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Attitude control for the rigid spacecraft with the improved extended state observer

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Abstract

In this article, a three-axis attitude manoeuvre spacecraft consisting of a central rigid body and a rotating solar array is studied. The rotating solar array is considered a disturbance to the spacecraft. In the design of the controller, the coupled terms and the rotating solar array are considered a disturbance. The improved extended state observer is proposed by combining the sliding mode observer with the originally extended state observer to estimate the disturbance. The sliding mode control method is adopted to adjust the attitude of the spacecraft. Numerical simulations are presented to demonstrate the outstanding performance of the present observer.

Keywords: rigid spacecraft, rotating solar array, disturbance, improved extended state observer, sliding mode control

AMS 2010 codes: ???

1 Introduction

Solar arrays on the spacecraft track and face the Sun to provide the power to the payload. The rotation of the solar array has effects on the attitude of the spacecraft, which causes a series of problems about multi-body dynamics and the accuracy of the attitude control.

There exists the relative movement between the central platform and the solar array due to the rotation of the solar array. The multi-body dynamics of such structure have attracted extensive attention of scholars. Gasbarri et al. [1] devoted themselves to study the structural uncertainty of a very large space structure. The model

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was established by the finite element method (FEM), and a nonlinear control scheme was adopted to solve the robustness of the system. In their another paper, the model was reconstructing and a new control concept called ‘control-oriented modelization’ was proposed to solve the problem of the model’s uncertainties and disturbances [2]. Sattar and Wei [3] established an accurate mathematical model for kinematic and coupled micro vibrations of a rigid aluminium plate driven with external excitation torque. Iwata [4] proposed a composite control scheme for compensating and rejecting the disturbance of the Advanced Land Observing Satellite (ALOS). Based on the adaptive-robust control law, Azadi et al. [5] used piezoelectric layers as actuators to suppress the residual vibration of the flexible appendages during the spacecraft attitude manoeuvring in space. Lu and Liu [6] designed the H_∞ optimal control to improve the robustness of the spacecraft with the rotating solar array, where the singularity of the system was also considered. Besides the above, the sliding mode control (SMC) method is widely used as a robust approach to solve the disturbance and uncertainty. Hu et al. [7] combined the variable structure output feedback and input shaping and considered adaptive law to design control law. The control scheme not only guaranteed the stability of the closed-loop system but also overcame the parameter uncertainty and external disturbances. Hu [8] studied the SMC scheme for the three-axis stabilised flexible spacecraft. Cao et al. [9] combined SMC with input shaping techniques to reduce the vibration of the driving solar arrays. The controller accomplished asymptotic attitude manoeuvring in the closed-loop system and performed good robustness. For the rigid spacecraft with the disturbance, the SMC with the adaptive algorithm was adopted to track the attitude and compensate the disturbance [10].

In this article, the flexible vibration of the appendage is not considered. Instead, the multi-rigid-body motion of the spacecraft with disturbance is studied. To estimate and compensate the uncertainty and disturbance, more and more researchers pay attention to the new control algorithm that involves the application of the state observer. The disturbance observer-based control (DOBC) is a promising approach, which can be used to eliminate the disturbance and improve the robustness of the system. For the spacecraft with an attitude adjustment and Sun-tracking missions, a feed-forward control based on a disturbance observer was designed [11]. Chen and Kun-Yung [12] proposed the adaptive sliding mode tracking scheme to control the two-link manipulator robot system, where the disturbance observer is used to stabilise and compensate the unknown time-varying disturbances. Lee [13] proposed a composite control technique with a nonlinear disturbance observer for the attitude tracking of the rigid spacecraft, where the uncertain inertia parameters were compensated through feedforward and the robustness performance of the spacecraft was enhanced. In addition to the above, there are two other state observers widely used, namely, sliding mode observer (SMO) [14–17] and extended state observer (ESO) [18–21].

The SMOs are widely used for finite-time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation [15]. The nonlinear sliding mode observers are adopted on the robot manipulators estimation [14]. An important algorithm for control and observation using SMO is the so-called super twisting algorithm described by the differential inclusion [16]. The second-order SMC was used to track the constant reference and had been validated on the unstable non-minimum phase inverted pendulum system [22]. Moreno and Osorio [17] gave the rigorous and detailed proof using the Lyapunov approach for the convergence of the SMO. The synchronous motor was controlled by the adaptive sliding mode method, where the estimation of the rotor speed was obtained by the SMO [23].

