Applied Mathematics and Nonlinear Sciences

<https://www.sciendo.com>

Risk contagion in financial markets based on copula model

Li Ma^{1*}, Fahad Abdullah Alqurashi², Mohammed Helmi Qeshta³

¹School of Finance, Guangdong University of Finance and Economics, Guangzhou, Guangdong, 510320, China

²Department of Computer Science, Faculty of Computing and Information Technology,
King Abdulaziz University, Jeddah, Saudi Arabia

³Applied Science University, Al Eker, Kingdom of Bahrain

Submission Info

Communicated by Juan Luis García Guirao

Received June 16th 2021

Accepted September 24th 2021

Available online December 30th 2021

Abstract

Economic globalisation and the development of financial trade liberalisation lead to a higher probability of financial crises. At the same time, the occurrence of financial crises has a particular risk of contagion. Based on this research background, this paper constructs a dynamic Copula model. It demonstrates the application of this model in financial market risk management based on the correlation changes between the US stock market and the Chinese stock market before and after the financial crisis. The results show that the Standard & Poor's Index and China before the crisis broke out. There is a specific correlation between the stock markets, which shows that the financial crisis has affected both the Chinese and American stock markets. Therefore, risks in the financial market are contagious.

Keywords: Financial crisis; structural Copula model; contagion effect; random variable.

AMS 2010 codes: 60B12

1 Introduction

Foreign scholars have done a lot of research on the contagion effect of the financial crisis and put forward many research methods. First, some scholars have applied the multivariate GRCH-M model to analyse crisis contagion. Second, some scholars have applied the multivariate extreme value theory method in statistics to analyse the crisis contagion, allowing asymmetry in the distribution of returns [1]. Third, some scholars use the ProBit model to test the contagion of financial crises empirically. The correlation during the crisis period is more significant than the correlation during the non-crisis period. Therefore, it can be considered that the two capital markets have crisis contagion or volatility spillover effect. For the stock market, it is believed that if

*Corresponding author.

Email address: mali19800913@163.com

the two countries' stock markets experience large fluctuations, the connection between the international markets will be significantly enhanced. Some scholars have analysed and studied several major international financial crises and found that the international capital market's relevance has increased significantly after the financial crisis. That is, there is a volatility spillover effect. However, some scholars pointed out that it is biased to test the correlation change without considering the conditional heteroscedasticity. Some scholars have studied several major international financial crises and found that after adjusting the conditional heteroscedasticity, there is no evidence that the correlation between markets has been destroyed in any financial crisis.

There are also many domestic scholars studying the contagious effects of financial crises [2]. Some scholars have used the VAR system to find a feedback mechanism for financial crisis contagion. Some scholars combined the Copula theory, Bayes time-series diagnosis and Z-test studies, which have shown that Asian countries have little correlation in financial markets during the Asian financial crisis. Some scholars have tested the existence of the contagion effect through Archimedes Copula's change-point detection method, which more comprehensively analyses the interdependence structure of the country's rate of return. They believe that the tail dependency index of the returns of two countries can be used to measure the degree of contagion.

In short, the primary methods to test the contagion effects of financial crises include cointegration analysis, asset price correlation test, GRACH model spillover effect test, ProBit model conditional probability test and VAR system method. The above analysis methods mainly analyse the degree of market correlation through variance, covariance matrix, or correlation coefficient. The primary purpose of these analysis methods is to test the stability of the parameters in data generation. However, if there are endogenous variables, ignoring variables and heteroscedasticity, the test results of parameter stability are often biased. Therefore, although corrections can be made in some exceptional cases, the results are not universal.

Moreover, none of these methods can reproduce the dynamic effects of infection, let alone quantify the intensity of the impact of infection. In addition, a large number of empirical studies have shown that the conditional fluctuation of asset prices exhibits long memory or large-scale persistence. Many scholars believe that this kind of persistence or long memory is that the existence of variable structure points in the fluctuation process aggravates the persistent structural changes [3]. Therefore, it is indispensable to describe the marginal distribution of financial variables using a volatility model without a variable structure. Considering the marginal distribution, using the variable structure Copula model to test the contagiousness of financial crises can avoid the above problems.

