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Financial accounting measurement model based on numerical analysis of rigid normal differential equation and rigid generalised functional equation

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Abstract

In order to solve the problems of financial accounting measurement quickly and accurately, this paper starts the analysis from the perspective of mathematics and finance and establishes the differential equation and the generalised functional equation for the related numerical analysis through mathematical knowledge. The results show that the limit and integral of rigid differential equation and the rigid generalised functional equation can improve their role and status in the financial accounting measurement environment so that they can be more widely used in the financial accounting measurement environment and promote the development of the financial accounting environment.

Keywords: rigid differential equation, rigid general functional subequation, numerical analysis, the financial accounting.**AMS 2010 codes:** 97B10

1 Introduction

With the expansion of the application field of the differential equation, it becomes very important to study the solution of the differential equation. In recent years, many famous mathematicians in the world have made more in-depth and extensive research on rigid differential equations and rigid generalised functional equations by analysing theories and methods. The solution of the differential equation not only contains a component attenuation that is very quick and contains a relatively slow weight change, when they try to change in the

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solution of the slow interval value to solve these problems, even as attenuation of the fast varying component values are insignificant, but also the disturbance of rapid change still seriously affects the stability and accuracy of the numerical solution, creating a lot of real difficulties for the calculation. These processes are called ‘rigid’, and the initial value problems of ordinary differential equations describing these processes are called ‘rigid’.

The research on rigid differential equations has been conducted for >200 years, and the research object is mainly in the pure theory field of financial accounting. With the widespread application of differential equations and generalised functional equations, the research process has gradually attracted the attention of the general financial accounting field. In recent decades, differential equations have achieved breakthrough development in the fields of biology, rheology, chemical data processing, signal processing identification, control theory and financial accounting. Their main principle is to convert them into equivalent integral equations using Green’s function, and when the nonlinear term satisfies certain conditions, we use the property of noncompactness measure and the fixed point theorem to prove the related problems of the solution, and the main idea used is the idea of transformation. The new results are of great significance in the theory and practical application of financial accounting. Differential boundary value problem is one of the most active research fields in analytical mathematics, which originated from various applied disciplines. A large number of physical models and natural phenomena can be simplified to solve the boundary value problem of the equation. Therefore, it is particularly important to study the boundary value problems of differential equations and generalised functional equations [1].

Researchers first coined the term ‘rigid’, and they were the first to come up with a class of numerical methods known as the BDF method. It is important to note that the BDF method they proposed was very mature at an early stage and is still an essential part of many highly successful numerical methods for some rigid problems. Since 1952, numerical methods for rigid problems have been widely used in the field of financial accounting, especially in the last two decades. Hunderts’ paper not only solves the problem of constructing efficient algorithms for financial accounting measurement but also provides financial accounting theoretical analysis for these algorithms. A large number of remarkable results have been obtained in the study of rigid differential equation algorithm theory. The early literature on rigidity focused on the linear stability of numerical algorithms in financial accounting, that is, the numerical stability of the scalar model equation $Y' = \lambda Y$, ($\lambda \in \mathbb{C}$) is studied. The related stability concepts include A – stability, A (α) – stability and rigid stability, etc. These theories and concepts based on linear model equations have been successfully used to guide more complex numerical problems, but from a strict mathematical point of view, these theories can only be applied to linear autonomous systems with constant coefficients, and cannot be used as the theoretical basis for studying the stability of numerical methods for rigid nonlinear problems. Moreover, the convergence analysis of numerical algorithms for rigid problems is even neglected in the early literature.

Two typical methods are the backward differential formula and the Runge-Kutta method when dealing with rigid problems. Backward differential formulas are particularly applicable to rigid differential equations and differential algebraic equations. The earliest methods of BDF can be traced back to 1952 by Yan Y and Loureno I although the name of the BDF method was not given at that time. However, for the rigid problem, the BDF method shows outstanding stability properties that allow larger step sizes compared with the explicit method [2,3]. Especially since Badri gave a classical introduction in 1971, the BDF method has attracted more attention from many scholars. Badri M proposed that the first and second-order methods of backward differential formula meet the A-stability and L-stability conditions. All methods of order 3 to order 6 satisfy the weaker A (α)-stability and rigid stability conditions, and there is no available BDF formula higher than order 6 [4].

