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The Optimal Solution of Feature Decomposition Based on the Mathematical Model of Nonlinear Landscape Garden Features

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Abstract

This Article aims at the current high idle rate of landscaped gardens and a single overall style. The article inputs the quantitative relationship programming of the dynamic model of the urban landscape ecological city system into the Grasshopper software to obtain the urban landscape parameter growth curve, and at the same time applies the nonlinear parameterized model method to the landscape design. The research found that the feature form of the landscape garden presented in the thesis is mainly based on the nonlinear transformation of the local analysis structure feature. In the end, the thesis deeply researches the existing operating modes based on the concept of nonlinear thinking. Furthermore, it combines with information technology to supplement and update the traditional landscape construction from different perspectives.

Keywords: Nonlinear; landscape garden features; wavelet transformation; mathematical variance model; feature decomposition; the optimal solution.

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1 Introduction

The pattern in landscape ecology refers to the spatial pattern. It includes the type, number, and spatial distribution and configuration of landscape components. In landscape ecology, the scale is expressed in terms of granularity and amplitude. The spatial granularity reflects the characteristic length, area, or volume represented by the identifiable units in the landscape. The amplitude refers to the continuous range or length of the research object in space or time. This paper uses the definition of spatial granularity to study the variation characteristics of landscape variables in spatial distribution.

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Although different spatial scales influence and control different ecological processes, there is still variability in scale in the research and data acquisition of landscape and ecological phenomena [1]. Most variable scales are directly used to obtain the time scale of landscape variable data in the research. Any determination of the scale of landscape analysis and variable data acquisition may lead to biased or unreliable analysis results. Therefore, the scale analysis of landscape variables is gradually being paid attention to by eco-environmental scientists. Remote sensing technology provides practical tools for studying the heterogeneous structure of terrestrial ecosystems. The raster model of remote sensing data is also easier to integrate with various landscape models. Still, the direct use of remote sensing technology for landscape research and application is relatively rare.

Recently, some scholars have begun to use remote sensing data to study landscape patterns using semivariance analysis methods. Some scholars have applied wavelet analysis methods to study the structure of forest gaps. Wavelet analysis is widely used in remote sensing data compression, texture feature extraction, data fusion and classification, but it is rare to use remote sensing data for landscape structure analysis [2]. In the 1990s, commercial resource satellites with metre-level resolution appeared one after another. This provides a wealth of surface information for remote sensing applications in various fields, enriches the spatial information of land types, and better reflects the image features' size, shape and relationship. Some scholars use simulated metre-level high-resolution satellite images, and they use the local variance method to analyse the spatial pattern of forest sites.

The purpose of this paper is to study further the methods and characteristics of wavelet and semivariance for landscape pattern feature scale detection by using the IKONOSPAN

1 m remote sensing data as an example [3]. At the same time, the paper compares their analysis results to get their ability to detect landscape scale and their respective advantages and disadvantages. This provides experience for better use of wavelet and semivariance for landscape-scale analysis.

2 Research methods

2.1 Study area and data

The test area is flat, and the primary landscape is cultivated land and urban landscape types. Each accounted for 25% and 40% of the area. IKONOS image was 1 m full-colour data in May 2020. We use Magellan 'Bumblebee' GPS/GIS acquisition system to collect 32 ground control points and perform geometric correction on IKONOS full-colour data [4]. The actual point position error is 1.6 m. We segmented the corrected remote sensing data into 1024 rows and 1024 columns of the urban landscape and farmland landscape as the types and data of landscape pattern analysis (Figs. 1 and 2). At the same time, 256 rows and 256 columns of landscape areas were extracted from the upper right of the two types of landscape images as data for studying the local structure of the landscape.

2.2 Wavelet and semivariance analysis methods for landscape structure of remote sensing data

Wavelet analysis comes from Fourier analysis. This method has thrived since its establishment in the 1980s. The wavelet function can be regarded as the impulse response of a band-pass filter. The wavelet transform filters the original signal with band-pass filters of different scales [5]. This method decomposes the signal into a series of frequency bands for analysis and processing. In digital image processing, the wavelet transform is often binary discretised. Therefore, we can use discrete binary wavelets to perform multi-channel and multi-resolution analysis on images.