After the subprime mortgage crisis, many financial institutions in the United States suffered huge losses and developed into a financial crisis. This situation intensified and led to the global financial crisis. By comparing the Chinese and American stock markets after the crisis, it can be found that the Chinese stock market has also experienced a certain degree of volatility, and the fluctuations of the Chinese stock market and the US stock market have the same trend. However, this is not enough to prove that the US financial crisis has a contagious effect on the Chinese stock market [4]. Therefore, this article uses a variable structure model to empirically test the contagion of the US financial crisis to China's stock market based on collecting relevant data on the Chinese stock market and the US stock market.

2 Research method and process

2.1 Setting of edge distribution

This article uses autoregressive GARCH, GARCH-t and GARCH-GED models to fit the sample data, respectively. The results show that the AR(n)-GARCH(1,1)-t model can better describe the daily return rate sequence of the Standard & Poor's 500 Index and the Shanghai Stock Exchange 300 Financial Index [5]. Through the ARCH-LM test and the residual square correlation plot test, we further confirmed the optimality of the marginal

distribution hypothesis. The expression of the marginal distribution model is as follows:

$$X_{nt} = u_n + \sum \phi_{nt} X_{t-p} + \varepsilon_{nt}, n = 1, 2, \dots, 8, t = 1, 2, \dots, T, p = 1, 2, \dots, 6 \quad (1)$$

$$\varepsilon_{nt} = h_{nt}^2 \xi_{nt} \quad (2)$$

$$h_{nt} = \omega_n + \alpha_n \varepsilon_{nt-1}^2 + \beta_n h_{nt-1} \quad (3)$$

$$\sqrt{\frac{v_n}{h_t(v_n - 2)}} \varepsilon_t \sim t(v_n) \quad (4)$$

where $t(v_n)$ represents the standard t distribution with v_n degrees of freedom.

2.2 The choice of the variable structure Copula model

We use the single-parameter and dual-parameter Copula models of the Archimedes to fit the data and compare them with the empirical distribution. It is found that the BB3 model has the best fit for the data. Therefore, the BB3 model is selected for the Copula model in this paper, and its expression is as follows:

$$C(u, v) = \exp\{-[\delta^{-1} \ln(e^{u\delta} + e^{v\delta})]^{1/\theta}\} \quad (5)$$

The generators of $\theta > 1, \delta > 0, u = -\ln u$ and $v = -\ln v$ are:

$$\phi(t) = \exp\{\delta(-\ln t)^\theta\} - 1 \quad (6)$$

2.3 Copula model parameters and correlation estimation

The parameter estimation of the Copula model generally adopts maximum likelihood estimation, moment estimation and semi-parametric estimation. The empirical studies of many scholars have shown that the best estimation can be obtained by using the two-stage maximum likelihood estimation method to estimate the parameters of the Copula model [6]. Therefore, this paper uses a two-stage maximum likelihood estimation method to estimate the Copula model. This paper adopts the Z test as the test method for the correlation difference in the financial crisis contagion test. The Z test statistics are:

$$Z = \frac{\bar{\rho}_1 - \bar{\rho}_2}{\sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}} \quad (7)$$

where $n_i, i = 1, 2$ is the number of observations $n_i \geq 10, \bar{\rho}_i, i = 1, 2$ of the two test samples. The Fisher transformation of $\bar{\rho}_i, i = 1, 2$ is expressed as $\bar{\rho}_i = \frac{1}{2} \ln \left(\frac{1+\rho_i}{1-\rho_i} \right)$ as follows. The statistic Z approximately obeys (0, 1) normal distribution. We give the critical value $z^*_{a/2} = \Phi^{-1} \left(1 - \frac{a}{2} \right)$ of the confidence level $1 - a$ at this time, where $\Phi^{-1}(\bullet)$ is the inverse function of the standard normal distribution function. If $|Z| \geq z^*_{a/2}$, reject $H_0: \rho_1 = \rho_2$ and consider that the difference ρ_1, ρ_2 is significant.