And all the formulas in BDF format are extremely stable at infinity. Although the convergence order to the BDF method is limited, it has outstanding advantages, namely high practical calculation efficiency, extreme stability at the point of infinity leads to the rapid attenuation of the error of the rigid component. This makes the BDF method known as the numerical method that has long been used to solve the rigid problem. For example, Xia Y Q et al. constructed A cyclic linear method that satisfies A (α)-stability from order 1 to order 7 in 1978, which is more stable than the BDF method [5]. In addition, the EBDF method proposed by Portioli F in 1980

and the modified EBD method proposed by Portioli F in 1983, the methods of order 2 to order 4 are A-stable, and the methods of order 5 to order 9 are A (α)-stable. Its stability is better than that of the BDF method, and its disadvantages are, in other words, the actual calculation amount of each step is larger than BDF, but the overall effect is better than BDF [6]. Swann J D systematically studied the stability of rigid V Olterra type functional differential equations and the shrinkage, asymptotic stability, B-stability and B-convergence of Runge-Kutta method and general linear method [7]. Chang M et al. obtained the conditions of stability and asymptotic stability of the Runge-Kutta method for solving nonlinear neutral delay integral and differential equations [8]. Plakisiy et al. solved the Runge-Kutta method, Pouzet-Runge-Kutta method and the asymptotic stability of the single-branch method for a class of nonlinear neutral delay integral-differential equations [9]. Koriga S et al. solved the stability of numerical solutions of several types of nonlinear neutral functional integral and differential equations [10].

2 Based on the general theory of rigid functional differential equations

Let X be a real or complex Banach space and $\|\bullet\|$ denote the norm therein. For any closed interval, the symbol $C_X(I)$ represents the Banach space formed by all continuous mappings $x: I \rightarrow X$, and the norm is defined as $\|x\|_\infty = \max_{t \in I} \|x(t)\|$. Consider the initial value problem

$$\begin{cases} y'(t) = f(t, y(t), y(\cdot)), & 0 \leq t \leq T, \\ y(t) = \phi(t), & -\tau \leq t \leq 0, \end{cases} \quad (1)$$

Here T, τ is a real constant, $T > 0, \tau \geq 0, \phi \in C_X[-\tau, 0]$, is a given initial function, $f: [0, T] \times X \times C_X[-\tau, T] \rightarrow X$ is a given continuous mapping that satisfies this condition

$$\begin{cases} (1 - \alpha(t)\lambda)G_f(0, t, u, v, \psi) \leq G_f(\lambda, t, u, v, \psi) \\ \forall \lambda \geq 0, t \in [0, T], u, v \in X, \psi \in C_X[-\tau, T], \\ \|f(t, u, \psi_1) - f(t, u, \psi_2)\| \leq \beta(t) \max_{t - \mu_2(t) \leq \xi \leq t - \mu_1(t)} \|\psi_1(\xi) - \psi_2(\xi)\| \\ \forall t \in [0, T], u \in X, \psi_1, \psi_2 \in C_X[-\tau, T], \end{cases} \quad (2)$$

where $\alpha(t)$ and $\beta(t)$ are continuous functions, the functions $\mu_1(t)$ and $\mu_2(t)$ satisfy the inequality $0 \leq \mu_1(t) \leq \mu_2(t) \leq t + \tau \quad \forall t \in [0, T]$.

Note that for the special case where X is a Hilbert space, condition (2a) is equivalent to the one-sided Lipschitz condition. We assume that there exists a true solution $y(t), 0 \leq t \leq T$ for problem (1), and denoted by $z(t)$ the solution of the perturbation problem obtained by replacing the initial function $\phi(t)$ with $\psi(t)$ in problem (1). We obtained the following results from the paper.

