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Application of Forced Modulation Function Mathematical Model in the Characteristic Research of Reflective Intensity Fibre Sensors

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Abstract

In order to discuss the application and mode of the forced modulation function in a sensor, the optical fibre emphasis function was established by referring to the geometric method, the tilt factor and the shape factor of the reflecting surface. These were introduced for the first time, and the corresponding mathematical model was established. The method of numerical simulation is systematically studied and multimode optical fibre parameters (including optical fibre of axial spacing, optical fibre core diameter and numerical aperture) are adopted. The reflective surface inclination and shape factors on the RIM-FOS intensity modulation characteristics are studied according to the obtained light quasi-Gaussian distribution model, establishing a general three-intensity modulation function of fibre optic sensor. The results show that the intensity modulation characteristic of specular reflection is obviously better than that of the diffuse reflection surface, and the peak value of the modulation function is five times that of diffuse reflection. The intensity modulation characteristic decreases with increase in the roughness of the reflection surface. The system can not only complete the RIM-FOS characteristic simulation and characteristic testing functions, but can also start-up the test and not be affected by the ambient light interference and power fluctuation of the light source. The test stability is good with high repeatability.

Keywords: Optical fibre sensor; reflection intensity modulation; mathematical model.

AMS 2010 codes: 30D99

1 Introduction

According to the function of optical fibre, optical fibre sensors can be divided into two types: the sensing type, or functional fibre optic sensor; and the non-functional fibre optic sensor, or the optical fibre sensor. The optical fibre of the sensing fibre sensor not only transmits the light wave, but also senses the change in the measured parameters, that is, the acquisition and transmission of information are in the optical fibre, which has the

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characteristics of transmission and sensing. Optical fibre sensing technology is a new optical sensing technology with the development of optical fibre and its communication technology. The basic principle of optical fibre sensing detection is to use optical fibre in the light wave parameters (such as light intensity, frequency, wavelength, phase and polarisation state) with a change in the law of the measured parameters. By measuring these optical wave parameters to achieve the detection of external physical quantity, it is an important application of optical fibre in the field of non-communication. Reflective intensity modulation fibre-optic sensor has high sensitivity, is a simple structure and is not subject to electromagnetic interference. It can be used under high pressure, high temperature and harsh environment, and has the advantages of fast response speed, low cost and has been widely applied to the displacement, velocity, acceleration, pressure, temperature, flow, the electromagnetic field, the surface roughness measurement [1] and other physical quantities. For example, F. J. Arregui from Spain developed a new type of optical fibre sensor which can measure both the temperature and humidity at the same time. This shows that the optical fibre sensor plays a significant role in temperature and humidity measurement. Golnabi from Iran has presented the design and operation of a fibre optic sensor for mass measurement using a pair of fibre optic and reflective coated lenses whose light emphasis system is based on the relative motion of the lens. The design demonstrates the potential application of fibre optic sensors in mass measurement using the principle of intensity modulation. Hua-Yong Yang and others studied the geometric analysis method of single fibre of reflective optical fibre sensor theory model. They studied the intensity modulation function of the measured object surface inclination factor and the shape factor influence on the intensity modulation function shows that with increase in the tilt reflector and shape factors, the strength of the fibre optic sensor modulation function decreases, The measurement sensitivity is also reduced.

Intensity modulation function (IMF) is an important basis for detecting the position shift of reflective optical fibre sensors. The measurement results are easily affected by the inherent structure parameters of the fibre, the performance parameters of the fibre and the reflection characteristics of the reflecting surface. The light intensity modulation characteristic function model of the reflective intensity modulated fibre sensor is established in this paper through theoretical analysis. The influence of the mirror surface (roughness $h = 0$) and diffuse reflective surface (roughness $h = 5$) on the optical stress control characteristics is analysed and studied, which provides a theoretical basis for the design of reflective intensity modulated optical fibre sensor.

