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A mathematical model of the fractional differential method for structural design dynamics simulation of lower limb force movement step structure based on Sanda movement

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Abstract

In order to promote the development of sports training robots and improve the training of lower limbs in Sanda, a new mechanical structure of Sanda training robots is designed. The thesis combines the mathematical method of fractional differential equations to design a new type of omnidirectional moving platform mechanism, which realises the movement in any direction in the plane and can turn at any radius during the movement. From the perspective of kinematics, the mathematical relationship between wheel speed and robot trajectory is analysed, and a virtual prototype technology combined with Pro/E and Adams joint modelling and simulation method is used to establish an accurate and reliable virtual prototype model. The experimental simulation results show that the designed virtual prototype model is consistent with the mathematical model, which verifies the practical feasibility of the mechanical structure of the lower limb power generation robot for Sanda movement, and provides a reliable basis for the establishment of the physical prototype.

Keywords: Sanda exercise, lower limb strength training, lower limb strength training, omnidirectional wheels, virtual prototype technology.

AMS 2010 codes: 12H99

1 Introduction

In martial arts Sanda (hand), the side leg is often regarded as a ‘heavy weapon’ with high strength and lethality (heavily enough to stun the opponent on the spot and win the entire game). However, when completing the side leg movements, due to the large moment of inertia of the entire leg, the time required to start is relatively long. In addition, the trajectory (route) travelled is long, so the concealment of the movement is relatively poor

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and is easy to be seen by opponents, often in actual combat and short-sale or counter-attack. Therefore, in order to improve the actual combat effect of the side legs, how to increase the concealment and suddenness of the athlete's starting, how to increase the swing angular velocity and the striking speed and how to improve the effectiveness and pertinence of the striking should be the problems to be solved by training and scientific research [1–3].

This article mainly studies the structural design of the lower limb power robot with wheeled Sanda and its virtual prototype dynamics simulation and analyses whether the centre of gravity trajectory moves according to the expected trajectory under the known input to make the rationality of the mechanical structure design of the robot. Judge and give suggestions for optimisation.

2 Kinematics of sanda's lower limb power robot

The robot adopts omnidirectional wheel technology, and the omnidirectional wheel is composed of an active hub and a passive roller. The direction of the passive roller is parallel to the axis of the active hub. The movement of the wheel is a combination of the movement of the active hub and the passive roller. The four omnidirectional wheels are in a zigzag shape in the positive direction of the car body, and the omnidirectional wheels are 45° or 135° in the forward direction. Each omnidirectional wheel is independently driven by a servo motor. The speed of the four motors is controlled by a program, and then, the robot moves according to the combination of speeds. The robot motion structure is shown in Figure 1 [4,5].

Fig. 1 Motion analysis chart.

The numerical solutions to the initial value problems of ordinary differential equations are given in a recursive form, that is, the recursive algorithm. According to the recursive algorithm, when calculating y_{k+1} , the approximate value y_i , $i = 0, 1, \dots, k$ at the previous moments has been obtained. The various numerical solutions introduced have one thing in common: when calculating y_{k+1} , only the 'information' y_k of the previous time (the current time t_k) is used to predict the 'state' $y(t_{k+1})$ of the t_{k+1} system at a certain time in the future. Such a numerical solution is called the single-step method for the state $y(t_{k+1})$ of a dynamic process $y = y(t)$ at time t_{k+1} . Not only the information y_k at the previous time affects it but also the information at the previous times usually also affects it. Obviously, the disadvantage of the single-step method is an underutilisation of the information obtained [6].

Based on the above considerations, the method given by appropriately taking the information y_i , $i = k, k-1, \dots, k-m$ at the previous moments and replacing the average change rate of $y = y(t)$ in the interval $[t_k, t_{k+1}]$ with a linear combination of $f(t_i, y_i)$ is called a linear multi-step method.