The ESO was first proposed together with the concept of the active disturbance rejection controller by Professor J.Q. Han. It was an effective way to estimate the so-called uncertainty from the unknown model and perturbation [21]. The key part of the ESO is that the uncertainty and disturbance of the system are considered as a new order state. Then, the estimations of all states including the added new one can be obtained by a specific nonlinear function, where the gains of the function need to be selected appropriately. The bandwidth and stability of the linear ESO (LESO) are determined by the gains, which can be ensured by allocating the poles rationally [24, 25]. The impact of the observed bandwidth changes on the performance of the LESO was discussed in detail by Godbole et al. [26]. However, the LESO was out of its depth in dealing with the nonlinear system. Then, the ESO with the nonlinear function must be adopted. The stability and convergence of nonlinear

ESO were given by using the piece-wise smooth Lyapunov function [18]. Huang and Han [19] and Huang [20] proposed the self-stable-region (SSR) theory. This theory could improve the ESO convergence performance and observation accuracy. Wang et al. [27] added an output signal filter to the ESO to eliminate measure noise. Then, the ADRC and feedback compensation were presented together in dealing with the uncertainty of the lateral thrust and aerodynamics blended control system [28]. Li et al. [29] adopted the ESO to estimate all states of the attitude stabilisation system for the spacecraft. Besides, the uniformly ultimately bounded stability of the ESO was assured by rigorous Lyapunov analysis. For the attitude tracking of the rigid spacecraft, the SMC scheme was designed to force the state variables to converge to the reference states [30]. In this case, the ESO was indispensable to estimate the disturbance. Zhu et al. [31] used the ESO to estimate the target acceleration and achieve the missile interception. The chattering was proved to be decreased by combing the SMC with the ESO. The third-order discrete ESO was employed on the antenna pointing system [32]. It made each channel decouple and linearise successfully.

In this article, a three-axis attitude motion of the spacecraft with a single rotating solar array is studied. The SMC method is designed to achieve the attitude adjustment. The improved ESO (IESO) is proposed to estimate the disturbance and uncertainty, where the sliding mode observer is adopted to replace the nonlinear function fal in the traditional ESO. This article is organised as follows: the dynamic model of the rigid spacecraft is established in Section 2, where the Euler attitude angle is defined and the kinematic equations are obtained. Then the double closed-loop control scheme is proposed in Section 3. The design of the IESO and the regulator of the angular position are presented in detail. Simulation results analysis of the proposed control scheme is given in Section 4. Some major conclusions about the present study are summarised in Section 5.

2 Dynamical model of the spacecraft with the rigid solar array

As shown in Figure 1, the satellite consists of a central rigid platform and one solar array. The attitude manoeuvring of the spacecraft is in a small range and the solar array is driven in a fairly low velocity, which means these two motions do not excite the elastic deformation of the solar array. Thus, the whole spacecraft is considered as rigid. The reference coordinate system $O - X_0Y_0Z_0$ is set on the core of the earth. The body coordinate system $C - X_1Y_1Z_1$ is fixed on the satellite. The origin C is the centre of the rigid body, which is also the centre of mass in the satellite. The floating coordinate system $A - X_2Y_2Z_2$ is embedded on the solar array that the origin A is set on the link point between the solar array and centre body.

Fig. 1 The model of the spacecraft with rotating solar array.

Let $\omega_C = [\omega_x, \omega_y, \omega_z]^T$ denotes the vector of the angular velocities of the spacecraft and $\omega_A = [0, \omega_A, 0]^T$ denotes the vector of the angular velocities of the solar array. The dynamic equations of the spacecraft with rotating solar array are expressed as follows [6, 10, 13, 30]:

$$\begin{cases} \mathbf{J}_t \dot{\omega}_C + \mathbf{H} \dot{\omega}_A + \tilde{\omega}_C (\mathbf{J}_t \omega_C + \mathbf{H} \omega_A) = \tau_C + \tau_d, \\ \mathbf{J}_b \dot{\omega}_A + \mathbf{H}^T \dot{\omega}_C = \tau_A, \end{cases} \quad (1)$$

where $\mathbf{J}_t = \mathbf{J}_C + \mathbf{C}_{CA}\mathbf{J}_b\mathbf{C}_{AC}$ is the rotary inertia of the whole spacecraft with respect to the body-fixed coordinate system, and is the rotary inertia of the solar arrays with respect to the rotation coordinate system. $\tilde{\omega}_C$ is the antisymmetric matrix of ω_C , τ_A and τ_d are the torques of the attitude control, solar array driving and external disturbance, respectively. The structural coupled matrix $\mathbf{H}$ is expressed as follows:

$$\mathbf{H} = \mathbf{C}_{CA}\mathbf{J}_b, \quad (2)$$

where $\mathbf{C}_{CA}$ is the coordinate transformation matrix from the rotation coordinate frame to the body-fixed coordinate frame. It is expressed as follows:

$$\mathbf{C}_{CA} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}, \quad (3)$$

