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Application of Lane-Emden differential equation numerical method in fair value analysis of financial accounting

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Abstract

In order to study the fair value analysis of financial accounting, the Euler wavelet method is proposed to solve the numerical solutions of a class of Lane-Emden type differential equations with Dirichlet, Neumann and Neumann-Robin boundary conditions. The results show that the fractional integral formula of Euler wavelet function under the Riemann-Liouville fractional order definition and the L∞ and L2 errors of Haar wavelet are derived by the analytic form of Euler polynomial. By fixing M=4 and increasing the resolution scale k of Euler wavelet, a stable convergence solution can be obtained. The Lane-Emden equation with boundary conditions is transformed into algebraic equations by Euler wavelet collocation method, and the numerical results are compared with the results and exact solutions of other methods. The application advantages of fair value can be exerted through financial accounting to promote the transformation and upgrading of enterprises and realise the stable economic growth.

Keywords: Fair value, Accounting practice, Functional differential equations.
AMS 2010 codes: 34A08

1 Introduction

Contrast is currently being implemented as the accounting standard for business enterprises; major changes have taken place in the new accounting standard for business enterprises to achieve the substantial convergence with international financial reporting standards in our country. This new revision of ‘accounting standard for business enterprises’ made great adjustment in measurement attributes, no longer emphasises the historical cost as the basic measurement attributes and comprehensively introduced the fair value and present value measurement attribute [1]. The new guidelines stipulate the use of fair value for measurement of financial instruments,

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investment real estate mergers that are not under common control and debt restructuring and non-monetary transactions. The rigid Volteera functional differential equation stability theory and its numerical method is applied to the rigid delay differential equations and numerical methods of research, obtained a new set of stability and convergence results from nonlinear rigidity of delay differential equations and numerical methods; these results are more general compared to existing results in the literature and profound theoretical results obtained acts as a pointer, push and recommend some efficient algorithms for rigid delay differential equations , which are also efficient algorithms for general rigid functional differential equations [2].

The main difficulty of numerical solution of Lane-Emden equation is the singularity at x=0. Some scholars have proposed some methods to solve the Lane-Emden equations with Dirichlet and Neumann-Robin types, such as collocation method, finite difference method, spline finite difference method, B-spline, cubic spline method, Adomian decomposition method, variational iteration method, homotopy analysis method, and wavelet method. However, there are few literatures on the numerical methods of Lane-Emden equations with Neumann type boundary conditions. In recent years, due to the characteristics of compact support, vanishing moment and orthogonality of wavelet, numerical techniques based on wavelet have been paid wide attention in the field of numerical approximation [3].

Among them, the wavelet generated by stretching and translation of polynomials is used to solve different types of linear and nonlinear (fractional order) ordinary differential equations and partial differential equations, such as Legendre wavelet, Chebyshev wavelet and Bernoulli wavelet. Recently, Euler wavelet was first proposed and applied to the numerical solution of nonlinear Volterra fractional integral and differential equations. At present, there are few literatures on Euler wavelet and no literature has applied it to the solution of Lane-Emden equation. In order to further extend the application of Euler wavelet, we use Euler wavelet configuration method to solve the Lane-Emden equation with Dirichlet, Neumann and Neumann-Robin boundary conditions [4]. The application of wavelet is to transform the equation into algebraic equations by means of wavelet integral operator matrix and differential operator matrix. The existing Euler wavelet literature constructs the fractional order integral operator matrix of Euler wavelet function with the aid of Block function. Different from the literature method, the author does not use the integral operator matrix, but converts the equation to algebraic equations for solving with the aid of the derived Euler wavelet function arbitrary fractional order integral calculation formula. Then Newton iterative method is used to solve the problem. Numerical experimental results show the effectiveness of the proposed method [5].

On 15 February, 2006 the accounting standards system was issued, which included 1 basic standard and 38 specific standards, not only on the original accounting standards system in the 16 content of major revisions and supplements, but also based on the actual economic situation and economic development trend in China, a reasonable increase of 22 new standards; It effectively updated the accounting items and accounting statement format of the standard edition, laid a foundation for the use of fair value and also focused on the concept of fair value [6]. At present, China is facing a severe economic environment and complex economic composition and so the use of fair value in financial accounting, not only can reduce the operating pressure of enterprises, but also can help the person in charge of enterprises to master the real value of financial assets and liabilities, help the person in charge of enterprises to make the right decision and to achieve the long-term healthy development of enterprises.