2 Forced modulation function model

The characteristic function curve of optical stress control is mainly determined by the ratio of the received light intensity value to the transmitted light intensity value and the distance between the reflected surface of the measured object and the end face of the optical fibre probe. The change in displacement can be calculated by a change in the value of the characteristic function of optical stress system. With applications in the actual measurement, the impact of optical fibre sensor light emphasises the function of many factors. Most of these factors are the inherent optical fibre sensor system, such as optical fibre sending and receiving optical fibre core diameter and numerical aperture, sending fibre and receiving optical fibre core centre distance, and the reflective surface properties and optical fiber probe end face to the object being measured the reflection surface distance. Therefore, the light stress control function can be expressed as a function related to the inherent factors of the system:

$$M = f(r_1, r_2, NA_1, NA_2, p, d, h) \quad (1)$$

where r_1 and r_2 are the core radii of the transmitting fibre and the receiving fibre respectively; NA_1 and NA_2 are their numerical apertures respectively; p is the distance between two optical fibre cores; d is the distance between the optical fibre probe and the reflecting surface; and h is the roughness of the reflecting surface [2, 3]. When the reflecting surface is a rough surface and the light is diffusely reflected on the surface of the measured object, it is assumed that the output light intensity of the transmitting fibre is a Gaussian distribution. The intensity modulation function under this assumption can be expressed as the ratio of the optical power received by the

receiving fibre to the optical power sent by the transmitting fibre, namely:

$$M = \frac{P_R}{P_T} \quad (2)$$

When the outgoing light intensity of the transmitting fibre is Gaussian, the variation of its light intensity with distance can be expressed as:

$$I(p, d) = \frac{2P_T}{\pi R^2(d)} \exp \left[-\frac{2p^2}{R^2(d)} \right] \quad (3)$$

In the formula, P_T is the optical power emitted by the transmitting fibre, $R(d)$ is the spot radius of the Gauss beam at the distance of d , p is the spot radial positioning coordinate. The outgoing light spot of sending fibre is at a distance of d , and the light spot radius is:

$$R(d) = r_T + d \tan \theta_N \quad (4)$$

where $\theta_N = \arcsin \text{NA}$ is the numerical aperture angle of the fibre, and the numerical aperture of the fibre. According to the Gaussian light intensity theory and Beckmann scattering theory, the light intensity model of Gaussian light reflected by the reflecting surface is as follows:

$$I_R(p, d) = I_T(p, d) \cdot \frac{\gamma}{1+h} = \frac{\gamma}{1+h} \cdot \frac{2P_T}{\pi R^2(d)} \exp \left[-\frac{2p^2}{R^2(d)} \right] \quad (5)$$

where h is the roughness of the reflecting surface. When $h = 0$, the reflecting surface is a rational mirror, and the reflected light follows the geometrical and optical laws. $0 < h + 1$, the reflecting surface is a slightly rough surface; $H + 1$, the reflecting surface is a relatively rough surface; and $h \gg 1$, the reflecting surface is a very rough surface, which can be interpreted as complete diffuse reflection. In actual measurement, reflection is not only related to the roughness of the reflecting surface, but also to the material of the reflecting object itself and its surface processing mode. These influencing factors are usually collectively referred to as reflectivity, which is represented by γ . The reflected light intensity received by the receiving optical fibre is equivalent to the product of the light intensity sent by the sending optical fibre at the distance of $2d$ and the reflectance of the reflecting surface, that is, the light intensity received by the receiving optical fibre is:

$$I_R(p, d) = I_T(p, 2d) \cdot \frac{\gamma}{1+h} = \frac{\gamma}{1+h} \cdot \frac{2P_T}{\pi R^2(2d)} \exp \left[-\frac{2p^2}{R^2(2d)} \right] \quad (6)$$