Euler angle is used to denote the attitude of the satellite usually. Three coordinate rotations in sequence can describe any rotation. The satellite rotates in a sequence of Z-Y-X in the body-fixed coordinate frame. The x -, y - and z -axes are defined as rolling, yawing and pitching axes, respectively. Correspondingly, the three attitude angles are defined as φ, θ and ψ . The angular velocity can be obtained as follows:

$$\omega = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \varphi & \cos \theta \sin \varphi \\ 0 & -\sin \varphi & \cos \theta \cos \varphi \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} \cos \theta \sin \psi \\ \cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi \\ -\sin \varphi \sin \psi + \cos \varphi \sin \theta \sin \psi \end{bmatrix} \omega_0, \quad (4)$$

where ω_0 is the orbital angular velocity. In this article, the spacecraft attitude manoeuvre is in a small range, the variations of the attitude angles are small enough that the linearisation form is as follows:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} \psi \\ 1 \\ -\varphi \end{bmatrix} \omega_0. \quad (5)$$

Thus for simplicity, Eq. (5) can be rewritten in the short form:

$$\dot{\Theta} = -\delta + \omega, \quad (6)$$

where $\Theta = [\varphi, \theta, \psi]^T$, $\delta = [-\omega_0 \psi, -\omega_0, \omega_0 \varphi]^T$ and $\omega = [\omega_x, \omega_y, \omega_z]^T$.

3 Attitude manoeuvre controller for the spacecraft

The control system for the spacecraft with the rotating solar array consists of double closed loops. As shown in Figure 2, the outer loop is for the attitude motion system with an angular position regulator while the inner loop is the attitude dynamic system. In Figure 2, the attitude dynamic system employs an ESO to estimate the state variable. The sliding mode controller is adopted to adjust the attitude. The attitude angular ω is not only the output of the inner loop but also the input of the outer loop. The angular error regulation output ω_d of outer loop is the reference input of the inner loop and ω is only the nominal control input.

If we set $\mathbf{q} = [\varphi, \theta, \psi, \theta_A, \omega_A]^T$, $\omega_A = \mathbf{D}\omega$, where $\mathbf{D} = [0, 1, 0]^T$, the state space form of Eq. (1) can be written as

$$\mathbf{M}_\omega \dot{\mathbf{q}} = \mathbf{A}_\omega \mathbf{q} + \mathbf{B}_u \mathbf{u} + \mathbf{B}_p \tau_d, \quad (7)$$

with $\mathbf{M}_\omega = \begin{bmatrix} \mathbf{J}_t & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}_{5 \times 5}$, $\mathbf{A}_\omega = \begin{bmatrix} -\tilde{\omega}_C \mathbf{J}_C & -\tilde{\omega}_C \mathbf{H} \\ \mathbf{0}_{2 \times 3} & \mathbf{A}_\alpha \end{bmatrix}_{5 \times 5}$, $\mathbf{B}_u = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{2 \times 3} & \mathbf{B}_\tau \end{bmatrix}_{5 \times 4}$, $\mathbf{B}_p = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{5 \times 4}$ and $\mathbf{u} = [\tau_1, \tau_2, \tau_3, \tau_A]^T$.

Fig. 2 The block diagram of the attitude control scheme.

The symbols in the above matrices are expressed as $\mathbf{A}_\alpha = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B}_\tau = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{M}_{22} = \begin{bmatrix} 1 & 0 \\ 0 & \mathbf{D}^T \mathbf{J}_b \mathbf{D} \end{bmatrix}$, $\mathbf{M}_{12} = \mathbf{M}_{21}^T = [\mathbf{0}_{3 \times 1} \quad \mathbf{H}\mathbf{D}]_{3 \times 2}$, respectively.