Theorem 1.1. *Problem (1) can have at most one solution.*

Theorem 1.2. *Stability inequalities*

$$\begin{aligned} \|y(t) - z(t)\| &\leq \exp(ct) \max_{-\tau \leq \xi \leq 0} \|\phi(\xi) - \psi(\xi)\| \\ \forall t &\in [0, T] \end{aligned} \quad (3)$$

That's true, right there

$$c = \left[\sup_{0 \leq t \leq T} (\alpha(t) + \beta(t)) \right]_+$$

Here and hereafter, we define

$$[x]_+ = \begin{cases} x & x > 0, \\ 0 & x \leq 0. \end{cases}$$

The following two theorems apply to the case where the integral interval of problem (1) is $[0, +\infty)$.

Theorem 1.3. When $\sup_{0 \leq t \leq +\infty} \alpha(t) < 0$, $\sup_{0 \leq t \leq +\infty} (\beta(t)/|\alpha(t)|) < 1$, for any given constant $\mu > 0$, there exists a sequence t_k that diverges strictly incrementing to $+\infty$ $\{t_k\}, k = 0, 1, 2, \dots$, where $t_0 = 0$ is such that

$$\max_{t_k \leq t \leq t_{k+1}} \|y(t) - z(t)\| \leq C_\mu^k \max_{-\tau \leq \xi \leq 0} \|\phi(\xi) - \psi(\xi)\|, k = 0, 1, 2, \dots, \quad (4)$$

Here,

$$\begin{cases} C_\mu = v + (1 - v) \exp(\alpha_0 \mu) \in (0, 1), \\ v = \sup_{0 \leq t \leq +\infty} (\beta(t)/|\alpha(t)|) < 1, \alpha_0 = \sup_{0 \leq t \leq +\infty} \alpha(t) < 0. \end{cases}$$

Theorem 1.4. Under the assumption of Theorem 1.3, there is

$$\lim_{t \rightarrow +\infty} \|y(t) - z(t)\| = 0. \quad (5)$$

These results represent the stability, generalised shrinkage and asymptotic stability of solutions of rigid functional differential equations. This work provides a unified theoretical basis for the stability analysis of solutions of rigid ordinary differential equations, rigid delay differential equations, rigid integral differential equations and other functional differential equations encountered in practical problems and is also the theoretical basis for the study of numerical methods of rigid functional differential equations [11].

As one of the special cases of the above general theory, for a nonlinear rigid delay differential equation with a single delay $\tau(t)$ in a finite dimensional Euclidean space C^m , in 1989, Torelli showed that the stability inequality (3) holds for $\sup_{0 \leq t \leq +\infty} (\alpha(t) + \beta(t)) \leq 0$. Zennaro previously obtained asymptotic stability results in 1997 (5). However, in addition to all the assumptions in the corresponding theorem mentioned above, their results also require the following more stringent additional assumptions.

There exists a constant $\tau_0 > 0$ such that

$$\tau(t) \geq \tau_0 \quad \forall t \in [0, +\infty);$$

$t - \tau(t)$ is a strictly increasing function on the interval $[0, +\infty)$.

Thus it can be seen that even for the well-studied differential equations at home and abroad, the results directly derived from the above general theory are more general and profound than the existing results.

3 Efficient numerical methods for rigid differential equations and rigid general-functional subequations

This section recommends several classes of numerical methods that can be used to solve rigid Volterra functional differential equations with good performance in all aspects.

(1) Backward Euler method

$$\begin{cases} y^h(t) = \Pi^h(t; \psi, y_1, y_2, \dots, y_{n+1}), -\tau \leq t \leq t_{n+1}, \\ y_{n+1} = y_n + h_n f(t_{n+1}, y_{n+1}, y^h). \end{cases} \quad (6)$$

This simplest and oldest method is B-stable, B-compatible, first-order best B-convergent, extremely stable at ∞ , has the simplest structure, the smallest amount of computation and storage and can use piecewise constant interpolation or linear interpolation. The only drawback is minimal first-order accuracy. It is suitable for the problem where the calculation accuracy is not high.