The optical power received by the receiving optical fibre is:

$$P(d) = \int \int_{S_R} I_R(p, d) dS_R \quad (7)$$

where S_R is the overlap between the reflected spot area and the receiving fibre. In the polar coordinate system, the area of the overlapping part can be expressed as:

$$dS_R = 2\beta p dp \quad (8)$$

Among them,

$$\beta = \arccos \left(\frac{p^2 + p^2 - r_R^2}{2pp} \right) \quad (9)$$

To sum up, it can be concluded that with a change in the distance d from the optical fibre probe to the reflected surface of the measured object, the intensity modulation function is expressed as:

$$\begin{aligned} R(2d) &\leq p - r_R \\ p - r_R &< R(2d) < p + r_R \\ R(2d) &\geq p + r_R \end{aligned} \quad (10)$$

Equation (10) is the optical stress control function of the optical fibre sensor when the output light intensity of the transmitting fibre is Gaussian distribution and the reflecting surface is a rough surface. According to Eq. (10), when the optical fibre probe of the optical fibre sensor is determined, the structural parameters and characteristic parameters of the optical fibre are fixed and unchanged. The intensity modulation function is only closely related to the distance d from the end face of the optical fibre probe to the reflective surface. The intensity modulation characteristic curve changes with the change of the distance between the optical fibre probe and the measured reflective surface. This is also the basis for the displacement measurement of the reflective intensity modulated fibre optic sensor [4,5].

2.1 Reflected light model

According to the light intensity distribution model of the outgoing end of the transmitting fibre obtained by diffraction analysis, and Beckmann's theory of the scattering of light on reflective surfaces of different properties, we can give the reflected intensity distribution model of the quasi-Gaussian light passing through the reflecting surface, that is,

$$I_R(p, d) = I_T(p, d) \cdot \frac{\delta}{1+g} = \frac{K_0 \Phi_0}{\pi \xi^2(d)} \cdot \exp[-p^2/\xi^2(d)] \cdot \frac{\delta}{1+g} \quad (11)$$

This is the general model of the light intensity reflected by a quasi-Gaussian beam emitted by a multi-mode transmitting fibre after passing through a reflective surface. When the reflecting surface is ideally smooth, due to $R_0 = \sqrt{2/\pi\sigma} = 0$, $g = 0$, then the reflected light intensity is

$$I_R(p, d) = \frac{\partial K_0 \Phi_0}{\pi \xi^0(d)} \cdot \exp[-p^2/\xi^2(d)] \quad (12)$$

Therefore, in the case of ideal smooth reflection surface, the reflected light field and the outgoing light field of the transmitting fibre are essentially the same, and their distribution is also of quasi-Gaussian type, but there is a difference of reflectivity coefficient. In general, when the intensity modulation function of RIM-FOS is studied, the reflecting surface is usually determined as the mirror reflecting surface. In this case, the intensity modulation function of RIM-FOS is studied on the basis of Eq. (1). When RIM-FOS is used in industrial measuring applications and the reflecting surface is rough, Eq. (2) is used and parameters G and δ are determined as the case may be.

2.2 Intensity modulation model of fibre bundle sensor

The inner ring is composed of several TF clusters and the radius is R_1 , the core diameter of TF is r_1 , the outer ring radius is R_2 and the stem core diameter is r_2 . The numerical apertures of both TF and RF are Na . For a coaxial type I fibre bundle, the T in the centre can actually be considered as a coarse transmitting fibre with a radius of R_1 and a numerical aperture of NA . The thick TF and the outer layer are matched into optical fibre pairs one by one to calculate the combination of light intensity. Assume that the thickness of the cladding and adhesive is t_1 , and assume that the fabrication of the fibre bundle is so strict that the RF of the outer layer is completely arranged in the form of concentric circles, then it can be deduced that the RF of the outer layer can be arranged in m circles as

$$m = INT[(R_2 - R_1)/(2(r_2 + t_0))] \quad (13)$$

$INT(\bullet)$ represents a truncated round function (the same below). So we can calculate the i th circle ($i = 1, \dots, M$) Rf as N_{ri}

$$\begin{aligned} N_{ri} &= INT(\pi/\theta_i) \\ \theta &= \sin^{-1}((r_2 + t_0)/p_i) \\ p_i &= R_1 + (2i - 1)(r_2 + t_0) \end{aligned} \quad (14)$$

