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Deformation and stress theory of surrounding rock of shallow circular tunnel based on complex variable function method

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Abstract

After reviewing many literature foundations, the thesis combines the basic methods of elastic mechanics with mathematical knowledge, sets the bipotential stress potential complex function and analyses the relationship between stress component, strain component and stress potential function, and applies the complex variable function. The expression of the relevant stress component is derived, and the displacement boundary conditions of the surrounding rock of shallow circular tunnel are obtained. Furthermore, the paper applies the basic theory of complex variable function to solve the boundary condition complex variable function for common tunnel sections, and obtains the analytical expression of the surrounding rock stress of shallow circular tunnel. The simulation is carried out by finite element method. The establishment of complex variable function has a good application value in solving the stress of surrounding rock of shallow tunnel.

Keywords: complex variable function, shallow-buried circular tunnel surrounding rock, deformation analysis, stress finite element analysis.

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1 Introduction

Shallow-buried tunnel refers to the fact that the tunnel is too shallow, the stress distribution of the surrounding rock is relatively complicated and the stress distribution around the surrounding rock is not uniform, so the solution process is more troublesome [1]. The problem of shallow tunnel excavation is often encountered

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during urban underground development and construction. The study of stress and deformation components of surrounding rock is more complicated than the calculation model of shallow tunnel. There is not much research in this area. In order to analyse the variation law of stress and displacement, this paper uses the complex variable function method to solve the shallow-buried circular tunnel retaining structure under radial deformation, and gives the analytical solution. The finite element software ABAQUS is used to evaluate the results and carry out their verification.

Based on the above problems, this paper applies the complex variable function method to give the exact solution of the shallow-buried tunnel, and applies the equivalent radius method to give the analytical solution of the circular shallow tunnel section. The analytic solution and the equivalent radius of the circular shallow tunnel section given by the complex variable function method are given. The applicable conditions of the equivalent radius solution are discussed [2]. The excavation of the tunnel boring machine for the circular shallow tunnel section is discussed. The section and lining section design provide a theoretical basis for reference. Then the complex solution method is used to give the analytical solution of the shallow-buried circular tunnel lining structure. The influence of the radial deformation generated by the shallow circular tunnel on the stress state of the surrounding rock and the deformation and control of the influence are considered. The theoretical basis of the strength of the supporting structure required for deformation is provided. In practical engineering, this solution can be applied to provide a reference for theoretical analysis.

2 Theoretical basis of the method of plane elastic complex function

2.1 The constitutive relation of elastoplastic increment

The elastoplastic problem is a material nonlinear problem, and the strain depends not only on the current stress state but also on the entire loading history. In practical terms, the relationship between stress and strain can only be incremental, and the corresponding iterative solution can only be an incremental form of solution [3]. For the elastoplastic problem, the strain increment can be divided into elastic strain increment $d\epsilon_e$ and plastic strain $d\epsilon_p$ increment, i.e.

$$d\epsilon = d\epsilon_e + d\epsilon_p \quad (1)$$

In general, the elastoplastic constitutive relationship of geomaterials includes the following four components: (1) Yield conditions and failure criteria to determine whether the material is plastically yielded or destroyed. (2) The law of hardening, indicating the change in yield conditions due to plastic strain. (3) The law of flow, which determines the direction of plastic strain. (4) Loading and unloading criteria, indicating the working state of the material.

(1) Yield conditions of geomaterials

Drucker-Prager guidelines:

$$k = \alpha J_1 + \sqrt{J_2'} \quad (2)$$

Plane strain state

$$\alpha = \frac{\sin \varphi}{\sqrt{3}\sqrt{3 + \sin^2 \varphi}}, \quad k = \frac{\sqrt{3}c \cos \varphi}{\sqrt{3 + \sin^2 \varphi}} \quad (3)$$

where c represents cohesion and φ represents internal friction angle.

(2) The law of flow

$$d\epsilon_p = d\lambda \frac{\partial F_0}{\partial \sigma} \quad (4)$$

where $\lambda \geq 0$ is the proportionality factor.