(2) Trapezoidal method

$$\begin{cases} y^h(t) = \Pi^h(t; \psi, y_1, y_2, \dots, y_{n+1}), -\tau \leq t \leq t_{n+1}, \\ y_{n+1} = y_n + h_n (f(t_n, y_n, y^h) + f(t_{n+1}, y_{n+1}, y^h))/2. \end{cases} \quad (7)$$

Its parent method is A-stable, weak stable at ∞ , classical order of convergence 2. This method has the same calculation speed as the backward Euler method, and linear interpolation can be used. For linear problems with constant coefficients, numerical instability may occur when solving nonlinear strong rigidity problems with a variable step size.

Both the backward Euler and trapezoid methods are bootstrapping and can be used to provide additional starting values for the following numerical methods.

(3) Second-order BDF method

$$\begin{cases} y^h(t) = \Pi^h(t; \psi, y_1, y_2, \dots, y_{n+2}), -\tau \leq t \leq t_{n+2}, \\ 3y_{n+2} - 4y_{n+1} + y_n = 2hf(t_{n+2}, y_{n+2}, y^h). \end{cases} \quad (8)$$

It is B-stable, B-compatible, second-order best B-convergent, extremely stable at ∞ , and has the same computational speed as the backward Euler method. Linear interpolation can be used. This method is generally more robust than the trapezoidal method and has better computational results, but it requires an additional starting value.

(4) One-to-third-order parallel multi-valued hybrid method

The k order parallel multi-valued mixed method for rigid ordinary differential equation is defined as

$$\begin{cases} y_{n+k} = \sum_{j=0}^{k-1} \alpha_j y_{n+j} + h\beta_k f(t_{n+k}, y_{n+k}) + h\beta_v f(t_{n+v}, y_{n+v}) \\ y_{n+v} = \sum_{j=-1}^{k-2} \alpha_j^* y_{n+j} + \alpha_v^* y_{n+v-1} + h\beta_v^* f(t_{n+v}, y_{n+v}) \end{cases} \quad (9)$$

The coefficients are determined by a system of equations

$$\begin{cases} \sum_{j=0}^{k-1} \alpha_j = 1, \sum_{j=-1}^{k-2} \alpha_j^* + \alpha_v^* = 1, \\ \sum_{j=0}^{k-1} j^p \alpha_j + pk^{p-1} \beta_k + pv^{p-1} \beta_v = k^p, p = 1, 2, \dots, k, \\ \sum_{j=-1}^{k-2} j^p \alpha_j^* + (v-1)^p \alpha_v^* + pv^{p-1} \beta_v^* = v^p, p = 1, 2, \dots, k, \end{cases}$$

The first- to third-order methods are A-stable and B-compatible, extremely stable at ∞ , with very small error constants, and for large-scale rigid problems have roughly the same computational speed as the backward Euler method when computing in parallel with two processors. Equipped with regular interpolation operators of the same order, these methods can be used to solve rigid Volterra functional differential equations. We still call them parallel multi-valued mixed methods. A large number of numerical experiments show that the computational efficiency of this numerical method is obviously better than that of other common algorithms of the same order; so it is worth popularising and using. But they are multivalued methods and require an additional starting value to start with.

(5) Implicit multi-step Runge-Kutta method with real eigenvalues of third- or fourth-order

They are B-stable, B-compatible, B-convergent and extremely stable at ∞ . Parallel computations for large-scale rigid problems have roughly the same computational speed as the backward Euler method, but additional starting values are needed. Because of its excellent stability, it is suitable for the high-precision calculation of nonlinear rigid problems.

Since the calculation speed of single-step fully hidden Runge-Kutta method above level 2 is relatively slow, for example, the number of computations for each integration step of the second-order and third-order one-step Radau2A Runge-Kutta method under Butcher transform is at least four or five times as much as that of the backward Euler method, respectively, and so we do not recommend the use of such methods. We also do not recommend the single-step fully implicit continuous Runge-Kutta methods. Although they are in practice from the beginning, they not only have the disadvantages of the single-step fully implicit Runge-Kutta methods mentioned above but also cause order reduction due to interpolation with internal level values.

