A., & Tasbozan, O., (2020), *Applied Mathematics and Nonlinear Sciences*, 5(1), 455-460.
- [15] Aziz, I., & Sarler, B., (2010), *Mathematical and Computer Modelling*, 52(9-10), 1577-1590.
- [16] Kaya, D., Yokuş, A., & Demiroglu, U., (2020), *In Numerical Solutions of Realistic Nonlinear Phenomena* (pp. 53-65). Springer, Cham.

- [17] Su-Ping, Q., & Li-Xin, T., (2007), Chinese Physics Letters, 24(10), 2720.
- [18] Darvishi, M., Arbabi, S., Najafi, M., & Wazwaz, A., (2016), Optik, 127(16), 6312-6321.
- [19] Rady, A. A., Osman, E. S., & Khalfallah, M., (2010), Applied Mathematics and Computation, 217(4), 1385-1390.
- [20] Sulaiman, T. A., & Bulut, H., (2019), Applied Mathematics and Nonlinear Sciences, 4(2), 513-522.
- [21] Durur, H., Şenol, M., Kurt, A., & Taşbozan, O., (2019), Erzincan University Journal of the Institute of Science and Technology, 12(2), 796-806.
- [22] Kumar, D., Seadawy, A. R., & Joardar, A. K., (2018), Chinese journal of physics, 56(1), 75-85.
- [23] Eskitaşcioglu, E. I., Aktaş, M. B., & Baskonus, H. M., (2019), Applied Mathematics and Nonlinear Sciences, 4(1), 105-112.
- [24] Baskonus, H. M., Bulut, H., & Sulaiman, T. A., (2019), Applied Mathematics and Nonlinear Sciences, 4(1), 129-138.
- [25] Dusunceli, F., (2019), Applied Mathematics and Nonlinear Sciences, 4(2), 365-370.
- [26] Yokus, A., (2020), Boletim da Sociedade Paranaense de Matematica, (in Press).
- [27] Yokus, A., & Yavuz, M., (2018), Discrete & Continuous Dynamical Systems-S, 0.
- [28] Yokuş, A., & Gulbahar, S., (2019), Applied Mathematics and Nonlinear Sciences, 4(1), 35-42.
- [29] Rezazadeh, H., Kumar, D., Neirameh, A., Eslami, M., & Mirzazadeh, M., (2020), Pramana, 94(1), 39.
- [30] Gao, W., Silambarasan, R., Baskonus, H. M., Anand, R. V., & Rezazadeh, H., (2020), Physica A: Statistical Mechanics and its Applications, 545, 123772.
- [31] Durur, H., Tasbozan, O., & Kurt, A., (2020), Applied Mathematics and Nonlinear Sciences, 5(1), 447-454.
- [32] Rezazadeh, H., Osman, M. S., Eslami, M., Mirzazadeh, M., Zhou, Q., Badri, S. A., & Korkmaz, A., (2019), Nonlinear Engineering, 8(1), 224-230.
- [33] Osman, M. S., Rezazadeh, H., & Eslami, M., (2019), Nonlinear Engineering, 8(1), 559-567.
- [34] Osman, M. S., (2019), Nonlinear Dynamics, 96(2), 1491-1496.
- [35] Osman, M. S., & Wazwaz, A. M., (2019), Mathematical Methods in the Applied Sciences, 42(18), 6277-6283.
- [36] Javid, A., Raza, N., & Osman, M. S., (2019), Communications in Theoretical Physics, 71(4), 362.
- [37] Goyal, M., Baskonus, H. M., & Prakash, A., (2019), The European Physical Journal Plus, 134(10), 482.
- [38] Prakash, A., & Verma, V., (2019), Pramana, 93(4), 66.
- [39] Prakash, A., Goyal, M., Baskonus, H. M., & Gupta, S., (2020), AIMS Mathematics, 5(2), 979.
- [40] Kumar, D., Singh, J., Prakash, A., & Swroop, R., (2019), Progr Fract Differ Appl, 5(1), 65-77.
- [41] Prakash, A., & Kumar, M., (2019), India Section A: Physical Sciences, 89(3), 559-570.
- [42] Gao, W., Ismael, H. F., Husien, A. M., Bulut, H., & Baskonus, H. M., (2020), Applied Sciences, 10(1), 219.
- [43] Garcia Guirao, J. L., Baskonus, H. M., Kumar, A., Rawat, M. S., & Yel, G., (2020), Symmetry, 12(1), 17.
- [44] Cattani, C., & Rushchitskii, Y. Y., (2003), International applied mechanics, 39(10), 1115-1145.
- [45] Cattani, C., (2003), International Journal of Fluid Mechanics Research, 30(5).
- [46] Cattani, C., Chen, S., & Aldashev, G., (2012), Mathematical Problems in Engineering, 2012.
- [47] Attia, R. A., Lu, D., & Khater, M. M., (2018), Phys. J, 1(3), 234-254.
- [48] Bruzon, M. S., Gandarias, M. L., Muriel, C., Ramirez, J., & Romero, F. R., (2003), Theoretical and mathematical physics, 137(1), 1378-1389.
- [49] Helal, M. A., Seadawy, A. R., & Zekry, M. H., (2013), Applied Mathematical Sciences, 7(65-68), 3355-3365.
- [50] Ping, L., Bao-Qing, Z., Jian-Rong, Y., & Bo, R., (2015), Chinese Physics B, 24(1), 010202.
- [51] Gao, W., Yel, G., Baskonus, H. M., & Cattani, C., (2019, June), In Book of Abstracts (p. 86).
- [52] Chun-Long, Z., & Jie-Fang, Z., (2002), Chinese Physics Letters, 19(10), 1399.

This page is intentionally left blank