(3) Consistency conditions

Reflects the consistency of the load increment is small, the stress point falls on the new loading surface.

$$dF_0 = \left(\frac{\partial F_0}{\partial \sigma} \right)^T d\sigma + \left(\frac{\partial F_0}{\partial \varepsilon_p} \right)^T d\varepsilon_p + \frac{\partial F_0}{\partial K} \left(\frac{\partial K}{\partial \varepsilon_p} \right) d\varepsilon_p = 0 \quad (5)$$

(4) Scale factor

$$d\lambda = \frac{1}{c_p} \left(\frac{\partial F_0}{\partial \sigma} \right)^T D_e d\varepsilon \quad (6)$$

In the formula, $c_p = \left(\frac{\partial F_0}{\partial \sigma} \right)^T D_e \frac{\partial F_0}{\partial \sigma} - \left(\frac{\partial F_0}{\partial \varepsilon_p} \right)^T \frac{\partial F_0}{\partial \sigma} - \frac{\partial F_0}{\partial K} \left(\frac{\partial K}{\partial \varepsilon_p} \right)^T \frac{\partial F_0}{\partial \sigma}$. (5) Stress-strain relationship of general elastoplastic increment

$$d\sigma = D_{ep} d\varepsilon \quad (7)$$

where $D_{ep} = D_e - D_p$ is an elastoplastic matrix.

2.2 Basic equations for plane strain problems

Since the tunnel is a cylindrical object in the longitudinal direction, the size and shape of the tunnel section do not change with the length of the axis. It can be regarded as the external force acting in the longitudinal direction, and the two ends of the cylinder are regarded as fixed constraints. In this way, the original structure can be simplified into an infinite elastic body with a single hole, which is analysed according to the plane strain problem [4, 5]. Since the shape and the external force of the elastic body do not change with the coordinate z , the displacement, strain and stress in the elastic body are both the coordinates z are independent of the coordinates. The geometric equation of the plane elasticity problem is

$$\begin{aligned} \varepsilon_x &= \frac{\partial u_x}{\partial x} \\ \varepsilon_y &= \frac{\partial u_y}{\partial y} \\ \gamma_{xy} &= \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \end{aligned} \quad (8)$$

$$\begin{aligned} \mu \nabla^2 u_x + (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) &= 0 \\ \mu \nabla^2 u_y + (\lambda + \mu) \frac{\partial}{\partial y} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) &= 0 \end{aligned} \quad (9)$$

where λ and μ are the Lamé constants.

2.3 Complex variable function of plane strain problem

Introducing the function $A = A(x, y)$, $B = B(x, y)$, according to the equilibrium differential equation in Eq. (9), the following relationship can be obtained

$$U(x, y) = U \left(\frac{z + \bar{z}}{2}, \left(\frac{z - \bar{z}}{2i} \right) \right) = U(z, \bar{z}) \quad (10)$$

For z , the second integral

$$2U(z, \bar{z}) = \bar{z}\phi_f'(z) + \chi(z) + z\phi_{f1}(\bar{z}) + \chi_1(\bar{z}) \quad (11)$$

Therefore, the stress function can be expressed as:

$$U(z, \bar{z}) = \text{Re}[\bar{z}\phi_f(z) + \chi(z)] \quad (12)$$

Complex variable function representation of stress component

$$\begin{aligned} \partial_y - \partial_z + 2i\tau_{xy} &= 2[\bar{z}\phi_f''(z) + \psi'(z)] = 2[\bar{z}\Phi'(z) + \Psi(z)] \\ \Phi(z) &= \frac{d\phi_f(z)}{dz} = \phi_f'(z) \\ \Psi(z) &= \frac{d\psi(z)}{dz} \end{aligned} \quad (13)$$

2.4 Incremental finite element theory of elastoplastic problem

In the incremental finite element method of elastoplastic problem, the incremental elastoplastic constitutive relation is limited by the condition of ‘simple loading’. In practical engineering, many elastoplastic problems are often coupled with large deformation and nonlinearity. The simple loading theory does not conform to the actual structural stress process, and the most suitable solution for large deformation problems is the incremental load iterative method. The finite element method is an ideal choice for calculating the elastoplastic problem [6, 7]. When the unit is in loading, whether in an elastic state, a plastic state or a transition state (i.e. one part is in an elastic state and the other part is in a plastic state), and when the incremental load is small enough, the corresponding stress-strain relationship is

$$\begin{aligned} \Delta\sigma &= D_e\Delta\varepsilon \\ \Delta\sigma &= D_{ep}\Delta\varepsilon \\ \Delta\sigma &= \bar{D}_{ep}\Delta\varepsilon \end{aligned} \quad (14)$$

