Applied Mathematics and Nonlinear Sciences

<https://www.sciendo.com>

Regression function model in risk management of bank resource allocation

Hongzhi Zhang[†]

School of Economics and Management, Shenyang Ligong University, Shenyang 110159, China

Submission Info

Communicated by Juan Luis García Guirao

Received June 17th 2021

Accepted September 24th 2021

Available online December 30th 2021

Abstract

Commercial bank asset-liability management is a method of overall risk control and resource allocation. It has an essential impact on commercial banks' operational safety and affects the stability of a country's overall financial system due to the domino effect. The article chooses the regression function model to measure the expected default frequency (EDF) in the resource allocation risk of Chinese banks. It analyses the transmission process of the impact of the main macroeconomic variables on the EDF of the resource allocation risk of Chinese commercial banks. The study found that under the significance level of 5%, standardised industrial output growth rate, money supply growth rate, and real estate price index are the main reasons for the changes in bank resource allocation risk EDF.

Keywords: ice EDF model, regression function model, bank resource allocation, credit risk, expected default frequency.**AMS 2010 codes:** 62G08

1 Introduction

The maturity and perfection of the Chinese stock market make it possible for commercial banks to use these ready-made, large-scale and huge-potential credit risk management tools. The most famous stock market credit risk measure is KVM's expected default frequency (EDF) model. It represents an innovative method of using stock market information to value debt.

KVM company created an EDF model that can estimate the company's probability of default. It extends the idea that the value of a company's stock has the characteristics of options to evaluate the company's credit risk [1]. At the same time, the article regards the value of the company's equity as a call option for the value of the company's assets. The company's owner holds a European-style call option with the face value of the company's debt as the strike price and the market value of the company's assets as the subject. If the market value of the company's assets is higher than its debt when the debt matures, the company will exercise the call option. At this time, the company's income is the difference between the asset's market value and the debt.

[†]Corresponding author.Email address: 912230452@qq.com

When the market value of the company's assets is less than its debt, the company chooses to default, and the equity value is zero.

On the other hand, debt is similar to selling a put option. When the bank recognises the possibility of default when lending to the company, it also sells shareholders a put right [2]. At maturity, if the asset value is less than its debt, the shareholder will execute the put option and sell the asset at the price of the debt. At this time, shareholders use the income they receive to pay debts. On the contrary, if the asset's value is greater than the debt, the shareholder will not execute the put option but directly sell the asset. In addition to debt service, shareholders' income has a surplus.

2 Model introduction

In credit analysis, the algorithm mainly focuses on analysing the value of call rights and the probability that they may be executed. The EDF model has three steps to determine the EDF value of a company:

(1) According to the option concept of Merton and Black-Scholes, the market value of a company's equity can be expressed as:

$$V_E = V_A N(d_1) - e^{-rt} X N(d_2) \quad (2.1)$$

In Formula (1), V_E is the market value of equity. X is the book value or strike price of the liability. V_A is the market value of the company's assets. T is the expiration time and r represents the risk-free interest rate. Here d_1 and d_2 are:

$$d_1 = \frac{\ln(V_A/X) + (r + \sigma_A^2/2)T}{\sigma_A \sqrt{T}} \quad (2.2)$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \quad (2.3)$$

In Formula (3), σ_A is the volatility of asset value. After deriving the two sides of Formula (1) and then seeking the expectation, the following equation can be obtained:

$$\sigma_E = \frac{V_A}{V_E} N(d_1) \sigma_A \quad (2.4)$$

In Formula (4), σ_E is the volatility of equity value. We can solve Formulas (1) and (4) to obtain the market value V_A of the company's assets and the volatility rate σ_A of the asset value.

(2) Calculation of the point of default (DP) and the distance to default (DD) [3]. The percentage of the asset value that must fall to reach the DP is the multiple of the standard deviation called the default distance. The default distance measurement is a standardised method, which can be used for comparison between different companies.

(3) Determine the mapping relationship between default distance and default rate. The default distance is a measure similar to the ordinal of the company's rating. It does not tell us what the probability of default is. The classic Merton method uses simplified assumptions such as the asset return distribution to be a normal distribution to establish the mapping relationship between DD and EDF. It uses the cumulative normal distribution to transform [4]. Merton EDF = N(-DD). Here N is the cumulative normal distribution function. However, empirical research by KVM shows that the EDF calculated by this method significantly underestimates the probability of default. The reasons for this result are as follows: (1) The company will change its liabilities when it is on the verge of trouble. (2) Default can occur at any time before expiration. (3) Compared with the normal distribution, the distribution of asset returns is fat-tailed. To solve this problem, KVM uses a large historical database of samples of defaulting companies. See Figure 1 for details.

Fig. 1 The relationship curve between default distance and expected default probability.