Therefore, on the premise that the optical fibre emphasises the control function M_{pair} on the light, the contribution of all Rf in Layer I to the total modulation function is:

$$M_I = \delta N_{ri} M_{pair}(R_1, r_2, NA, p_i, d)$$

3 Strength compensation experiment

To compare the intensity of the above two kinds of method of compensation effect, we adopt the spectroscopy spectral compensation method and n type optical fibre beam compensation method based on the spectral form experiment system (the fibre bundle design, produced by Beijing optical fibre technology research institute), and the optical fibre end to reflect the distance is unchanged, fixed point light intensity compensation experiments [6, 7]. In the experiment, a He-Ne laser with Brewster window was used as the light source. The optical path was set according to Figure 5.1 and Figure 5.8, respectively, and a polariser was inserted into the main optical path. The polarised plate is rotated in the vertical direction of the beam during each measurement, and the intensity and polarisation direction of the incident light are changed simultaneously. The same signal processing system was used in both experiments.

As can be seen from the experimental results in Table 1, in the light intensity compensation experiment using a spectroscopy, when the amplitude of the measured signal was nearly doubled due to the rotation of the polariser (from 0.6380 V to 1.1301 V), the ratio after compensation changed by about three times, indicating that the purpose of light intensity compensation could not be achieved at all in this case. However, in the intensity compensation experiment using 'II' fibre bundle, the output variation after compensation is still less than 3.0%, even when the measured intensity variation is nearly ten times. It can be seen that the 'II' type optical fibre bundle has a higher compensation accuracy compared with the spectroscopic method, and the former does not have harsh requirements on the polarisation characteristics of the incident light [8].

The influence of specular reflection and diffuse reflection on the modulation function is analysed by changing the roughness h of the reflecting surface when other parameters remain unchanged. $h = 0$ can be regarded as an ideal specular reflection, and $h = 5$ can be regarded as an extremely rough surface on which light is diffusely reflected. The influence of specular reflection and diffuse reflection on the characteristic curve of light stress control function was analysed by MATLAB simulation. The parameters are set as reflectivity $\gamma = 0.5$, fibre numerical aperture $NA = 0.4$, core diameter radius $R_1 = R_2 = 100 \mu\text{m}$, core axis spacing $P = 300 \mu\text{m}$, and simulation step spacing is set as $1 \mu\text{m}$. The simulation results are shown in Figure 1.

Figure 1 shows that the modulation function curve of the optical fibre sensor is significantly higher than that of the diffuse reflecting surface when the optical fibre sensor is reflected specular on the reflective surface, and the modulation function values are all higher than those of the diffuse reflecting surface, and the linear interval of the front and rear slopes and the sensitivity are better than those of the production function curve of the diffuse reflecting surface. When the reflection surface contains both specular reflection and diffuse reflection, the roughness H of the reflection surface is changed successively. When other parameters remain unchanged, that is, reflectivity $\gamma = 0.5$, fibre numerical aperture $NA = 0.4$, core diameter radius $R_1 = R_2 = 100 \mu\text{m}$, $P = 300 \mu\text{m}$ between fibre core axes, and simulation step spacing is set as $1 \mu\text{m}$. The simulation results are shown in Figure 2.