(1) Explicit of elastoplastic matrix

$$D_p = \frac{1}{H' + 3G} \left(\frac{3G}{\bar{\sigma}} \right)^2 SS^T \quad (15)$$

$$D_{ep} = D_e - D_p \quad (16)$$

where $H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}_p} = K'$, S represents the deviatoric stress tensor,

$$G = \begin{bmatrix} 1-\nu & & & & \\ \nu & 1-\nu & & & \\ \nu & \nu & 1-\nu & & \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \\ & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}.$$

(2) Transition state $\bar{D}_{ep}$

$$\bar{D}_{ep} = mD_e + (1-m)D_{ep} \quad (17)$$

In the formula, $m = \frac{\Delta \bar{\varepsilon}_{eT}}{\Delta \bar{\varepsilon}_{eI}} = \frac{\Delta \bar{\sigma}_{eT}}{\Delta \bar{\sigma}_{eI}}$, ($0 < m < 1$), $\Delta \bar{\varepsilon}_{eT}, \Delta \bar{\sigma}_{eT}$ represents the equivalent strain increment and equivalent stress increment required to reach the yield; $\Delta \bar{\varepsilon}_{eI}, \Delta \bar{\sigma}_{eI}$ represents the equivalent strain and stress increment caused by the current load increment ΔP .

(3) Elastoplastic incremental finite element equation

$$K \Delta d = \Delta P \quad (18)$$

In the formula, $K = \sum K_e$,

$$K_e = \begin{cases} \int_{V_e} B^T D_e B dV_e \\ \int_{V_e} B^T \bar{D}_{ep} B dV_e \\ \int_{V_e} B^T D_{ep} B dV_e \end{cases} \quad (19)$$

3 Research on surrounding rock support scheme for shallow circular tunnel

3.1 Support scheme

In order to effectively control the deformation of the tunnel, the effect of the retaining wall and the effect of the lime-soil compacted pile are studied. Taking the K75+750 section as an example, the specific two supporting schemes are shown in Figure 1. In the analysis of the supporting effect, the initial thickness of the C25 shotcrete is 25 cm, and the second lining is used as a safety reserve; the length of the anchor is 3.5 m and 1 m $\times$ 1 m [8].

Fig. 1 Two different circular tunnel support schemes.

The general large-scale structural analysis software ANSYS10.0 and continuous medium model are used for numerical simulation. For different supporting schemes, the surrounding rock deformation, stress, tunnel perimeter (vertical and horizontal deformation) and initial branch are studied in the tunnel excavation process. The force and safety factor are used to judge the safety of the retaining structure, and a reasonable supporting scheme is determined to ensure the stability of the tunnel.

3.2 Calculation model

On the premise of fully considering the geological environment near the tunnel and minimising the so-called 'boundary effect', the distance between the boundary of the calculation model and the centreline of the tunnel is 45 m, and the distance from the bottom boundary to the top of the tunnel is 30 m. Surface. According to the characteristics of the bias tunnel, the upper boundary is the free boundary, the left and right boundaries are horizontal constraints and the upper boundary is the vertical displacement constraint [9, 10]. There are 1876

quadrilateral elements and the total number of nodes is 1847. The calculation model is shown in Figure 2. The pre-support of the tunnel in the calculation considers the overall reinforcement effect of the surrounding rock, so that the reinforcement of the pre-support can improve the physical and mechanical parameters in the reinforcement area. The hardened area can be simplified to a solid unit. The tunnel adopts full-section excavation, and the calculated load release rate is 40%, i.e. the support bears 60% of the surrounding rock pressure.

Fig. 2 Two different supporting models.

3.3 Design and calculation parameters

The initial support has the following characteristics: 20 cm thick C25 shotcrete, 3.5 m side wall anchor (1 m × 1 m), plum-shaped arrangement, 4φ12 steel grille arch. The physical and mechanical parameters used in the calculation are shown in Table 1.