Some terms in Eq. (7) are the products of two or more angle velocity vectors, so they can be ignored in the calculation as higher-order small quantities. Then the simplified equation is obtained as follows:

$$\dot{\mathbf{q}} = \mathbf{A}_q \mathbf{q} + \mathbf{B}_{ctrl} \mathbf{u} + \mathbf{B}_d \tau_d, \quad (8)$$

where $\mathbf{A}_q = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{A}_\alpha \end{bmatrix}_{5 \times 5}$, $\mathbf{B}_{ctrl} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}_{5 \times 4}$ and $\mathbf{B}_d = [\mathbf{B}_{d1}, \mathbf{B}_{d2}]_{5 \times 3}^T$

$$\mathbf{B}_{11} = \mathbf{J}_t^{-1} + \mathbf{J}_t^{-1} \mathbf{C}_{CA} \mathbf{J}_b \mathbf{D} (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D})^{-1} \mathbf{D}^T \mathbf{J}_b \mathbf{C}_{AC} \mathbf{J}_t^{-1},$$

$$\mathbf{B}_{12} = -\mathbf{J}_t^{-1} \mathbf{C}_{CA} \mathbf{J}_b \mathbf{D} (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D})^{-1},$$

$$\mathbf{B}_{21} = \begin{bmatrix} \mathbf{0}_{1 \times 3} \\ -(\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D})^{-1} \mathbf{D}^T \mathbf{J}_b \mathbf{C}_{AC} \mathbf{J}_t^{-1} \end{bmatrix}_{2 \times 3}^T,$$

$$\mathbf{B}_{22} = \begin{bmatrix} 0 \\ (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D})^{-1} \end{bmatrix}_{2 \times 1},$$

$$\mathbf{B}_{d1} = \mathbf{B}_{11}, \quad \mathbf{B}_{d2} = \mathbf{B}_{21}.$$

3.1 Improved ESO

According to Eq. (5), three channels are decoupled and the rotation of the solar array has effects on the pitching axis directly. Taking the pitching channel as an example, the controller is designed in this part. The state variables are defined as follows:

$$x_1 = \omega_c, \quad x_2 = w(t), \quad (9)$$

where x_1 is the state of the system while x_2 is the extended state. $w(t)$ represents the sum of the model and disturbance parts calculated from Eq. (8). Then the attitude dynamic model of a single axis can be obtained from Eq. (8) as follows:

$$\begin{cases} \dot{x}_1 = x_2 + bu, \\ \dot{x}_2 = -\rho. \end{cases} \quad (10)$$

where $-\rho$ is the derivative of the $w(t)$.

Based on the concept of ESO [21], second-order ESO is defined as follows:

$$\begin{cases} e_{11} = z_{11} - x_1, \\ \dot{z}_{11} = z_{12} - \beta_1 e_{11} + bu, \\ \dot{z}_{12} = -\beta_2 fal(e_{11}). \end{cases} \quad (11)$$

where z_{11} and z_{12} are the tracking state of x_1 and x_2 , respectively. e_{11} is the output error between the observed value and state output of the inner-loop. The double number of the subscript is defined to distinguish from the ESO in the solar array driving system, which will be mentioned later. The nonlinear function fal is introduced as follows [18, 21]:

$$fal(e_{11}) = \begin{cases} |e_{11}|^\alpha \text{sign}(e_{11}), & |e_{11}| \geq \delta, \\ \frac{e_{11}}{\delta^{1-\alpha}}, & |e_{11}| < \delta. \end{cases} \quad (12)$$

where the appropriate values of coefficients and can make the state observation error converge rapidly. $\text{sign}(\bullet)$ is the sign function.

To improve the performance of the observer, this article proposed the IESO. It is combined with the SMO as follows:

$$\begin{cases} e_{11} = z_{11} - x_1, \\ \dot{z}_{11} = z_{12} - k_1 |e_{11}|^{\frac{1}{2}} \text{sign}(e_{11}) - k_2 e_{11} + bu, \\ \dot{z}_{12} = -k_3 \text{sign}(e_{11}) - k_4 e_{11}. \end{cases} \quad (13)$$

where $k_i (i = 1, 2, 3, 4)$ are the selected positive parameters for the gains of the observer. Compared with the SMO [16], z_{11} of IESO is the estimation of the state of the dynamic system. z_{12} uses a similar form of the SMO to estimate the disturbance instead of the higher-order state.

The error dynamic equations are obtained by Eqs (10) and (13):

$$\begin{cases} \dot{e}_{11} = e_{12} - k_1 |e_{11}|^{\frac{1}{2}} - k_2 e_{11}, \\ \dot{e}_{12} = -k_3 \text{sign}(e_{11}) - k_4 e_{11} + \rho. \end{cases} \quad (14)$$

where $e_{12} = z_{12} - x_2$.

The stability analysis in the same form as Eq. (14) has been discussed in detail by Moreno and Osorio [16, 17]; thus some useful conclusions are cited directly in this article.