The Kendal rank correlation coefficient in the correlation measure is

$$\tau = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1 \quad (8)$$

The Spearman rank correlation coefficient is

$$\rho = 12 \int_0^1 \int_0^1 uv dC(u, v) - 3 \quad (9)$$

3 Empirical analysis

This article selects the Shanghai Stock Exchange 300 Financial Index and the Standard & Poor's 500 Index daily return rates from 5 January 2015 to 29 June 2019 as the sample. The original data are all from RESET. Due to the mismatch between the Sino-US transaction dates, this paper excludes the mismatched data in the analysis, a total of 1048 data. This article divides the research into four periods [7]. The first period is from 5 January 2015 to 12 February 2017, the pre-crisis period. At this time, the financial crisis did not appear. The second period is the first stage of the financial crisis, from 13 February 2017 to 18 September 2018. At this time, the impact of the crisis gradually increases. The third period is from 19 September 2018 to 28 November 2018. This is the second stage of the financial crisis. At this time, the global impact of the financial crisis appears. The fourth period is from 1 December 2018 to 29 June 2019. At this time, countries worldwide have taken measures to resist the negative impact of the financial crisis. Among them, Sh represents the daily return rate of the SSE 300 Financial Index. Bp represents the daily return rate of the US Standard & Poor's 500 Index.

First, we use the ADF unit root test method to get the ADF value of -2.8673 when the confidence level is 5% before the crisis occurs. When the confidence level of the first stage of the financial crisis is 5%, the ADF value is 2.8690. When the confidence level of the second stage of the financial crisis is 5%, the ADF value is -2.9297. Finally, in the third stage of the financial crisis, when the confidence level is 5%, the ADF value is -2.8830. Therefore, we can infer that the daily yields of China and the United States were stable before the financial crisis and in the three phases after the financial crisis.

Secondly, the descriptive statistics for each sample period show that the average returns of the two markets before the financial crisis are both positive crises. After the occurrence, the average returns of the two samples in the fourth period were both positive. In the third period, the daily return rate of the SSE 300 Financial Index was positive, and the rest were negative. The standard deviation of the daily return rate of the SSE 300 Financial Index after the financial crisis was more significant than before the financial crisis. The standard deviation of the S&P 500 index yield in the three periods after the financial crisis was more significant than that in the first period [8]. At the same time, we can also see that there are partial peaks and thick tails in the four-period returns of the sample, and statistics do not obey the standard normal distribution.

It can be seen from Figure 1 that during the sample period, the volatility of the daily yield of the S&P 500 index during the financial crisis was more significant than that before the financial crisis. Still, the volatility only began to increase in the last two stages of the financial crisis. Among them, there are several outliers of fluctuations in yields in 2015.

Fig. 1 S&P 500 daily yield (January 2015 to June 2019)

Comparing Figures 1 and 2, we can find that the volatility of the SSE 300 Financial Index's daily return rate during the sample period is greater than that of the Standard & Poor's 500 Index daily return rate [9]. The volatility of the daily return rate of the SSE 300 Financial Index after the financial crisis was more incredible

Fig. 2 SSE 300 Financial Daily Yield (January 2015 to June 2019)

than before the financial crisis, and 2018 was a year of relatively high volatility. The volatility of 2019 has been slower than the previous year.

3.1 Granger causality test

We carried out a Granger causality test on the four periods of the two samples and selected the 5% confidence level as the reference standard. The causality of the four periods is shown in Tables 1–4.