4 Numerical experiments of rigid differential equations and rigid general-functional equations

The initial boundary value problem of one-dimensional parabolic differential equations with a delay term is considered

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} = 2e^{-t} - e^{t/2}u(x, \frac{t}{2}) + \int_0^t u(x, \tau) d\tau, \\ 0 < x < 1, 0 < t \leq 1, \\ u(0,t) = u(1,t) = 0, & 0 \leq t \leq 1, \\ u(x,0) = x - x^2, & 0 < x < 1, \end{cases} \quad (10)$$

The problem has a unique true solution to $u(x,t) = (x - x^2)e^{-t}$. Therefore, we can replace the second-order spatial derivative with the second-order central difference quotient in the network domain $x_i = i/N, i = 1(1)N - 1$; there is no local truncation error, where N is any given natural number. Denote $u_i(t) = u(x_i, t), \Delta x = 1/N$, thus obtaining the semi-discrete approximation of problem (10)

$$\begin{cases} \frac{du_i(t)}{dt} = \frac{1}{\Delta x^2}(u_{i-1}(t) - 2u_i(t) + u_{i+1}(t)) + 2\exp(-t) \\ \quad - \exp(\frac{t}{2})u_i(\frac{t}{2}) + \int_0^t u_i(\tau) d\tau, \\ u_0(t) \equiv u_N(t) \equiv 0, \\ u_i(0) = i\Delta x(1 - i\Delta x), i = 1, 2, \dots, N - 1, \end{cases} \quad (11)$$

Make

$$f(t, u, \psi(\cdot)) = [f_1(t, u, \psi(\cdot)), f_2(t, u, \psi(\cdot)), \dots, f_{N-1}(t, u, \psi(\cdot))]^T,$$

Among them

$$\begin{aligned} f_i(t, u, \psi) &= \frac{1}{2\Delta x^2}(u_{i+1} - 2u_i + u_{i-1}) + 2e^{-t} - e^{t/2}\psi_i(\frac{t}{2}) + \int_0^t \psi_i(\tau) d\tau, \\ \psi &= [\psi_1, \psi_2, \dots, \psi_{N-1}]^T \in C_{R^{N-1}}[0, 1], u_0 = u_N = 0. \end{aligned}$$

Then problem (11) can be written in the form of initial value problem (1) of Volterra functional differential equation in space R^m ; according to the standard inner product, it satisfies the condition (2)'; here, $m = N - 1, \tau = 0, T = 1, \alpha = -4N^2 \sin^2 \frac{\pi}{2N} \cong -\pi^2$ (when N is large), $\beta = 1 + \sqrt{e}$ (here e is the base of the natural log). Therefore, the B theory of the numerical method of Volterra functional differential equations introduced in Section 2 of this paper can be directly applied to this special case, and various efficient numerical methods recommended in Section 3 of this paper can be as well used to solve the problem (11) [12].

As an example, we consider the second-order and third-order real eigenvalue implicit multistep Runge-Kutta method (RMRK3) for solving rigid ordinary differential equations proposed in the literature.

$$\begin{cases} Y = hC_{11}F(Y) + C_{12}y^{(n)}, \\ y_{n+1} = h\gamma^T \tilde{F}(Y) + \alpha^T y^{(n)}, \end{cases} \quad (12)$$

Here

$$\begin{cases} y^{(n)} = [y_{n-2}^T, y_{n-1}^T, y_n^T]^T \approx [y(t_{n-2})^T, y(t_{n-1})^T, y(t_n)^T]^T, \\ Y = [y_1^T, y_2^T]^T \approx [y(t_n + \mu_1 h)^T, y(t_n + \mu_2 h)^T]^T, \\ \tilde{F}(Y) = [f(t_n + \mu_1 h, Y_1)^T, f(t_n + \mu_2 h, Y_2)^T]^T, \\ C_{11} = [c_{ij}^{11}] \in R^{2 \times 2}, C_{12} = [c_{ij}^{12}] \in R^{2 \times 3}, \\ \gamma^T = [\gamma_1, \gamma_2], \alpha^T = [\alpha_1, \alpha_2, \alpha_3], \end{cases}$$