Lemma 1 [16]. *Suppose that the perturbation term of Eq. (14) is globally bounded by*

$$\rho = \delta_1 |e_{11}| + \delta_2, \quad (15)$$

for the constants $\delta_1 \geq 0$ and $\delta_2 \geq 0$. The gains k_i can be selected large enough so that the system is strongly globally asymptotically stable, where the origin is an equilibrium point. Then k_i can be given by

$$\begin{aligned} k_1 &> 2\sqrt{\delta_2}, \quad k_2 > \frac{\sqrt{2\delta_1}}{2}, \quad k_3 > \delta_2, \\ k_4 &> \frac{k_1^3(2k_2^2 - \delta_1) + (5k_2^2 + 2\delta_1)p_1}{2p_1 - k_1^3}. \end{aligned} \quad (16)$$

Where $p_1 = k_1 (\frac{1}{4}k_1^2 - \delta_2) + (\frac{1}{2}k_1 - \delta_1) (2k_3 + \frac{1}{2}k_1^2)$.

The controller and observer for the other two channels can be designed according to the above-mentioned steps.

Fig. 3 The block diagram of the outer-loop control algorithm.

3.2 Angular position regulator

As shown in Figure 3, if the controller and the ESO operate well, then the inner loop can be considered as a pathway. The attitude motion system is presented as an error feedback control system.

For the attitude motion formulation in Eq. (6), the ω is considered as the nominal input $\mathbf{u}^*$ of this system; the attitude motion is rewritten as follows:

$$\dot{\Theta} = -\delta + \mathbf{u}^* \quad (17)$$

where $\mathbf{u}^*$ is designed as

$$\mathbf{u}^* = \beta_0 \cdot \mathbf{Fal}(\theta_d - \theta, \alpha, \gamma) + \delta, \quad (18)$$

Where $\beta_0 = \text{diag}[\beta_0, \beta_0, \beta_0]$, $\mathbf{Fal} = [fal_1(\bullet), fal_2(\bullet), fal_3(\bullet)]^T$.

Thus, the nonlinear angle position error feedback law is designed as

$$\dot{\Theta} = \beta_0 \cdot \mathbf{Fal}(\theta_d - \theta, \alpha, \gamma). \quad (19)$$

Without loss of generality, taking the pitching channel as an example to discuss the stability of the regulator, the input error for the outer-loop system is

$$e_0 = \theta_d - \theta. \quad (20)$$

As the desired attitude angle velocity $\dot{\theta}_d$ must be zero when the manoeuvre finishes, the derivative of the error is as follows:

$$\dot{e}_0 = -\dot{\theta} = -\beta_0 fal(e_0, \alpha, \gamma). \quad (21)$$

To show the stability of the ESO, Eq. (21) can be linearised as follows:

$$\dot{e}_0 = -\beta_0 F(e_0) e_0 \quad (22)$$

by the expression

$$F(e_0) = \frac{fal(e_0)}{e_0}. \quad (23)$$

When $e_0 \neq 0$, setting the Lyapunov function as

$$V_1 = \int_0^{e_0} P(e_0) e_0 de_0 \quad (24)$$

Fig. 4 The driving control algorithm of the solar array.

it is easy to ensure that $V_1 > 0$ by first mean value theorem for integration. Differentiating V_1 with respect to time and considering Eq. (24), one obtains

$$\begin{aligned}\dot{V}_1 &= \left(\int_0^{e_0} P(e_0) e_0 de_0 \right)' \\ &= P(e_0) e_0 \dot{e}_0 \\ &= -\frac{1}{\beta_0} \dot{e}_0^2\end{aligned}\quad (25)$$

Only when $e_0 = 0$, one has $\dot{V}_1 = 0$. That means converges to zero asymptotically and the ESO is stable. It demonstrates that the angular position regulator can track the desired attitude angle effectively.