3 Linear multi-step method: Starting from the two-step method

In the improved Euler method,

$$y_{k+1}^0 = y_k + hf(t_k, y_k) \quad (1)$$

$$y_{k+1} = y_k + \frac{h}{2} [f(t_k, y_k) + f(t_{k+1}, y_{k+1}^0)] \quad k = 1, 2, \dots, n \quad (2)$$

If prediction methods are improved, such research results are obtained using numerical differentiation:

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_{k-1}))}{2h} + O(h^2) \Rightarrow y(t_{k+1}) = y(t_k) + f(t_k, y(t_k))2h + O(h^3) \quad (3)$$

Then, since the local truncation error of (3) is $O(h^3)$ and the accuracy is higher than the local truncation error $O(h^2)$ of the Euler formula, the following prediction-correction method is obtained:

$$y_{k+1}^0 = y_{k-1} + 2hf(t_k, y_k) \quad (4)$$

$$y_{k+1} = y_k + \frac{h}{2} [f(t_k, y_k) + f(t_{k+1}, y_{k+1}^0)] \quad k = 1, 2, \dots, n \quad (5)$$

Although the step size is increased, the accuracy is improved. The numerical methods given by formulas (4) and (5) are called non-starting two-step methods. The reason is that when using this method to solve the initial value problem of differential equations, it cannot provide the required initial value y_{-1} by itself.

3.1 Further improvement of the linear two-step method

In the above discussion, we have only considered the improvement of prediction methods, thinking from a methodological point of view, we should also consider the improvement of correction, and we can hope that the appropriate improvement will bring the return of increased accuracy. The following recommended solutions are commonly used [7]:

$$y_{k+1}^0 = y_{k-1}^m + 2hf(t_k, y_k^m) \quad (6)$$

$$y_{k+1}^m = y_k^m + \frac{h}{2} [f(t_k, y_k^m) + f(t_{k+1}, y_{k+1}^{i-1})] \quad i = 1, 2, \dots, m \quad (7)$$

where y_{k+1}^m is the final calculation result at time t_{k+1} , and the number of corrections m is determined according to the following formula

$$|\epsilon_a| = \left| \frac{y_{k+1}^i - y_{k+1}^{i-1}}{y_{k+1}^i} \right| \times 100\% \leq \epsilon \quad (8)$$

where is the pre-specified error accuracy.

3.2 Error analysis and step size control of the linear two-step method

3.2.1 Prediction error

$$\begin{aligned} y(t_{k+1}) &= y(t_{k-1}) + f(t_k, y(t_k))2h + \frac{1}{3}h^3 f'''(t_k, \xi_p) \\ \Rightarrow E_p &= \frac{1}{3}h^3 f'''(t_k, \xi_p) \end{aligned} \quad (9)$$

The following table indicates the prediction error, which indicates the following:

$$y(t_{k+1}) = y_{k+1}^0 + \frac{1}{3}h^3 f'''(t_k, \xi_p) \quad (10)$$

3.2.2 Calibration Error

Similarly, available

$$\begin{aligned} y(t_{k+1}) &= y_{k+1}^m - \frac{1}{12}h^3 f'''(t_k, \xi_c) \\ \Rightarrow E_c &= -\frac{1}{12}h^3 f'''(t_k, \xi_c) \end{aligned} \quad (11)$$

3.3 Step control criteria

Estimated with errors (8)-(11)

$$E = \frac{y_{k+1}^0 - y_{k+1}^m}{5} = -\frac{1}{12}h^3 f'''(t_k, \xi) \quad (12)$$

If $|E|$ is greater than the previously specified error accuracy, then decrease the step size.

3.4 Further extension of the linear multi-step method

In integral formula,

$$\begin{aligned} \int_{y_{k-n}}^{y_{k+1}} y' dy &= y(t_{k+1}) - y(t_{k-n}) = \int_{t_{k-n}}^{t_{k+1}} f(t, y(t)) dt \\ \Rightarrow y_{k+1} &= y_{k-n} + \int_{t_{k-n}}^{t_{k+1}} f(t, y(t)) dt \end{aligned} \quad (13)$$

By replacing the integrand $f(t, y(t))$ with the n -degree interpolation polynomial $P_n(t)$ of $n+1$ nodes $(t_i, f_i(t_i, y_i))$, $i = k-n, k-n-1, \dots, k+1$ the general linear multi-step method is obtained.

As when $n = 1$,

$$y_{k+1} = y_{k-1} + 2hf(t_k, y_k) \quad (14)$$

As when $n = 2$.