By analysing the application of fair value in financial accounting, it can be found that the application of fair value in the process of financial accounting is based on the enterprise actual economic situation; Furthermore it helps enterprise decision-makers with financial report issued by the financial and accounting controls over a period of enterprise economics to take decisions, helps to make accurate projections for enterprise development and also the direction, strengthens the work level and ability of enterprise financial accounting and helps to realise the transformation of enterprise financial accounting to management accounting [7]. From the traditional way of enterprise financial accounting work, it will be very difficult for the decision makers to know the real financial information and the risks involved in real-time and moreover it will take certain time to realise that. With the introduction of fair value, the enterprise decision-makers will know enterprise financial risk in real-time, and the application of fair value can provide effective feedback about trading enterprise economy, reveals the fair value of the information provided by the predictive value of the enterprise value and feedback value, and helps enterprises to avoid risks. In the 1980s, the savings and loan crisis happened in the United States, which led to the bankruptcy of more than 400 financial institutions. Due to the condition of insufficient solvency, some enterprises and financial companies were paid more than 100 billion dollars in total by the Federal Reserve Fund of the federal government, thus maintaining the normal operation of the American economy. Since then, the American financial community has effectively expanded the application scope of fair value in financial accounting, effectively mastered real-time real financial information, and improved the ability to respond to the financial crisis.

The application of fair value in financial accounting should strictly follow the application principle, which can reduce financial risks to a certain extent. First, the principle of authenticity is used. The application of fair value in financial accounting is carried out according to the real-time transaction information of enterprises [8]. It fully follows the principle of authenticity to combine the fair value with the reality and avoid the application of fair value in financial accounting. The second one is gradual principle; the current application of fair value in our country still has certain limitations as China's economic environment is relatively complex and the market dynamic changes frequently. In order to effectively guarantee the stability of enterprise economic development, enterprises must follow the principle of step by step, in combination with the practical situation of the enterprise and the main development direction. The economic and reasonable application of fair value gradually formed a perfect supporting system and the legal standard. Through financial accounting will provide full play to the application advantages of fair value, promote the transformation and upgrading of enterprises and achieve stable economic growth.

2 Research methods

2.1 Euler wavelet function and convergence analysis

The Euler wavelet function defined in the interval [0,1) has four parameters, k is any positive integer, n = 1, 2, 3, ..., 2k-1, m is the order of the Euler polynomial, x is the independent variable, the Euler wavelet is defined as follows:

The function $y(x)$L2 [0,1) defined on the interval [0,1] can be expanded by Euler wavelet as follows.

2.2 Algorithm Description

The Euler wavelet configuration method for solving Lane-Emden equation under three different boundary conditions is given [9, 10]. For the convenience of description, the fractional order integral of Euler wavelet function sequence (8) is written as I$\alpha\Psi$(x).

2.2.1 Dirichlet type boundary conditions

First, let the second derivative of the unknown function in Equation (1) be expressed by the Euler wavelet function:

$$\mathrm{Y}(\mathrm{x}) = \mathrm{CT}\Psi(\mathrm{x}) \tag{2.1}$$

condition (2) to obtain

$$\mathrm{Y}'(\mathrm{x}) = (1 - \mathrm{alphabeta1} - \mathrm{CTI2}\Psi(1)) + \mathrm{CTI}\Psi(\mathrm{x}) \tag{2.2}$$

$$\mathrm{Y}(\mathrm{x}) = \mathrm{alpha1} + \mathrm{x}(1 - \mathrm{alphabeta1} - \mathrm{CTI2}\Psi(1)) + \mathrm{CTI2}\Psi(\mathrm{x}) \tag{2.3}$$

Substitute Eqs (4)–(6) into Eq (1) to get

$$\begin{aligned} &\mathrm{CT}\Psi(\mathrm{x}) + \mathrm{alphax}(1 - (\mathrm{betaalpha1} - \mathrm{CTI2}\Psi(1)) + \mathrm{CTI}\Psi(\mathrm{x})) \\ &+ \mathrm{f}(\mathrm{x}, \mathrm{alpha1} + \mathrm{x}(1 - \mathrm{alphabeta1} - \mathrm{CTI2}\Psi(1)) + \mathrm{CTI2}\Psi(\mathrm{x}) = 0 \end{aligned} \tag{2.4}$$

Defining configuration points according to the configuration method, we get

$$\mathrm{Xj} = 2\mathrm{j} - 12\mathrm{kM}, \mathrm{j} = 1, 2..., 2\mathrm{k} - 1\mathrm{m}$$

By substituting xj into Eq. (7), a nonlinear system of equations with 2k-1m variables is obtained. Newton iterative method is used to solve these equations and unknown vector Ct can be obtained. By substituting Ct into Eq. (6), the numerical solution of Lane-Emden equation can be obtained.