3.3 Modelling for the rotating solar array

As the solar array is restricted to rotating around the y-axis, the dynamic equation is obtained by Eq. (8)

$$\ddot{\theta}_A = (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D}^{-1}) (\tau_A - \mathbf{D}^T \mathbf{J}_b \mathbf{C}_{AC} \mathbf{D} \dot{\omega}_c) \quad (26)$$

Where θ_A is the rotation angle of the solar array. The driving system is an independent and complete control system, which consists of the drive controller, the stepper motor with two-phase (A and B) and the motor [33–35]. Considering Eq. (26) and ignoring the effects of the harmonic torque and the static friction, the driving system expressions are as follows [36, 37]:

$$\begin{cases} \frac{di_A}{dt} = \frac{1}{L} [U_A - Ri_A + K_m \dot{\alpha} \sin(Z_r \theta_A)], \\ \frac{di_B}{dt} = \frac{1}{L} [U_B - Ri_B - K_m \dot{\alpha} \cos(Z_r \theta_A)] \\ \ddot{\theta}_A = (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D}^{-1}) (\tau_A - \mathbf{D}^T \mathbf{J}_b \mathbf{C}_{AC} \mathbf{D} \dot{\omega}_c) \\ \tau_A = -K_m i_A \sin(Z_r \theta_A) + K_m i_B \cos(Z_r \theta_A) - \sigma_0 \omega_A. \end{cases} \quad (27)$$

where U_A , U_B , i_A and i_B are winding voltages and currents, respectively. L and R are the winding self-inductor and resistance, respectively. K_m is the coefficient of the electromagnetic moment. σ_0 is the viscous friction coefficient. Z_r is the teeth number of the rotor.

It can be seen that there exists the attitude motion coupled term $\mathbf{D}^T \mathbf{J}_b \mathbf{C}_{AC} \mathbf{D} \dot{\omega}_c$ in the third formulation of Eq. (27). In this case, it is a perturbation term to the rotation of the solar array. The compensation scheme based on the ESO of Eq. (11) is adopted to estimate and eliminate the perturbation, which is shown in Figure 4.

In Figure 4, the second-order ESO is employed on the feedback path. The estimation of the rotating velocity z_{21} is transformed into a motor current signal i_{real} by the winding current modules. The error between the current signals i_{real} with reference current i_{ref} is the input for the control torque modules, where the is as follows [33, 34]:

$$\begin{cases} \bar{i}_A = I_m \cos [2\pi \text{round} (Z_r \omega_d t)], \\ \bar{i}_B = I_m \sin [2\pi \text{round} (Z_r \omega_d t)]. \end{cases} \quad (28)$$

where I_m is the rated current and ω_d is the desired rotation speed. The control torque τ_A generated by the disturbance estimation z_{22} is obtained as follows:

$$\tau_A = \tau_0 - \frac{z_{22}}{b}, \quad (29)$$

where $b = (\mathbf{D}^T \mathbf{J}_b \mathbf{D} - \mathbf{D}^T \mathbf{J}_b \mathbf{J}_t^{-1} \mathbf{J}_b \mathbf{D})^{-1}$ is shown in Eq. (8).

3.4 Sliding mode controller for the attitude dynamics

In this article, the attitude manoeuvre is achieved by the sliding mode method. With the help of the IESO, the perturbation can be eliminated as much as possible. The sliding surface for the three-axis manoeuvre is selected as follows:

$$\mathbf{s} = \mathbf{c}\mathbf{e}_1 + \mathbf{e}_2, \quad (30)$$

where $\mathbf{s} = [s_1, s_2, s_3]^T$. $\mathbf{c} = \text{diag}[c_1, c_2, c_3]$ is a designed parameter to stabilise the system. should meet the Hurwitz conditions that the system is stable as long as the trajectory of the system reaches the sliding surface in finite time. $\mathbf{e}_1$ and $\mathbf{e}_2$ are defined as

$$\begin{aligned} \mathbf{e}_1 &= \boldsymbol{\omega}_d - \mathbf{z}_1, \quad \mathbf{e}_2 = -\mathbf{z}_2, \\ \boldsymbol{\omega}_d &= [\omega_{d\varphi}, \omega_{d\theta}, \omega_{d\psi}]^T, \quad \mathbf{z}_1 = [z_{1\varphi}, z_{1\theta}, z_{1\psi}]^T, \quad \mathbf{z}_2 = [z_{2\varphi}, z_{2\theta}, z_{2\psi}]^T, \end{aligned} \quad (31)$$

where $\boldsymbol{\omega}_d$ is the vector of desired angular velocities. $\mathbf{z}_1$ and $\mathbf{z}_2$ are the vectors of the estimations obtained by the ESO.