Among them

$$\begin{cases} c_{11}^{11} \approx 0.915809216730669, c_{12}^{11} \approx -0.453336810549659, \\ c_{21}^{11} \approx 0.0149519583665559, c_{22}^{11} \approx 0.741962909796344, \\ c_{11}^{12} \approx 0.193708609271523, c_{12}^{12} \approx 0, c_{13}^{12} \approx 0.806291390728476, \\ c_{21}^{12} \approx -0.121542565918549, c_{22}^{12} \approx 0, c_{23}^{12} \approx 1.12154256591855, \\ \gamma_1 \approx 0.653011933174220, \gamma_2 \approx 0.398988066825780, \alpha_1 = 0.026, \\ \alpha_2 = 0, \alpha_3 = 0.974, \mu_1 \approx 0.075055187637962, \mu_2 = 1. \end{cases}$$

These coefficients are calculated according to the techniques provided and the relevant formulae in the literature. In these two papers, it is proved that the RMRK3 method is algebraically stable, diagonally stable, with order 2 and weak order 3. It is easy to know from B theory that the numerical method of Volterra functional differential equation obtained by assembling piecewise quadratic polynomial interpolation operator is B-stable, B compatible and has the best B convergence order 2 and B convergence order 3. For convenience, we still denote the obtained method as RMRK3. In the actual calculation of each integral step, the linear equations caused by the simplified Newton iteration can be decomposed into two independent low-order linear equations by Butcher transform for solving, which can be either serially calculated or parallel calculated by two processors to improve the computational efficiency.

In addition, for comparison, we also consider Volterra's Dirk3 method for functional differential equations, which is derived from the well-known 2nd and 3rd order single-step diagonal implicit Runge-Kutta method for solving rigid ordinary differential equations

$$\begin{cases} Y_1 = y_n + h\gamma f(t_n + \gamma h, Y_1) \\ Y_2 = y_n + h[(1 - 2\gamma)f(t_n + \gamma h, Y_1) + \gamma f(t_n + (1 - \gamma)h, Y_2)] \\ y_{n+1} = y_n + \frac{h}{2}[f(t_n + \gamma h, Y_1) + f(t_n + (1 - \gamma)h, Y_2)] \end{cases} \quad (13)$$

Here $\gamma = (3 + \sqrt{3})$ is assembled by piecewise quadratic polynomial interpolation operator. Note that the Dirk3 method for functional differential equations is neither B-stable nor B-convergent, since its parent method (13) is only A-stable and classically convergent to the third-order.

In order to make a fair and reasonable comparison, the two methods are used to solve the problem (11) ($N=5000$) on the same Dell Optiplex GX270 computer with the same fixed step size h , and both are calculated in a serial way, and the simplified Newton iteration is used to solve the nonlinear equations. The direct method is used to solve the linear equations, and the same iteration termination strategy is adopted. The maximum overall error E and the calculation time T of the obtained numerical results can be seen in Table 1. It can be clearly seen that the computational efficiency of the B-stabilised method RMRK3 is indeed higher than that of the DIRK3 method.

Table 1 Integral step size h , corresponding maximum overall error e and calculation time t

Integral step/ h	RMRK3		DIRK3	
	e	t (s)	e	t (s)
1/200	1.70e-8	7.20	2.57e-7	6.8
1/400	2.18e-9	14.5	4.26e-8	13.7
1/600	6.52e-10	21.7	1.48e-8	20.5
1/800	2.77e-10	29.8	6.93e-9	27.4
1/1000	1.42e-10	37.9	3.84e-9	34.2

RMRK3, real eigenvalue implicit multistep Runge-Kutta method.

Note that for large-scale rigid problems, the RMRK3 method can be computed in parallel with both processors, which is close to the backward Euler method and faster than any other commonly used multistage methods, such as the DIRK3 method, which is significantly faster because it can be computed only serially.

5 Conclusion

In this paper, based on the rigid normal differential equation and rigid general functional equation of financial accounting measurement research, effectively solving the numerical solution of the two equations and the solution of the equation are expressed; then the reliability and practicability of the two equations in financial accounting measurement are expressed.

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