It can be seen from Fig. 2 that when the reflector roughness increases sequentially, the modulation function curve tends to decline, the peak value of the modulation function decreases sequentially, and the linear interval and the sensitivity of the front and rear slopes decrease with the increase of the reflector roughness. According to the simulation results, when the roughness of the reflecting surface is a smooth mirror, slightly rough surface, relatively rough surface and very rough surface, the intensity modulation characteristics of the optical fibre sensor decrease successively, and the linear interval used for measurement decreases, and the sensitivity decreases, which is not good for the actual measurement of the optical fibre sensor. As can be seen from the simulation results, on the premise that other parameters remain unchanged: (1) With the increase of core spacing, the signal

Table 1 Experimental data of two kinds of spectral compensation

	The experimental sequence	1	2	3	4	5	6	7	8	9
The town of spectroscopic Spectroscopic fill Their data	Measure the signal (v)	0.6380	0.7075	0.8263	0.9326	0.9873	1.0646	1.1069	1.1241	1.1301
	Reference signal (v)	0.2554	0.3271	0.4896	0.6750	0.8049	1.0217	1.1940	1.2644	1.3470
	The ratio of	2.498	2.163	1.688	1.382	1.227	1.042	0.927	0.889	0.839
Type II fibre beam splitting compensation	Measure the signal (v)	0.3776	0.91325	1.4256	2.0683	2.7413	3.2959	3.9183	4.3254	4.6840
	Reference signal (v)	0.2819	0.6692	1.5036	1.5437	20.569	2.4815	2.9666	3.2935	3.5949
	The ratio of	1.339	1.365	1.353	1.340	1.333	1.328	1.321	1.313	1.303

Fig. 1 The influence of specular and diffuse reflection on the characteristic curve of light stress control function

light received by the receiving fibre decreases, and the light emphasis control function decreases accordingly; With the increase of the distance between the core and the shaft, the curve of the optical stress characteristic function decreases and moves backward, and the peak distance gradually increases with the increase of the dis-

Fig. 2 Influence of reflector roughness on modulation function

tance between the core and the shaft. And the sensitivity of the foreslope gradually decreases [9,10]. From this analysis, it is concluded that the distance between the fibre core axis of the optical fibre probe should be as small as possible, that is, the sending fibre and the receiving fibre should be closely arranged within the range of conditions. The results show that the foreslope sensitivity and the peak modulation function of the intensity modulation characteristic curve of the fibre sensor are mainly affected by the core spacing of the fibre probe. Therefore, the sensor sensitivity can be improved by reducing the core spacing of the fibre probe in practical application.

4 Conclusion

Sent via theoretical analysis to establish when the optical fibre end light intensity of Gaussian distribution and reflective surface diffuse reflection occurs fibre optic sensor light emphasises the system function model. The intensity modulation characteristic analysis summarised in the design of the actual optical fibre sensor, in order to improve the precision of luminescent fibre core shaft spacing, shall be as far as possible in small, send the fibre core diameter should be reduced, The receiving fibre and the numerical aperture of the fibre should be as large as possible under the premise of meeting the measurement range. Different optical fibre parameters correspond to different intensity modulation characteristics. In the practical application of reflective intensity modulation optical fibre sensor, appropriate optical fibre parameters should be selected according to the specific requirements to achieve the ideal measurement requirements. In this paper, the research and application progress of rim-fos since the invention of the first patent for the application of reflective strength optical fibre sensor are reviewed comprehensively, and the important achievements in the research of this kind of optical fibre sensor in the past 40 years as well as the existing theoretical and technical problems are pointed out. For the basic structure of RIM-FOS fiber pair, the tilt factor and shape factor of the reflector are introduced for the first time in this paper. The optical fibre parameters (including the spacing p of the fibre pair, the core diameters r of Tf and RE, and r of the fibre numerical aperture NA), the tilt factor of the reflector and the shape factor r are studied systematically and theoretically. The influence of these parameters on the sensor characteristics is summarised one by one. These rules have important guiding significance for users when designing RIM-FOS. Both theoretical research and simulation experiments show that the change

of polarisation direction of incident light will inevitably lead to the reverse fluctuation of transmitted light and reflected light. Therefore, measures must be taken to keep the polarisation direction of the incident light constant when the spectroscopic compensation is adopted. The results show that the system can not only complete the characteristics of RIM-FOS simulation and testing functions, but can also be switched on the test without the influence of ambient light interference and power fluctuation of the light source, the test stability is good, high repeatability.

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