The exponential rate reaching law is applied as follows:

$$\dot{\mathbf{s}} = -\boldsymbol{\varepsilon} \cdot \text{sign}(\mathbf{s}) - \mathbf{k}\mathbf{s}, \quad (32)$$

where

$$\begin{aligned} \boldsymbol{\varepsilon} &= \text{diag}[\varepsilon_1, \varepsilon_2, \varepsilon_3], \quad \mathbf{k} = \text{diag}[k_1, k_2, k_3], \quad \varepsilon_i, k_i > 0, \\ \text{sign}(\mathbf{s}) &= [\text{sign}(s_1), \text{sign}(s_2), \text{sign}(s_3)]^T. \end{aligned}$$

Combing with Eq. (31), the derivative of $\mathbf{s}$ can also be written as

$$\begin{aligned} \dot{\mathbf{s}} &= \mathbf{c}\dot{\mathbf{e}}_1 + \dot{\mathbf{e}}_2 \\ &= \mathbf{c}(-\dot{\mathbf{z}}_1) + (-\dot{\mathbf{z}}_2) \\ &= \mathbf{c}(-\mathbf{z}_2 + \boldsymbol{\beta}_1 \mathbf{e}_1 - \mathbf{B}_{ctrl} \mathbf{u}) + (\boldsymbol{\beta}_2 \mathbf{e}_1), \end{aligned} \quad (33)$$

where $\boldsymbol{\beta}_i = \text{diag}[\beta_i, \beta_i, \beta_i]$.

The control law is obtained by Eqs (32) and (33) as follows:

$$\mathbf{u} = (\mathbf{c}\mathbf{B}_{ctrl})^{-1} (\mathbf{k}\mathbf{s} + \boldsymbol{\varepsilon} \cdot \text{sign}(\mathbf{s}) + \boldsymbol{\beta}_2 \mathbf{e}_1 + \mathbf{c}\boldsymbol{\beta}_1 \mathbf{e}_1 - \mathbf{c}\mathbf{z}_{12}). \quad (34)$$

To discuss the stability of the control law, a candidate Lyapunov function with the linear sliding surface is given as

$$V_2 = \frac{1}{2} \mathbf{s}^T \mathbf{s}. \quad (35)$$

Taking the derivatives of Eq. (35) and considering the Eqs (33) and (34) yield

$$\begin{aligned} \dot{V}_2 &= \mathbf{s}^T \dot{\mathbf{s}} \\ &= \mathbf{s}^T \mathbf{c}(-\mathbf{z}_2 + \boldsymbol{\beta}_1 \mathbf{e}_1 - \mathbf{B}_{ctrl} \mathbf{u}) + (\boldsymbol{\beta}_2 \mathbf{e}_1) \\ &= \mathbf{s}^T (-\mathbf{k}\mathbf{s} - \boldsymbol{\varepsilon} \cdot \text{sign}(\mathbf{s})) \\ &= -\sum_{i=1}^3 (k_i s_i^2 + \varepsilon |s_i|). \end{aligned} \quad (36)$$

Table 1 Control parameters selected for numerical analysis

Observer	Coefficients	Control Scheme	Gains
ESO	$\alpha = 0.5, \gamma = 0.01,$ $\beta_1 = \text{diag}(20, 20, 20),$ $\beta_2 = \text{diag}(100, 100, 100).$	SMC	$c = \text{diag}(10, 10, 10),$ $K_s = \text{diag}(10, 10, 10),$ $D_s = \text{diag}(8, 8, 8).$
IESO	$k_1 = \beta_1, k_2 = \beta_2,$ $k_3 = k_4 = \text{diag}(100, 100, 100).$	Error regulator	$\beta_0 = \text{diag}(1, 1, 1)$

Table 2 Solar array driving system selected of numerical analysis

Parameters	Value	Parameters	Value
Rated winding current I_m (A)	0.3	Viscous friction coefficient σ_0	26
Number of teeth Z_r	300	Proportional coefficient k_p	10
Winding self-induction L (H)	0.2	Integration coefficient k_i	300
Winding resistance R (Ω)	68	Desired angular velocity θ_d ($^\circ/s$)	0.06

It is obvious that $\dot{V}_2 < 0$ because both k_i and ε_i are positive according to Eq. (32). Only when $s_i = 0$, $\dot{V}_2$ equals zero and the trajectory reaches the sliding surface. Therefore, the present sliding mode motion is asymptotically stable.

4 Simulation results analysis

The numerical simulation of the proposed control scheme to the rigid spacecraft is given by the MATLAB software. The rotary inertia of the spacecraft consists of $\mathbf{J}_C$ and $\mathbf{J}_b$ as follows:

$$\mathbf{J}_C = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 150 \end{bmatrix} \text{ kg} \cdot \text{m}^2, \quad ; \mathbf{J}_b = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 50 & 2 \\ 0 & 2 & 30 \end{bmatrix} \text{ kg} \cdot \text{m}^2$$

The parameters of the control scheme for the spacecraft and the solar array system driving system are summarised in Tables 1 and 2, respectively.