3 Functional nonparametric partial autoregressive model

The form of the functional nonparametric partial autoregressive model is as follows:

$$Y_i = g(X_i(t)) + \sum_{k=1}^p \varphi_k Y_{i-k} + \varepsilon_i, i = p+1, p+2, \dots, n \quad (3.1)$$

$X(t)$ is a functional covariate defined on an interval I that is squarely integrable and fully observed; $g(\cdot)$ is the real-valued function from the square-integrable space L^2 to the actual number domain R and $g : L^2 \rightarrow R, Y$ is a scalar. Also, p is the lag order of the model; ε is the error term and satisfies $E_\varepsilon = 0, \text{Var}\varepsilon = \sigma^2$. This model overcomes the linear constraints of functional covariates and response variables. The functional linear model is a particular case of this model (the model has no lag term). $\varphi_1 = \dots = \varphi_p = 0$ and $g(X(t)) = \int_I X(t)\beta(t)dt$. The autoregressive model is also a particular case of this model (i.e.) $g(X(t)) = 0$. Therefore, the model has muscular flexibility. It has a crucial significance in practical research [5]. We use the method of combining *profile* least square estimation and nonparametric kernel estimation to estimate the model. The specific steps are as follows:

$$\tilde{Y}_i = Y_i - \sum_{k=1}^p \varphi_k Y_{i-k} \quad (3.2)$$

Then the model (5) is transformed into

$$\tilde{Y}_i = g(X_i(t)) + \varepsilon_i \quad (3.3)$$

We use *Nadaraya – Watson* kernel to estimate that and we can get

$$\tilde{g}(u) = \sum_{j=p+1}^n \omega_j(u) \tilde{Y}_j \quad (3.4)$$

Among them

$$\omega_j(u) = \frac{K_h(d(u, X_j(t)))}{\sum_{s=p+1}^n K_h(d(u, X_s(t)))} K_h(\cdot) = \frac{K(\bar{h})}{h} \quad (3.5)$$

$K(\cdot)$ is the unary kernel function; h is the width of the window; $d(\cdot, \cdot)$ is a distance measure of two functional variables (for example, $d(X(t), Z(t)) = \left\{ \int_I (X(t) - Z(t))^2 dt \right\}^{\frac{1}{2}}$). So we follow the principle of *profile* least

squares. We can minimise

$$\sum_{i=p+1}^n \left\{ Y_i - \sum_{k=1}^p \phi_k Y_{i-k} - \sum_{j=p+1}^n [\omega_j(X_i(t)) (Y_j - \sum_{k=1}^p \phi_k Y_{j-k})] \right\}^2 \quad (3.6)$$

We get an estimate of the parameter $\phi_1, \dots, \phi_p$.

$$Y = (Y_{p+1}, \dots, Y_n)^T \Phi = (\phi_1, \dots, \phi_p)^T W = (\omega_{ij})_{(n-p) \times (n-p)} \quad (3.7)$$

Among them

$$\begin{aligned} \omega_{ij} &= \omega_{j+p}(X_{i+p}(t)) i, j = 1, \dots, n-p \\ Z_1 &= (Y_p, \dots, Y_{n-1})^T \\ Z_2 &= (Y_{p-1}, \dots, Y_{n-2})^T, \dots \\ Z_p &= (Y_1, \dots, Y_{n-p})^T \\ Z &= (Z_1, \dots, Z_p)^T \\ Y^* &= (I - W)Y Z^* = Z(I - W) \end{aligned} \quad (3.8)$$

I is the $n - p$ order unit matrix. Then Eq. (9) can be simplified to $(Y^* - Z^*{}^T \Phi)^T (Y^* - Z^*{}^T \Phi)$. Then

$$\hat{\Phi} = (Z^* Z^{*T})^{-1} Z^* Y^* \quad (3.9)$$

We substitute Eq. (9) back to Eq. (7) to get

$$\hat{g}(u) = \sum_{j=p+1}^n \omega_j(u) (Y_j - \sum_{k=1}^p \hat{\omega}_k Y_{j-k}) \quad (3.10)$$

So we get an estimate of the nonparametric function $g(\cdot)$. When solving the least square estimation, we use the improved *BFGS* algorithm of *YUAN* and *WEI*. Of course, the selection of the parameters h and p is involved in the estimation. It embodies the autoregressive form of the historical moment of the scalar response variable [6]. Therefore, the model itself has greater applicability and flexibility.

4 Establishment of empirical EDF function of listed companies in China

The establishment of the functional relationship between DD and EDF of listed companies in China requires full consideration of the actual situation in China in the selection of samples and the determination of parameters. Due to historical reasons, tradable shares and non-tradable shares in the Chinese securities market [7]. In the empirical research, the author found that the significant fluctuation of stock return on the ex-right and ex-dividend date greatly influences equity value volatility. This will make the EDF value of listed companies significantly overestimated.