2.2.2 Neumann type boundary conditions

Let the second derivative of the unknown function y(x) in Eq. (1) be expressed by the Euler wavelet function

$$\mathrm{Y}(\mathrm{x}) = \mathrm{CT}\Psi(\mathrm{x}) \tag{2.5}$$

Integrate Eq. (8) from 0 to x twice respectively and combine with boundary conditions to obtain

$$\mathrm{Y}'(\mathrm{x}) = \mathrm{alpha2} + \mathrm{CTI}\Psi(\mathrm{x}) \tag{2.6}$$

$$\mathrm{Y}(\mathrm{x}) = \mathrm{alpha2x} + \mathrm{y}(0) + \mathrm{CTI2}\Psi(\mathrm{x}) \tag{2.7}$$

Taking y(0) as an unknown variable, and substituting Eqs (7)–(9) into Eq. (1), we get

$$\mathrm{CT}\Psi(\mathrm{x}) + \mathrm{alphax}(\mathrm{alpha2} + \mathrm{CTI}\Psi(\mathrm{x})) + \mathrm{f}(\mathrm{x}, \mathrm{alpha2x} + \mathrm{y}(0) + \mathrm{CTI2}\Psi(\mathrm{x}) = 0 \tag{2.8}$$

According to the boundary condition (3)

$$\mathrm{Alpha2} + \mathrm{CTI}\Psi(1) = \mathrm{beta2} \tag{2.9}$$

The configuration points

$$\mathrm{Xj} = 2\mathrm{j} - 12\mathrm{kM}, \mathrm{j} = 1, 2...2\mathrm{k} - 1\mathrm{m}$$

Substituting the configuration points into Eq. (20) and combine with Eq. (12) to obtain a nonlinear system of equations containing 2k-1m +1 variables. Newton iterative method is used to solve these equations to obtain unknown vectors Ct and Y (0). Substitute Ct and Y (0) into Eq. (9) to obtain the numerical solution of Lane-Emden equation.

3 Research Results

In order to test the validity of Euler wavelet configuration method, the following Lane-Emden equation with three different boundary conditions is considered and the maximum absolute error is defined

$$\mathrm{L}\infty = \mathrm{maxx}[0, 1]\,\mathrm{y}(\mathrm{x}) - \mathrm{yw}(\mathrm{x}),$$

Wherein, y(x) is the exact solution, and yw(x) is the Euler wavelet numerical solution.

Example 1 considers the following Lane-Emden equation with Dirichlet boundary conditions

$$\mathrm{Y}(\mathrm{x}) + 0.5\mathrm{x}, \mathrm{y}'(\mathrm{x}) + \exp(2\mathrm{y}(\mathrm{x})) \text{ to } 0.5\exp(\mathrm{y})\,(\mathrm{x}) = 0, \mathrm{y}(0) = \ln 2, \mathrm{y}(1) = 00 < \mathrm{x} < 1,$$

The exact solution to this equation is y(x)=ln2 $\times$ 2+(1).

The L∞ and L2 errors between Haar wavelet and the proposed algorithm are given. As can be seen from Table 1, a stable convergence solution is generated by increasing the resolution scale k of the Euler wavelet by fixing the value M=4. Table 2 shows the L∞ errors of Euler wavelet under different parameters. As can be seen from Table 2, the numerical solution with higher precision can be obtained by fixing the resolution scale of Euler

Table 1 Comparison of L∞ and L2 errors under different parameters

Error method	L∞	L∞	**L2**	**L2**
k=1	Haar	Euler	Haar	Euler
k=2	7.80e−4	5.36e−5	1.63e−3	1.68e−4
k=3	1.94e−4	3.97e−6	5.74e−4	1.21e−5
k=4	4.84e−5	2.73e−7	2.03e−4	8.57e−7
k=5	1.21e−5	1.76e−8	7.18e−5	5.50e−8
k=6	3.02e−6	1.17e−9	2.54e−5	3.66e−9
k=7	7.55e−7	7.39e−11	8.97e−6	2.32e−10
k=8	1.89e−7	4.59e−12	3.17e−6	1.44e−11