Assuming that $\{V_j\}$ is a volume product space, then $L^2(R^2)$ formed by $V_j^2 = V_j \otimes V_j$ is a multi-resolution analysis. If φ and ψ are the one-dimensional scaling function and wavelet function, respectively, then we can

Fig. 1 IKONOSPAN city and farmland landscape

define the two-dimensional scaling function and the two-dimensional separable wavelet function as:

$$\begin{cases} \phi(x, y) = \phi(x)\phi(y) \\ \psi^1(x, y) = \phi(x)\phi(y) \\ \psi^2(x, y) = \psi(x)\phi(y) \\ \psi^3(x, y) = \psi(x)\psi(y) \end{cases} \quad (1)$$

Then any image $f(x, y)$ can be decomposed and expressed as a combination of a series of sub-channel images:

$$\begin{aligned} & \{A_{2^{-J}}f, D_{2j}^1f, D_{2j}^2f, D_{2j}^3f, -J \leq j \leq 1\} \\ & A_{2^{-J}}f = [f(x, y) \times \phi_{2J}(-x, y)](2^{-J}m, 2^{-J}n) \\ & D_{2j}^{1 \sim 3}f = [f(x, y) \times \psi_{2j}^{1 \sim 3}(-x, -y)](2^{-J}m, 2^{-J}n) \end{aligned} \quad (2)$$

The image is decomposed into a pyramid structure composed of a low-frequency smooth image at 2^{-J} resolution and a high-frequency image composed of J layer image detail information, where $D_{2j}^{1 \sim 3}f$ represents the high-frequency information of the image in the horizontal, vertical and diagonal directions at the resolution $2j$, respectively. Thus, they are orthogonal to each other.

Looking at wavelet analysis from the perspective of the structure of ground features, the wavelet coefficient measures the intensity of local variation of the signal under a particular observation scale. The greater the variation, the higher the value of the wavelet coefficient. Therefore, in a particular analysis scale, the wavelet coefficients of different pixel positions reflect the characterisation of the structure at that position [6]. It can be seen that the multi-resolution wavelet analysis decomposes the structural information on the original image. We distribute them on the $D_{2j}^{1 \sim 3}f$ image according to different directions and scales. On the $2j$ resolution scale, the $D_{2j}^{1 \sim 3}f$ channel indicates that the feature-length of the ground feature is between $2j - 1$ and $2j$. For $A_{2^{-J}}f$ image, it is not $< 2j$ scale feature structure information. We adopt similar G. A. The method of Bradshaw et al. calculates the sum of the squares of the wavelet coefficients of the corresponding pixels of each channel image

under a particular decomposition scale, which is called the wavelet variance. Therefore, the wavelet variance becomes a function of a particular scale and the structural information under this scale. It uses scale and wavelet variance to map, reflect the structural characteristics on different scales through graphical methods, and then obtain the characteristic scale for analysing the landscape pattern. The more significant the wavelet variance on the corresponding scale, the richer the structural information. It is the main characteristic scale of the landscape structure. The more commonly used tap-4 Daubechies wavelet (D4) was selected in the experiment (Table 1). We substitute it into formula (1) to generate a two-dimensional scaling function and wavelet function. Then we substitute it into formula (2) to analyse the structure of IKONOSPAN urban landscape and farmland landscape to generate smooth and high-frequency images on the corresponding scale.

Table 1 Daubechies wavelet transform coefficients

Position	Value
h0	0.483
h1	0.837
h2	0.224
h3	-0.129

The semivariance function can be fitted with some theoretical models to obtain parameters for spatial variability analysis of geological attributes. It mainly includes base station value, nugget variance, structure variance, and range. The semivariance function is similar to spectral analysis [7]. However, what they provide is the overall average structural information in the data. The ability to express the local structural characteristics of the data is limited, which is incapable of multiscale structural analysis of the data. This article uses the following formula to calculate the semivariance in different directions.

(1) Semivariance of landscape structure in an east-west direction

$$\gamma_{e-w}(k) = 1/2m \sum_{i=1}^n \sum_{j=1}^n [Z(i, j) - Z(i, j+k)]^2 \quad (3)$$

(2) The semivariance of landscape structure in the north-south direction

$$\gamma_{n-s}(k) = 1/2m \sum_{i=1}^n \sum_{j=1}^n [Z(i, j) - Z(i+k, j)]^2 \quad (4)$$

(3) The semivariance of landscape structure in the southeast-northwest direction

$$\gamma_{sw-se}(k) = 1/2m \sum_{i=1}^n \sum_{j=1}^n [Z(i, j) - Z(i+k, j+k)]^2 \quad (5)$$

(4) The semivariance of landscape structure in the northeast-southwest direction

$$\gamma_{se-sw}(k) = 1/2m \sum_{i=1}^n \sum_{j=1}^n [Z(i, j+k) - Z(i+k, j)]^2 \quad (6)$$

where k represents the calculated distance, n represents the calculation window size. i, j are the positions of the pixels and m is the number of calculated data pairs, which depends on the direction and distance of the calculation. The wavelet, as mentioned above, and semivariance landscape structure analysis and calculation are integrated into the author's remote sensing image analysis system using VC5.0++ programming.