4.1 Selection of samples

This article selects 178 effective samples of companies listed on A-shares in Shanghai and Shenzhen from 2018 to 2019. The audit results of the last two fiscal years show that the net profit is negative. Listed companies whose net assets per share are lower than the stock's par value or loss for two consecutive fiscal years will be specially treated [8]. Therefore, ST companies have a higher credit risk than standard listed companies.

4.2 Determination of parameters

According to the characteristics of the Chinese securities market and the results of Chinese EDF model research in recent years, this article adjusts the relevant parameters of the EDF model.

- (1) Select the daily closing price of Chinese stocks from 2 January 2018 to 31 December 2019 to calculate the daily stock return and volatility, and then convert it into 1-year volatility. The volatility of the equity value is expressed as (σ_E) . The return on the ex-dividend date is excluded from the calculation.
- (2) Because of the particularity of the equity split in the Chinese securities market, we select the average closing price of the first ten trading days at the end of 2019 as the market value of tradable shares. We select the adjusted net assets per share at the end of 2019 as the market value of non-tradable shares [9].
- (3) We use current liabilities plus 75% of long-term liabilities as to the default point (X). We put the result and X value into Formula (5) to get the default distance. The DD values of the sample companies are shown in Figure 2 (listed after sorting).

Fig. 2 DD value of listed companies. DD, distance to default.

4.3 Empirical results and analysis

1) Empirical results. The relationship curve between KVM's default distance and expected default probability (see Figure 1) is similar to an exponential function graph. Therefore, we use the above calculation results and the Excel index fitting regression method to fit the EDF function of the default distance of 178 listed companies in China. The relevant result is shown in Figure 3.

The logarithm of the fit is $\ln y = 5.3908 - 1.3426x(8.1680^*)(-5.2219^*)$, $wR^2 = 0.9317$, $F = 27.2687^*$ here y represents the EDF value; x represents the DD value. The t value is given in parentheses. *Indicates significance at the 5% level. It can be seen from the R^2 value that the goodness of fit of the fitting equation is relatively high. The F value indicates that the regression equation has good significance. The corresponding exponential function is $y = 219.38e^{1.3426x}$.

(2) Result from the analysis. From the DD value of listed companies (see Figure 2), we can see that the company's default distance is mainly concentrated between 2 and 4. This is consistent with the actual situation. The worst companies and the best companies are relatively small. Another reason for the small number of companies with low DD values in the sample is that ST companies were excluded from the sample screening.

Our simplified assumption of using ST company as the defaulting company may make EDF valuation too large because not all ST company loans will eventually default, which may cause the DD-EDF curve to shift upward [10]. At the same time, due to the limitation of sample size, the number of listed companies with low

Fig. 3 The EDF function representing the company's default distance. DD, distance to default; EDF, expected default frequency.

DD value and high DD value in the sample is small. This may cause a specific error in the mapping curve of DDEDF, whose DD value is <1.5 and >4 in Figure 3. We see that the fitting curve still has a high degree of goodness of fit to the actual data, which shows that the model has a significant effect on distinguishing the credit quality of listed companies.

5 Test of empirical EDF function

To test the effect of the EDF as mentioned in the above method on the credit risk measurement of listed companies in China, this paper uses a pilot test method to test its predictive ability.

Fangda Carbon New Material Technology Co., Ltd. is a company limited by shares in the graphite and carbon products industry. It was listed on the Shanghai Stock Exchange on 30 August 2002. The total share capital is 40 million shares, of which 176.77 million A-shares are outstanding. As of the end of 2020, its bank loans due and outstanding were 362.5 million yuan and owed 9.36 million yuan. On 26 April 2021, the Shanghai Stock Exchange implemented a delisting risk warning, and the stock abbreviation was changed to '*ST Hailong.' We used the EDF model method and combined it with the empirical EDF function shown in Figure 3 to analyse its credit status 3 years before default. The change in credit quality is shown in Figure 4.

It can be seen from Figure 4 that the frequency of overdue defaults of the company has risen sharply since December 2019. The EDF model well predicted Fangda Carbon New Material Technology Co., Ltd. 16 months before the default date (April 2021) the deterioration of credit quality.

Sihuan Pharmaceutical Co., Ltd. is a joint-stock company in the chemical pharmaceutical preparation industry. It was listed on the Shenzhen Stock Exchange on 13 September 1996, with a total share capital of 93.325 million shares, including 31.35 million A-shares in circulation. As of the end of 2021, the company has accumulated losses of 53.133 million yuan, and the company has overdue borrowings of 210 million yuan. We used the EDF model method and combined it with the empirical EDF function shown in Figure 4 to analyse the credit status of Sihuan Pharmaceutical in the 3 years before default. The credit change is shown in Figure 5.