In particular, by replacing the integrand $f(t, y(t))$ with the cubic interpolation polynomial $P_3(t)$ at four nodes $(t_i, f_i(t_i, y_i))$, $i = k-3, k-2, k-1, k$ we get the following:

$$y_{k+1} = y_k + \frac{h}{24}(55f_k - 59f_{k-1} + 37f_{k-2} - 9f_{k-3}) \quad (15)$$

The four-node $(t_i, f_i(t_i, y_i))$, $i = k-2, k-1, k, k+1$ and cubic interpolation polynomial $P_3(t)$ are substituted and we get the following [8]:

$$y_{k+1} = y_k + \frac{h}{24}(9f_{k+1} + 19f_k - 5f_{k-1} - f_{k-2}) \quad (16)$$

The errors are $\frac{251}{720}h^5 y^{(5)}(\xi)$ and $-\frac{19}{720}h^5 y^{(5)}(\xi)$, respectively. The accuracy of formula (16) is higher than that of formula (15).

Adams Forecast-Correction Formula:

$$\begin{cases} \bar{y}_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) \\ y_{n+1} = y_n + \frac{h}{24}(9\bar{f}_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}) \end{cases} \quad (17)$$

Among them, $f_n(t_n, y_n)$, $\bar{f}_{n+1}(t_{n+1}, \bar{y}_{n+1})$.

Note. Formula (18) cannot provide the three initial values required by itself and requires the cooperation of Runge-Kutta method.

4 Sanda movement of the lower limb power robot model

Adams software has strong analysis capabilities, but Adams' modelling capabilities are generally not suitable for the construction of complex models; Pro/E has powerful modelling capabilities, which can build a

variety of complex models, but Pro/E cannot perform complex kinematics and dynamics simulations. Combining the advantages of both, this paper uses Pro/E to model and then performs simulation analysis in Adams. The combined model of Pro/E and Adams is a process of continuous improvement. The 3D solid modelling includes two phases of part modelling and general assembly. According to the structural characteristics and functional requirements of the training robot, the modelling and assembly of each part are completed, and a three-dimensional solid model of the training robot is obtained. Through the interface program Mechanic/Pro between Pro/E and Adams software, rigid body and partial constraints are defined, and then, the 3D solid assembly model created by Pro/E is converted into Adams, which further improves the Adams dynamics model. The steps are shown in Figure 2 [9].

Fig. 2 Flow chart of modelling.

4.1 Pro/E modelling

Pro/E was used to model the mechanical structure of the new Sanda lower limb powering robot. Figure 3 shows the parts model of the training robot, and Figure 4 shows the assembly model in Pro/E.

4.2 Adams virtual prototype modelling

The Pro/E model is imported into Adams, and the Adams model is generated through interface software of mechpro2005 of Adams and Pro/E [10], and the simulation analysis is performed in Adams. Model transformation includes three steps: defining rigid body, defining constraints and model transformation. Add necessary constraints and forces to the system in the Adams environment and improve the imported Adams model. The necessary forces and drives are added to define the reference quantities of the material density, stiffness, elastic deformation and acceleration of gravity of each rigid body. Finally, the interaction forces between various rigid bodies are defined, including the contact force between the hub and the ground, the acceleration of gravity and the motion drive. The built Adams dynamic model is shown in Figure 5.

The specific statistics of each component, rigid body and constraint involved in the modelling process are shown in Table 1.

Fig. 3 Part models of rehabilitative robot.

Fig. 4 Rehabilitative robot model in Pro/E.

Fig. 5 Adams model.

5 Adams dynamics simulation and analysis

5.1 Rotation simulation

Given an omnidirectional wheel angular velocity of $\pi 6$ rad/s, the curve of omnidirectional wheel angle change in the rotating state is shown in Figures 6, and 7 shows the robot trajectory curve and rotation angular velocity curve. Figure 6 samples the angular velocity of each wheel. Since the simulation purpose is to analyse the relationship between the angular velocity of each wheel and the robot trajectory, this sampling is used as a