Table 1 SH1 and BP1 Granger causality test

Lag length	Null hypothesis	<i>F</i>	<i>P</i>	In conclusion
1	SH1 is not a Granger reason for BP1	1.08434	0.29825	Accept
	BP1 is not a Granger reason for SH1	0.63241	0.42686	Accept
2	SH1 is not a Granger reason for BP1	0.87401	0.41793	Accept
	BP1 is not a Granger reason for SH1	2.1805	0.1141	Accept

Table 2 SH2 and BP2 Granger causality test

Lag length	Null hypothesis	<i>F</i>	<i>P</i>	In conclusion
1	SH2 is not a Granger reason for BP2	0.08311	0.77329	Accept
	BP2 is not a Granger reason for SH2	5.77063	0.01678	Refuse
2	SH2 is not a Granger reason for BP2	0.17162	0.84236	Accept
	BP2 is not a Granger reason for SH2	3.17334	0.043	Refuse

Table 3 SH3 and BP3 Granger causality test

Lag length	Null hypothesis	<i>F</i>	<i>P</i>	In conclusion
1	SH3 is not a Granger reason for BP3	2.65028	0.11119	Accept
	BP3 is not a Granger reason for SH3	0.00298	0.95676	Accept
2	SH3 is not a Granger reason for BP3	0.97062	0.38804	Accept
	BP3 is not a Granger reason for SH3	1.084	0.34847	Accept

From the above causal analysis, it can be seen that there is no causal relationship between the two variables before the financial crisis. In the first stage after the financial crisis, the S&P 500 index has a causal relationship

Table 4 SH4 and BP4Granger causality test

Lag length	Null hypothesis	<i>F</i>	<i>P</i>	In conclusion
1	SH4 is not a Granger reason for BP4	3.76151	0.05459	Accept
	BP4 is not a Granger reason for SH4	0.10389	0.74773	Accept
2	SH4 is not a Granger reason for BP4	2.99813	0.0534	Accept
	BP4 is not a Granger reason for SH4	0.04023	0.96058	Accept

with the SSE 300 financial index. On the other hand, there is no causal relationship between the SSE 300 Financial Index and the Standard & Poor's 500 Index in the latter two stages of the financial crisis.

3.2 Estimation of marginal distribution model and Copula model

We substitute the sample data into the marginal distribution model and use maximum likelihood estimation to obtain the parameter values of each model [10]. A total of eight models need to be estimated for the four samples before and after the crisis. The parameter estimates of each sample are given in Tables 5–8.

Table 5 Estimated values of the parameters of each marginal distribution model before the crisis broke out

BP1	μ_1	$\phi_{1,1}$	$\phi_{1,2}$	ω_1	α_1	β_1	ν_1
	0.0005	-0.0474	-0.0637	4.83E-05	0.1287	0.0538	3.3915
	(0.0002)	(0.0449)	(0.0328)	(8.53E-06)	(0.0839)	(0.1311)	(0.2186)
Sh1	μ_2	$\phi_{2,6}$	ω_2	α_2	β_2	ν_2	
	0.0019	-0.0762	1.86E-06	0.0457	0.9537	8.1928	
	(0.0007)	(0.0456)	(4.32E-06)	(0.0215)	(0.029)	(2.9599)	

Table 6 Parameter estimation of each marginal distribution model in the first stage after the outbreak of the crisis

BP2	μ_3	$\phi_{3,1}$	α_3	β_3		ν_3
	0.0002	0.1511	2.15E-06	0.0863	0.9107	7.4628
	(0.0005)	(0.0607)	(2.43E-06)	(0.0366)	(0.037)	(4.0494)
Sh2	μ_4	$\phi_{4,5}$	ω_4	α_4	β_4	ν_4
	0.0006	-0.1208	0.0003	0.0145	0.6166	8.394
	(0.0013)	(0.0494)	(0.0005)	(0.0513)	(0.6567)	(4.7757)

Table 7 Parameter estimation of each marginal distribution model in the second stage after the outbreak of the crisis

BP3	μ_5	ω_5	α_5	β_5	ν_5
	-0.0055	0.0015	-0.1642	0.4349	20.02
	(0.0073)	(0.0007)	(0.0363)	(0.4032)	(90.079)
Sh3	μ_6	ω_6	α_6	β_6	ν_6
	-0.007	0.0004	-0.2405	1.0267	21.6
	(0.0047)	(0.0002)	(0.0806)	(0.171)	(103.3)