Table 2 L∞ error under different parameters

k	**M=3**	**M=4**	**M=5**	**M=6**	**M=7**	**M=8**
2	7.83e−4	5.36e−5	2.86e−5	1.79e−6	5.77e−7	7.33e−8
3	3.73e−5	3.97e−6	3.59e−7	4.66e−8	3.07e−9	5.78e−10

Table 3 Comparison of numerical solutions and exact solutions with different parameters

x		**k=2, M=4**	**k=3, M=5**	**k=5, M=6**
0	0.693147180559945	0.693147180559945	0.693147180559945	0.693147180559945
0.1	0.683196849706777	0.683227245796262	0.683197075022063	0.683196849437443
0.2	0.653926467406664	0.653969148173502	0.653926783980363	0.653926466991036
0.3	0.606969484318893	0.607019006611345	0.606969843582379	0.606969483813016
0.4	0.544727175441672	0.544780344557389	0.544727529512898	0.544727174911210
0.5	0.470003629245736	0.470056791556372	0.470003963513028	0.470003628756537
0.6	0.385662480811985	0.385704191793314	0.385662747357822	0.385662480415734
0.7	0.294371060602578	0.294402255148925	0.294371256591398	0.294371060311363
0.8	0.198450938723838	0.198471255975016	0.198451064512775	0.198450938537864
0.9	0.099820335282211	0.099829667365162	0.099820397248811	0.099820335194224
1.0	0	0	0	0

wavelet and increasing the M value, which is also consistent with the conclusion of Theorem 1. Table 3 shows the comparison of numerical and exact solutions with different parameters.

Considering the following Lane-Emden equation with Neumann-Robin boundary conditions

$$\mathrm{Y}(\mathrm{x})+2\mathrm{xy}'(\mathrm{x})+\mathrm{y}5(\mathrm{x})=0, \mathrm{y}'(0)=0, 0<\mathrm{x}<1, \mathrm{y}(\mathrm{x})+2\mathrm{xy}'(\mathrm{x})+\mathrm{y}5(\mathrm{x})=0,$$

the L∞ and L2 errors of Haar wavelet are given in Table 4. As can be seen from Table 4, fixing the value of M=4, increasing the resolution scale k of Euler wavelet generates stable convergence solution. Table 5 shows the L∞ errors of Euler wavelet under different parameters. As can be seen from Table 5, the numerical solution with higher precision can be obtained by fixing the resolution scale of Euler wavelet and increasing the value M.

4 Conclusion

In this paper, an Euler wavelet method for numerical solutions of nonlinear singular Lane-Emden equations is proposed. Using the analytical form of Euler polynomials and the Riemann-Liouville definition of fractional integral, the general formula for calculating the fractional integral of Euler wavelet function is derived. The

Table 4 L∞ and L2 errors under different parameters

	Haar	Euler	Haar	Euler
k=2	1.97e−3	1.05e−6	4.71e−4	2.28e−7
k=3	4.91e−4	6.99e−8	1.65e−4	1.97e−8
k=4	1.23e−4	6.41e−9	5.84e−5	2.79e−9
k=5	3.12e−5	4.32e−10	2.06e−5	2.72e−10
k=6	8.92e−6	2.75e−11	7.28e−6	2.46e−10
k=7	2.96e−6	1.73e−12	2.56e−6	2.19e−11
k=8	1.29e−7	1.08e−13	8.81e−7	1.93e−12

Table 5 L∞ error under different parameters

k	M=3	M=4	M=5	M=6	M=7	M=8
2	2.69e−5	1.05e−6	5.27e−7	1.13e−8	8.59e−9	2.01e−10
3	1.76e−6	6.99e−8	7.55e−9	4.89e−10	2.63e−11	1.61e−12

solution of Lane-Emden equation is transformed into a set of nonlinear algebraic equations by using the deduced formulas and the collocation method, and then the Newton iterative method is used to solve the equations. The validity and applicability of the proposed method are verified by comparing numerical examples with relevant literatures and exact solutions. The algorithm is simple and effective in programming and the approximate solution with high precision can be obtained by using very small sum values.

Although there are some problems in the application of fair value in our country, but from the accounting context and for the long-term development trend and the direction of our country economy development, the introduction of new accounting standards with fair value measurement attribute and its adaptation to the economic development of our country is actually a right choice; further, as a result, the process of accounting practice in our country also has a convergence to a certain extent with international accounting practice. Therefore, we should face it and meet it with a positive attitude, with each one performing his own duties and fulfilling his own responsibilities, so as to cultivate a more favourable environment for the further promotion and use of fair value.

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