Table 1 Model calculation of mechanical parameters

Material	Density/kg/m ³	Elastic Modulus/MPa	Poisson's ratio	Cohesion/kPa	Internal friction angle/°
Expansive Soil	2000	50	0.3	35	23
Sandy loess	2000	50	0.3	28	45
Leading support	2000	60	0.3	33	53
Bolt reinforcement ring	2000	60	0.3	42	28
Initial support	2500	29.5E3	0.2	—	—
Retaining wall	2500	29.5E3	0.2	—	—
Lime-soil pile	2000	150	0.3	33.8	48

3.4 Calculation results

3.4.1 Support scheme 1

(1) Deformation of surrounding rock caused by tunnel construction

Since the left side slope weakens the lateral constraint on the left side of the tunnel, the displacement cloud diagram is shown in Figure 3.

(2) Initial stress state

From Figure 4, we are able to infer the following: If the slope is not treated, due to the bias effect, axial force concentration occurs at the left and right foot legs; the bending moment also shows the left and right asymmetrical distribution, the right side is larger than the left side, the support The bearing capacity is reduced; the retaining structure is in a very unfavourable state of force, so a retaining wall must be provided.

It can be seen from Figure 5 that under the condition that the slope is not treated, the stress is unevenly

Fig. 3 Construction caused vertical and horizontal displacement.

Fig. 4 Supporting axial force and supporting bending moment (/N·m).

Fig. 5 Supporting the first principal stress and supporting the third principal stress (/N).

distributed due to the bias effect: the maximum tensile stress appears on the outer side of the left arch, which is 6.05 MPa, exceeding the ultimate tensile strength of C25 concrete. The compressive stress appears on the inside of the right wall, which is 11.50 MPa, which is less than the ultimate compressive strength of C25 concrete (19.00 MPa). Therefore, the slope must be treated to ensure that the supporting structure is reasonably stressed and safe.

(3) Initial safety check

The typical section safety factor of the retaining structure is shown in Table 2.

Table 2 Typical section safety factor of initial support structure

Key location	Dome	Left arch waist	Right arch waist	Left arch	Right arch	Left wall	Right wall	Left wall foot	Right wall foot	Inverted arch
Axial force/kN	-124.7	-348.5	-294.7	-380.1	-573.1	-488.9	-614.4	-311.9	-443.8	-146.1
Moment/kN-m	31.6	-8.562	0.069	52.57	-19.5	-0.49	-13.54	19.511	-41.7	-27.4
Safety factor	0.8	7.29	11.84	0.92	4.01	7.05	4.25	5.72	3.32	1.22

It can be seen from Table 2 that the structural force is extremely uneven and does not meet the safety factor requirement. Therefore, the retaining wall and the pipe shed must be set.

(4) Plastic zone distribution characteristics

Fig. 6 Distribution of plastic zone of surrounding rock.

Since the left side slope weakens the lateral constraint on the left side of the tunnel, the surrounding rock around the tunnel is in a state of shear stress, and a plastic zone appears at the left and right arch waists of the tunnel. The depth of the left arch waist plastic zone reaches >5 m, and the right arch waist has 3.5 m. In the unstable state, the slope must be treated with a retaining structure to ensure the stability of the surrounding rock of the tunnel.

3.4.2 Support scheme II

(1) Deformation of surrounding rock and retaining structure caused by tunnel construction

Based on the retaining wall and the advanced pipe shed, the tunnel construction begins.

Fig. 7 Vertical displacement and horizontal displacement caused by tunnel construction.

Comparing Figure 3 and Figure 7, it can be clearly seen that compared with the first scheme, the vertical displacement and horizontal displacement in scheme 2 are obviously improved, the arch settlement is reduced

from 4.73 cm to 1.86 cm and the horizontal displacement is greatly improved (the left arch is 4.40 cm $\rightarrow$ 1.73 cm, and the right arch is 1.70 cm $\rightarrow$ 1.38 cm), but the settlement at the foot of the slope is too large, which brings potential risks to the tunnel. The retaining wall strengthens the lateral restraint of the left side of the tunnel, the vertical settlement becomes smaller, the horizontal displacement becomes smaller and the asymmetry is basically asymmetrical. The retaining wall and the pipe shed can ensure the stability of the tunnel.