Table 1 Modelling quantity statistics

Name	Quantity	Note
Components	42	1 main frame, 4 motor components, 4 main wheels, 32 hubs, 1 ground
Defining rigid bodies	43	1 main frame, 4 motor components, 4 main wheels, 32 auxiliary wheels, 1 ground, 1 rigid body by default
Constraint	37	36 rotating pairs, 1 locking pair
Contact force	32	Is the force between the auxiliary wheels (32) and the ground
Motion driver	4	Main wheels (4) rotation drive
Gravitational acceleration	1	y-direction acceleration

known input. Figure 7 reflects the change of the centre of gravity of the robot in space during the rotation. On the horizontal plane, the centre of gravity of the robot exhibits a sinusoidal change on the x and z axes, which meets the system control requirements. There is a 1 mm disturbance on the y-axis change, indicating that there is a vibration phenomenon during the rotation of the robot waiting for buffering [11]. Judging from the maximum partial velocity in the three directions of X, Y and Z, each joint has the largest X direction, the Z direction is second and the Y direction is the smallest. For example, for the toe, the maximum speed in the X direction is 13.02 m/s, the Z direction is 11.55 m/s and the Y direction is only 7.09 m/s. Another example is the knee joint, which is also the largest in the X direction, with a maximum average speed of 6.43 m/s, followed by the Z direction with 5.09 m/s and Y direction with only 4.19 m/s.

Fig. 6 Angle of omni-directional wheel vs. time under rotating state.

Fig. 7 Trajectory and angular velocity of the robot under a rotating state.

5.2 Walking forward simulation

Given the angular velocity of the omnidirectional wheel as $\pi 6$ rad/s, the curve of the omnidirectional wheel angle in the forward running state, that is, the input curve, is shown in Figure 8. The trajectory and speed curve of the robot in the forward state, that is, the change curve of the centre of gravity of the robot corresponding to Figure 7, are obtained under a rotating state. It can be seen from the figure that among the velocity components, only the Y direction (lateral or striking direction) has the highest velocity value now on striking, and the maximum values of other components are before the striking point. Obviously, the Y direction is the effective strike direction. Now of the strike, it is necessary to strike at the maximum speed value in order to produce a crisp striking sound or to achieve the purpose of damaging the opponent, and the other directions are ineffective speed directions. From the perspective of kinematics, the magnitude of other velocity components is only meaningful for shortening the swing time but not significant for the effectiveness of the strike. Therefore, in order to improve the effectiveness of the strike, the Y direction (side leg direction) and the maximum speed should be in the direction of the athlete's efforts.

Fig. 8 Angle of omni-directional wheel vs. time under advancing state.

Fig. 9 Trajectory and velocity of robot under an advancing state along z axis.

Figure 9 reflects the disturbance in the x-axis direction during the starting process. The maximum peak-to-peak value of the disturbance is 4 mm, and it becomes gentle after 15 s. According to the analysis, the cause of the disturbance is that the centre of gravity of the robot and its geometric centre are not in the same vertical direction, and the forces on the wheels are uneven when starting; it reflects the 1 mm disturbance on the y-axis change; the z-axis travels at a constant speed. Since the disturbance peaks sampled in Figure 9 are relatively small for the entire system, they have little effect on the overall system effect. Now of impact, the three velocity components of the hip and knee joints are relatively small, all < 2 m/s. Among the X, Y and Z speed components of the ankle joint point, the speed in the Y direction is the smallest when it hits 4.03 m/s, while the speed component values in the X and Z directions have no significant difference ($P > 0.05$). The average is about 5.4 m/s. At the toes, there was no significant difference between the three speeds of X, Y and Z ($P > 0.05$), as now of on impact, with an average of about 7 m/s.

According to the dynamic simulation results obtained in Adams, it can be analysed that the centre of gravity of the robot is not at its geometric centre, which will certainly cause some interference to the control of the robot. Improving the design structure to make it more symmetrical can reduce this tendency of interference. From the perspective of the dynamic mathematical model analysis, Adams simulation results are consistent with the dynamic mathematical model. When the given input is the same, the dynamic simulation of the design is consistent with the mathematical model results, indicating that the design structure meets the training requirements.

6 Conclusion

The correspondence between wheel speed and trajectory was derived from the perspective of mathematical modelling. The feasibility of designing a lower limb power robot for wheeled Sanda is analysed by the method of joint modelling of Pro/E and Adams, and the consistency between the structural design and the mathematical model is verified. Adams kinematic analysis shows that the design meets the training requirements of lower limb strength training in Sanda, and a corresponding improvement plan is proposed for the shortcomings in the design. The vibration of the system is reduced by adding cushioning structures such as cushions; the symmetry of the system is improved to eliminate the trajectory shift caused by uneven force between the wheels.

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