Tables 5–8 are the parameter estimates of the marginal distribution of the sample. Among them, Bp1 is the AR(1,2)-GARCH(1,1)-t model. Sh1 is the AR(6)-GARCH(1,1)-t model. Bp2 is the AR(1)-GARCH(1,1)-t model. Sh2 is the AR(5)-GARCH(1,1)-t model. The marginal distributions of the remaining samples are all the GRCH(1,1)-t model. The K-S statistics and their probability statistics in Table 9 are based on estimated marginal distributions.

Table 8 Parameter estimation of each marginal distribution model in the third stage after the outbreak of the crisis

BP4	μ_7	ω_7	α_7	β_7	ν_7
	8.33E-05	5.89E-05	0.0316	0.8417	10.24
	(0.002)	(5.59E-05)	(0.0643)	(0.1244)	(11.2066)
Sh4	μ_8	ω_8	α_8	β_8	ν_8
	0.0037	-7.38E-06	-0.0019	1.0107	19.81
	(0.0018)	(1.12E-05)	(0.0359)	(0.0521)	(35.4823)

Table 9 Four-stage K-S test of the marginal distribution

	K-S	K-S probability
Bp1	0.011	1
Bp2	0.012	1
Bp3	0.197	0.0523
Bp4	0.079	0.3508
Sh1	0.011	1
Sh2	0.009	1
Sh3	0.189	0.0704
Sh4	0.084	0.2909

The autocorrelation test also found that there is no autocorrelation in the transformed sequences [11]. Therefore, we can consider the transformed sequence to be independent. We use it to describe the sequence of daily return rates of the two indices as appropriate.

We use the Archimedes Copula model to fit the data and compare it with the empirical distribution. We use the BB3 model. The paper uses formula (9) given above to find the Spearman rank correlation coefficient and uses formula (8) to calculate the Kendal rank correlation coefficient at each stage. At the same time, the article uses formula (7) to find the value of the Z test statistic, as shown in Table 10.

Table 10 Correlation structure between the Standard & Poor's 500 Index and the SSE 300 Financial Index

k	v	Date	τ	ρ	Z statistics
1	490	2017/2/12	0.0382	0.0573	-2.5852
2	868	2018/9/18	0.1563	0.2308	0.2257
3	913	2018/11/28	0.1303	0.191	0.0056
4	1048	2019/6/29	0.1271	0.1901	

According to the Kendal rank correlation coefficient and the Spearman correlation coefficient, we can see that the correlation between the two cities has become more prominent after the crisis [12]. The Z test found that the US financial crisis had a contagious effect on the Chinese stock market only in the first phase after the crisis. Its contagious effects in the second and third phases after the financial crisis have not been confirmed.

After the outbreak of the financial crisis, the impact of the US stock market on the Chinese financial industry has increased. The reasons can be roughly divided into the following three aspects: First, with the deepening of financial globalisation and integration, the amount of mutual investment between China and the United States increases, and the financial ties between the two countries are relatively enhanced. However, due to stricter controls on the Chinese stock market, only smaller B shares, N shares, H shares and S shares are open to foreign investors, while the larger A-share market has not been opened. Therefore, the Sino-US stock markets were less relevant before the crisis. Secondly, after the crisis, because the United States was Chinese leading export partner, the financial crisis reduced Chinese exports and affected the operating performance of some

listed companies [13]. At the same time, some financial institutions in China also hold a certain proportion of “toxic assets” and suffer losses, coupled with the expected effect of market psychology. Therefore, the financial crisis in the United States has a certain degree of spillover effect on the Chinese financial industry in the first stage after the outbreak of the crisis. Finally, after the crisis, the Chinese government has decisively adopted an investment increase of 4 trillion yuan to expand domestic demand and ease the impact of the crisis. These macroeconomic policy changes and related information are also reflected in the stock market. Therefore, the contagious effects of the second and third stages after the financial crisis have not been confirmed.