(2) Retaining wall foundation settlement

Since the retaining wall is subjected to the thrust of the right-side slope, the foundation is subjected to a relatively large force, and a large settlement occurs. It can be seen from Figure 8 that the thrust of the slope is large and the settlement of the retaining wall foundation is 2.96 cm. Therefore, the foundation must be treated, i.e. the lime-soil compacted pile needs to be applied to ensure the stability of the retaining wall and the entire structure.

Fig. 8 Retaining wall foundation settlement diagram.

(3) Initial stress state

Comparing Figure 4 with Figure 9, the slope is treated with the retaining wall and the pipe shed above the tunnel. The biasing effect is obviously improved. The axial force and bending moment are basically symmetrically distributed, and the bearing capacity of the support is increased.

Fig. 9 Supporting axial force and supporting bending moment ($\text{N}\cdot\text{m}$).

It can be seen from Figure 9 that the slope is treated with the retaining wall and the pipe shed above the tunnel, which significantly improves the biasing effect (uneven distribution of stress): the maximum tensile stress appears on the outside of the left arch, relative to no treatment. The 6.05 MPa without treatment was

reduced to 4.67 MPa, and the unfavourable stress state was obviously improved; the maximum compressive stress appeared on the inner side of the right wall, from 11.50 MPa to 9.80 MPa MP. Therefore, the retaining wall retaining structure of the slope and the pipe shed above the tunnel must be treated to improve the stress state of the tunnel retaining structure and the stability of the tunnel.

(4) Initial safety check

The typical section safety factor of the retaining structure is shown in Table 3.

Table 3 Typical section safety factor of initial support structure

Key location	Dome	Left arch waist	Right arch waist	Left arch	Right arch	Left wall	Right wall	Left wall foot	Right wall foot	Inverted arch
Axial force/kN	-137.5	-316.3	-238.1	-403.4	-489.4	-425.8	-537.1	-371.7	-407.6	-205.6
Moment/kN·m	11.87	-9.035	10.386	5.5608	15.619	-1.833	11.876	38.058	-36.01	-22.84
Safety factor	8.06	7.69	8.8	7.16	4.8	7.71	4.86	3.66	2.6	2.99

It can be inferred from Table 3 that the structural force is greatly improved and that all relevant safety factor requirements are met. Therefore, the retaining wall and the pipe shed must be provided to ensure the safety of the tunnel retaining structure.

(5) Plastic zone distribution characteristics

Fig. 10 Distribution of plastic zone of surrounding rock.

The slope is supported by the retaining wall support + pipe shed to greatly reduce the plastic zone around the tunnel. The plastic depth of the front arch waist is not more than 5 m, and the right arch waist is 3.5 m. There is basically no plastic zone around the hole after the treatment; however, the plastic zone appears at the joint of the slope pipe and the retaining wall. The main reason is that the settlement occurred at the retaining wall foundation. Therefore, based on the retaining wall and the pipe shed retaining structure, the mechanism of the lime-soil compacted pile of the retaining wall foundation is analysed.

3.5 Summary of key point displacement calculations for different supporting schemes (cm)

Table 4 Summary of displacement calculation of key points of different supporting schemes (cm)

Support scheme	Vault settlement	Left arch horizontal displacement	Right arch horizontal displacement	Retaining wall foundation settlement
No retaining wall without crowded pile	4.72	4.4	1.7	—
Retaining wall + pipe shed	1.86	1.73	1.38	2.96

The non-retaining wall without the compacted pile makes the tunnel stable, and the structure meets the requirements of standard safety, but the settlement of the retaining wall foundation is too large, which brings engineering risks to the construction phase and the tunnel operation phase; the lime-soil compacted pile can make the retaining wall foundation settle Small, and the deformation of the key points of the hole wall is more uniform and reduced, and the horizontal constraint on the left side of the tunnel is strengthened, so that the overall safety factor of the slope is reduced, so the retaining wall + pipe shed is a reasonable construction plan.

4 Conclusion

In this paper, first the basic theory of the plane elastic complex function method is deduced, and then the analytical expression of the surrounding rock stress of shallow circular tunnel is given by using the complex variable function method. Then, the equivalent expression radius is used to give the analytical expression of the surrounding rock stress of shallow circular tunnel. Finally, the complex variable function method is applied to study the self-supporting ability of surrounding rock through deformation under the action of shotcrete support, and solve the complex variable function solution of shallow tunnel under given deformation.

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