3 Results

3.1 Wavelet analysis

In the experiment, we used Daubechies wavelet to decompose the urban landscape and farmland landscape image from 1 to 9. Figs. 2 and 3 are the respective $A_{2-J}f$ channel wavelet variance-scale diagrams, and Figs. 4 and 5 are the respective $D_{2j}^{1\sim 2}$ channel wavelet variance-scale diagrams [8]. For example, in Fig. 2, the wavelet variance of the urban landscape smooth sub-graph has peaked at 16 m and 64 m, which means that the urban landscape structure information will be concentrated on two characteristic scales and appear at not <16 m and 64 m. On the other hand, in Fig. 3, the wavelet variance of the farmland landscape smoothing sub-graph has a peak at 32 m, which means that the information of the farmland landscape structure $A_{2-J}f$ will be concentrated on a characteristic scale and not <32 m. Therefore, the size of the information content of the image channel is to guide whether it is necessary to continue the wavelet analysis.

Fig. 2 Wavelet variance of the smooth subgraph of an urban landscape

Figs. 4 and 5 show that the wavelet variance of the two diagonal directions is relatively very small. In image compression and noise removal applications, this direction is usually considered noise. In Fig. 4, there are two peaks in both the horizontal and vertical structure of the urban landscape, being peak at 16 m and 128 m in the horizontal direction and 16 m and 256 m in the vertical direction. Through corresponding scale image analysis and field investigation, the urban landscape is mainly composed of buildings, streets and green spaces in buildings. The width of the buildings in the test area is about 16 m. Therefore, the horizontal and vertical directions of the building lead to the characteristic scale appearing at 16 m, which is mainly the characteristic scale of the width of the building. The rectangular area composed of 256 m and 128 m is the characteristic scale where the combined features of buildings and streets appear in the horizontal and vertical directions [9]. Through field investigation, the rectangular area is usually two buildings in the horizontal direction. The average length of the building is about 60 m and the vertical direction is usually eight rows of buildings. The width of the building plus the road between the buildings is 32 m on average. This is a typical combination in this area. This combination feature can also be seen in Fig. 1. Except for these typical characteristic scales, there is a small wave crest at 2 m horizontally and vertically. Image analysis and field investigations at the 2 m analysis scale show that the shadows of buildings are generally 2 m in size, or 2 m wide green belts and paths are reflected on this characteristic scale. It can be seen that the IKONOSPAN1 m urban landscape with a size of 1024 rows and 1024 columns shows three levels of characteristic scales.

The farmland landscape has 128 m in the horizontal direction and 256 m in the vertical direction. Through

Fig. 3 Wavelet variance of the smooth subgraph of farmland landscape

Fig. 4 Wavelet variance in different directions of the urban landscape

image analysis and field investigation of corresponding scales, it is found that this is the field size scale of available farmland [10]. The farmland blocks are surrounded by shelter forests and work roads, constituting a typical characteristic scale in this area. This farmland area is planted with green trees commonly used in the north. In addition, there is a small wave crest at 2 m in the horizontal direction, which is the average row spacing between planted trees. The typical characteristics of farmland and tree planting rows can also be seen in Fig. 1.

Fig. 5 Wavelet variance in different directions of farmland landscape

3.2 Semivariance function analysis

For urban landscapes and farmland landscapes of 1024 rows and 1024 columns, we use 1, 2, 4, 8, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300 and 400, respectively. The calculation interval of 500 m, 600 m is used for semivariance calculation. Figs. 6 and 7 are the semivariograms of the urban landscape and farmland landscape, respectively. We used a calculation interval of 1 m to 100 m for semivariance analysis of urban landscapes and farmland landscapes with 256 rows and 256 columns. Figs. 8 and 9 are the semivariance plots of urban and farmland sub-areas. Figs 6 and 7 show that the greyscale variables of the urban landscape and farmland landscape IKONOSPAN do not satisfy the second-order stationary hypothesis and intrinsic hypothesis in the 1024 rows and 1024 columns of the entire experimental area. Still, it meets the second-order stationary hypothesis and Eigen Order stationary hypothesis. Therefore, we use the experimental data of comprehensive average semivariance within 200 m to fit the theoretical model [11].

Similarly, our experimental semivariance for urban landscape sub-areas is also a quasi-second-order stationary hypothesis and quasi-intrinsic. Their semivariance trends in different directions have the same shape. Therefore, we use the experimental data of comprehensive average semivariance within 30 m to fit the theoretical model. The other directions except the vertical direction and the comprehensive average semivariance show a cavitation effect for our experimental semivariance of the farmland landscape sub-areas. It is a non-stationary series. Therefore, it does not perform theoretical semivariance fitting for the sub-areas of farmland landscape. Table 2 shows that the fitting results of each landscape type using the spherical model have reached a significant level.