In the beginning of the manoeuvre, the attitude angle is assumed to be $[\varphi_0, \theta_0, \phi_0]^T = [5, -2.1]^T$ and the angle position of the solar array is zero relative to the central rigid body.

For the analysis of the validity and advantages of the IESO, Figures 5 and 6 show the observer errors obtained from Eqs (11) and (13) under the SMC law. E_{1i} ($i = 1, 2, 3$) represents the estimated errors of three inertial angular velocities and E_{2i} represents the estimated errors of the disturbance, respectively. As shown in Figures 5 and 6, both the cases estimate the states successfully. The error of each observer reaches a stable stage of < 3 s. The right columns of Figures 5 and 6 are the partially enlarged image of the observer errors at the stable stage. They demonstrate the high precision of the ESO was guaranteed. Obtained from the observed errors, it is obvious that the ones which adopted the improved ESO converge to zero have little fluctuation. The ranges of the errors obtained by the original ESOs are , and $|E_{13}| < 1$, respectively. However, the ones of the IESOs are $|E_{11}| < 0.01$, $|E_{11}| < 0.02$ and $|E_{13}| < 0.01$, respectively. The reduction of the amplitude is an order of magnitude. From Figure 6, even though the error ranges for the disturbance observations of two kinds of the ESO are roughly similar, the fluctuation frequency of the IESO is much lower than the original ESO. The relative stable estimations are always easy to be observed and make use of in engineering.

Figures 7 and 8 show the inertial angular velocities and the Euler attitude angles, respectively. The right

Fig. 5 The state observation error $E_{1i}(i = 1, 2, 3)$ of three axes.

Fig. 6 The disturbance observation error $E_{2i}(i = 1, 2, 3)$ of three axes.

columns of these two figures are the partially enlarged drawing of the corresponding steady stage. From the left columns of Figures 7 and 8, both the inertial angular velocity and the Euler attitude angle under two kinds of ESO reach the commands < 5 s. However, the oscillation performance is much different from these two observers. From the partially enlarged drawing in Figure 7, the amplitude with original ESO is nearly 10 times more than that under improved ESO. Correspondingly, the curves of three attitude angles have an analogous trend. The amplitudes of the Euler angles obtained from IESO are $< 2 \times 10^{-4}$, which is much lower than the reference ones (obtained from ESO) about 1×10^{-2} . Even though the steady-state values are not zero, the precision is sufficient to meet the engineering requirement.

Figure 9 shows the rotation of the solar array under the driving torque, where the two control schemes are considered. Figure 9(a) is the variation of the angle position while Figure 9(b) is the angle velocity. At the initial stage, the angle is negative, which represents the solar array rotates in the opposite direction to the objective. This phenomenon depends on the coordinate frame selected for the observation. The attitude driving torque is large enough, and there exists a lag in the rotation of the solar array while offsetting its driving torque. However, with the attitude motion of the spacecraft turning to stable, the solar array driving performs well and the velocity

Fig. 7 The three-axis angular velocity about the body-fixed coordinate frame.

Fig. 8 The Euler angles of the reference coordinate frame.

Fig. 9 The variation of the angle and angular velocity of the solar array.

tends to steady. Similar to the variation of attitude angles, the angle velocity of the solar array obtained by the IESO is more stable than that of the ESO. It demonstrates the better performance of the IESO over the original ESO from the perspective of solar array.

5 Conclusion

In this article, a three-axis attitude manoeuvre spacecraft with a rotating solar array is studied from the attitude dynamic point of view. Considering the rigid solar array, the multi-body dynamic model is obtained.

To decouple the system, the coupled terms of the dynamical equation are considered the disturbance. The SMC scheme with second-order ESO is adopted to adjust the attitude and reduce the disturbance of the spacecraft. The scheme helps the spacecraft to compensate the disturbance effectively and reach the target rapidly. From the simulation results, the IESO presents good performance to reduce the disturbance and achieve the attitude manoeuvre. The IESO has higher accuracy than the original ESO on the premise of ensuring the convergence time. Moreover, the solar array driving system is also benefitted from the IESO. The fluctuation of the driving is so little that the attitude adjustment can be smooth. In addition, the good robustness of the IESO is also presented by the results that the inertia moments variation within a certain range.

However, it should be mentioned that the solar array driving is simplified by neglect some electromagnetic effects. If they have been fully considered, the complex driving scheme may be a subject for future researches.

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