4 Conclusion

This article divides the period from 5 January 2015 to 29 June 2019 into four different periods. By estimating its marginal distribution and using the variable structure Copula model, it conducts an empirical analysis of the correlation between the Chinese stock market and the US stock market before and after the US financial crisis. The research results show that the correlation between the volatility of the Shanghai Stock Exchange 300 Financial Index and the Standard & Poor’s 500 Index volatility presents different characteristics at different stages. Specifically, the correlation between the two cities before the financial crisis was minimal. After the financial crisis, the correlation between the two cities began to increase, and as the US financial crisis evolved into a global financial crisis, the correlation further increased. However, the contagious effects of the second and third stages after the financial crisis have not been confirmed. Further analysis of Granger causality found that the US stock market had a unidirectional causal relationship to China’s finance in the first stage after the crisis. Before the crisis, there was no causal relationship between the two markets.

References

- [1] Karmakar, M., & Paul, S. Intraday portfolio risk management using VaR and CVaR: A CGARCH-EVT-Copula approach. *International Journal of Forecasting*, 2019. 35(2): 699-709
- [2] Li, T., Zhong, J., & Huang, Z. Potential dependence of financial cycles between emerging and developed countries: Based on ARIMA-GARCH Copula model. *Emerging Markets Finance and Trade*, 2020. 56(6): 1237-1250
- [3] De Luca, G., Riviuccio, G., & Corsaro, S. Value-at-Risk dynamics: a copula-VAR approach. *The European Journal of Finance*, 2020. 26(2-3): 223-237
- [4] Taleblou, R., & Davoudi, M. M. Calculating Value at Risk: DCC-GARCH-Copula Approach. *Iranian Journal of Economic Research*, 2020. 25(82): 43-82
- [5] Jeribi, A., & Fakhfekh, M. Portfolio management and dependence structure between cryptocurrencies and traditional assets: evidence from FIEGARCH-EVT-Copula. *Journal of Asset Management*, 2021. 22(3): 224-239
- [6] Trucíos, C., Tiwari, A. K., & Alqahtani, F. Value-at-risk and expected shortfall in cryptocurrencies’ portfolio: A vine copula-based approach. *Applied Economics*, 2020. 52(24): 2580-2593
- [7] Zhang, Z., Zhang, D., Wu, F., & Ji, Q. Systemic risk in the Chinese financial system: A copula-based network approach. *International Journal of Finance & Economics*, 2021. 26(2): 2044-2063
- [8] Sharifi, M. & Raesi, B. Vortex Theory for Two Dimensional Boussinesq Equations. *Applied Mathematics and Nonlinear Sciences*, 2020. 5(2): 67-84
- [9] Zhao, Y. Analysis of Trade Effect in Post-Tpp Era: Based on Gravity Model and Gtap Model. *Applied Mathematics and Nonlinear Sciences*, 2020. 5(1): 61-70
- [10] Ab Razak, R. U. Z. A. N. N. A., & Ismail, N. O. R. I. S. Z. U. R. A. Dependence modeling and portfolio risk estimation using GARCH-Copula approach. *Sains Malaysiana*, 2019. 48(7): 1547-1555
- [11] Bruneau, C., Flageollet, A., & Peng, Z. Economic and financial risk factors, copula dependence and risk sensitivity of large multi-asset class portfolios. *Annals of Operations Research*, 2020. 284(1): 165-197
- [12] Bhatti, M. I., & Do, H. Q. Recent development in copula and its applications to the energy, forestry and environmental sciences. *International Journal of Hydrogen Energy*, 2019. 44(36): 19453-19473
- [13] Zhang, Z., Zhang, D., Wu, F., & Ji, Q. Systemic risk in the Chinese financial system: A copula-based network approach. *International Journal of Finance & Economics*, 2021. 26(2): 2044-2063