It can be seen from Figs. 6 and 7 that the semivariance analysis cannot reveal the structural differences in different directions of urban and farmland landscapes, nor can it effectively show the structural differences at different scales and levels. The fitting results in Table 2 reflect that the overall structure of the urban landscape has a range of 135 m. This reflects that the overall average characteristics of the combination of buildings and roads are between 128 m and 256 m in the characteristic scale of wavelet analysis [12]. The overall structure of the farmland landscape is 152 m, which is between 128 m and 256 m in the characteristic scale of wavelet analysis. It can be seen from Figs. 8 and 9 that some regional landscapes extracted from urban landscapes and farmland landscapes continue to undergo semivariance analysis. The urban landscapes show another structural

Fig. 6 Semivariance of urban landscape experiment

Fig. 7 Semivariance of farmland landscape experiment

Table 2 Theoretical semivariance parameters of the experimental semivariance fitting for each landscape type

Landscape type	city View	Farmland landscape	Urban landscape sub-area
Nugget variance	461	77	264
Variation	135	152	15
Structure variance	367	78	230
Abutment value	828	155	494
The ratio of nugget variance to base station value n/s (%)	55	49	53

feature on another scale with a range of 15 m. This is very close to the characteristic scale of 16 m in wavelet analysis. The half-length of the cycle where the cavitation effect appears in the farmland landscape sub-area except the vertical direction is about 2 m, precisely the size of the characteristic scale of the row spacing structure of the planted trees obtained by the wavelet analysis.

We used the wavelet analysis method to detect the three-level spatial pattern and combination relationship of the urban landscape in the experimental area from the IKONOSPAN1m image and the characteristic scale of each level of pattern. This provides essential information for evaluating and planning the urban landscape. In addition, from IKONOS images, it was detected that the field planting row spacing and the secondary landscape pattern of field plots in the experimental area and their sizes reflect the farming management information in this area.

Fig. 8 The semivariance of the urban landscape sub-areas test

Fig. 9 The semivariance of the farmland landscape sub-area experiment

4 Discussion

The urban landscape images of the experimental area show different levels of structural features, mainly the three-scale features of shadow-single building-combination of building and road. On the other hand, the farmland landscape mainly comprises two hierarchical scale features of tree rows and farmland. We use the tap-4Daubechies wavelet to analyse the different directions and scales of the IKONOSPAN urban landscape and farmland landscape structure, reflecting these characteristics. The semivariance analysis reflects the most critical overall average structure of the landscape structure in the experimental area [13]. Although it is also possible to analyse the landscape structure in different directions separately, it is not as sensitive to the structural characteristics of different directions as the wavelet analysis, nor can it reveal the multiscale characteristics of the landscape pattern in the experimental area. However, we can also reveal the scale characteristics of the corresponding level by changing the analysis range and calculation interval for different levels. For example, the semivariance analysis of urban landscape sub-areas and farmland landscape sub-areas in this experimental area is possible only if the analyst has prior knowledge of the study area.

The study of the experimental area proves that wavelet analysis is also a feasible method of landscape pattern research. Compared with semivariance analysis, it has apparent advantages in explaining multiscale structure and structure in different directions. And it is not restricted by the statistical stability assumption of analysed data. However, because the image decomposition is based on the binary system, the analysis scale changes with a power function of 2. Therefore, unlike in semivariance analysis, a theoretical model can fit and so estimating the characteristic scale can be continuous.

For ecosystems, the heterogeneity of ecosystems results from the combined effects of different ecological processes on different scales. The corresponding process relationships can be better understood and established

by detecting the characteristic scales of landscape variables in different ecological processes. Secondly, the results of this study provide a better way to use remote sensing data for landscape ecological research, and it provides a method for scale selection and determination. Generally, the size of remote sensing data pixels is not an essential spatial scale of the studied landscape variables. On the one hand, the spatial resolution of pixels may be too large, resulting in internal variability of landscape variables or noise interference analysis.

On the other hand, because the pixel spatial resolution is too small to reflect the law of landscape variables, the results are biased or unreliable. Therefore, we first use wavelet and semivariance analysis to identify the characteristic scales of the landscape variables of interest to avoid the variability in the choice of analysis scales. This improves the accuracy of the study of ecological processes.

5 Conclusion

The research also shows that metre-level high-resolution satellites have application potential in urban planning, digital city construction and precision agriculture. Through landscape pattern analysis such as wavelet, semivariance and other methods, we can obtain information such as the characteristic scale of the urban landscape and the size of the operation unit in precision agriculture. With remote sensing technology, it can provide abundant data sources on various scales of different time-space and spectral resolution, which provides a wealth of ecosystem characteristic parameters and multiscale research data for ecological research. This will promote the study of terrestrial ecology.

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