It can be seen from Figure 5 that the frequency of overdue defaults by Sihuan Pharmaceutical has risen sharply from June 2020, and the EDF will even exceed 20% after March 2021. It can be seen that the EDF

Fig. 4 Fangda carbon New Material Technology Co., Ltd. EDF. EDF, expected default frequency.

Fig. 5 Sihuan Pharmaceutical Co., Ltd. EDF. EDF, expected default frequency.

model well predicted the deterioration of Sihuan Pharmaceutical's credit quality in the 22 months before the default date.

From the above empirical results, it can be seen that the EDF model predicts the decline in the credit quality of listed companies 1–2 years before default. We give an absolute measure of the credit quality of listed companies (EDF). The dynamic measurement of the credit quality of the EDF model enables commercial banks to grasp the changes in the company's credit quality in time.

6 Applicability of EDF model to Chinese commercial banks

(1) The EDF model does not assume that the securities market is efficient. This may be good news for a capital market that needs to be improved and matured in China. Market effectiveness usually means that all

relevant information about the company's value is reflected in the current price. The EDF model is difficult to continuously select stocks and know when the market underestimates or overestimates the value of a company. The market reflects a general expectation of many investors, and it is usually impossible to say that a particular person or a particular institution's prediction is better.

(2) For Chinese commercial banks, introducing the EDF model to manage credit risk has many advantages over traditional methods. The excellent characteristics of the EDF model make it possible to be an effective tool for commercial bank credit risk measurement. The traditional method cannot calculate daily or monthly EDF value monitoring levels like the EDF model method. Continuous monitoring is the only effective method for early warning of credit quality deterioration. In addition, because the EDF value is an actual probability, it is also critical data for commercial bank pricing and performance measurement.

Commercial banks have consistently ignored the application of market prices in lending decisions. The introduction of the EDF model can effectively promote the deepening of the credit culture of commercial banks. At the same time, it can effectively reduce the higher fixed costs brought about by traditional credit analysis. It improves the competitiveness of commercial banks and can ensure the consistency of credit risk assessment and credit pricing.

7 Conclusion

The research results of this paper show that the EDF model has a good predictive ability for credit risk. We choose the equity value volatility calculated by the daily closing price of stocks for 1 year, making the EDF model have good stability for the company's credit risk measurement. This effectively reduces the impact of the noise of sharp short-term fluctuations in stock prices on a credit evaluation.

References

- [1] Bernstein, S., Colonnelli, E., & Iverson, B. Asset allocation in bankruptcy. *The Journal of Finance*, 2019. 74(1): 5-53.
- [2] Vidyarthi, H. Dynamics of intellectual capitals and bank efficiency in India. *The Service Industries Journal*, 2019. 39(1): 1-24.
- [3] Bayar, Y. Macroeconomic, institutional and bank-specific determinants of non-performing loans in emerging market economies: A dynamic panel regression analysis. *Journal of Central Banking Theory and Practice*, 2019. 8(3): 95-110.
- [4] Umar, Z., Shehzad, C. T., & Samitas, A. The demand for eurozone stocks and bonds in a time-varying asset allocation framework. *The European Journal of Finance*, 2019. 25(11): 994-1011.
- [5] Dar, Q. F., Ahn, Y. H., & Dar, G. F. The Impact of International Trade on Central Bank Efficiency: An Application of DEA and Tobit Regression Analysis. *Statistics, Optimization & Information Computing*, 2021. 9(1): 223-240.
- [6] Al-Homaidi, E. A., Ahmad, A., Khaled, A. S., & Qaid, M. M. External factors and banks' performance: An empirical examination of commercial banks listed on Bombay Stock Exchange (BSE). *International Journal of Emerging Technologies and Innovative Research*, 2019. 6(6): 368-371.
- [7] Acar, E., Zgi, A., & Serenbay, S. K. Note on jakimovski-leviatan operators preserving $e-x$. *Applied Mathematics and Nonlinear Sciences*, 2019. 4(2): 543-550.
- [8] Rahaman, H., Kamrul Hasan, M., Ali, A. & Shamsul Alam, M. Implicit Methods for Numerical Solution of Singular Initial Value Problems. *Applied Mathematics and Nonlinear Sciences*, 2020. 6(1): 1-8.
- [9] Angraini, R., & Prastiwi, M. Determinants of Bank Profitability: The Case of Listed Bank on Indonesian Stock Exchange. *EkBis: Jurnal Ekonomi dan Bisnis*, 2020. 3(2): 274-283.
- [10] Nkechi, O. T., & Sunday, F. A. Effect of assets utilization on net worth of big cap companies quoted in Nigeria. *Journal of Accounting, Business and Social Sciences*, 2020. 3(1): 1-17.