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Application of numerical method of functional differential equations in fair value of financial accounting

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Abstract

In order to improve the information quality of financial accounting reports, this paper puts forward a functional differential equation, establishes the determination model of fair value by mathematical analysis method and studies the correlation of financial accounting information of fair value through this model. The results show that according to the fair value determination model, in the free accounting environment, fair value financial accounting information has an impact.

Keywords: Functional differential equation; Fair value; Financial accounting information; Value relevance.**AMS 2010 codes:** 34K35

1 Introduction

With the continuous development of capital markets, derivatives of continuous innovation, based on the historical cost measurement and the highlighted drawbacks of the accounting information system, fair value accounting becomes a difficult and tough problem at the same time for all accounting boards including the International Accounting Standards Board (IASB), American accounting standards board (FASB) or our country's accounting standards board. They are being studied in depth. In 2001, FASB successfully studied and drafted more than 30 drafts on fair value measurement and finally published the No. 157 Financial Accounting Standards Bulletin – SFAS-157 'Fair Value Measurement' in September 2006, which clearly standardised the general framework of measuring and disclosing assets and liabilities at fair value. It provides a model for measuring all assets and liabilities at fair value. The Ministry of Finance in China also promulgated the accounting standards for enterprises on 15 February 2006, which introduced the measurement model of fair value on the premise of improving the quality of accounting information.

China's accounting standards and accounting system reform and change depend on its unique national conditions, that is, China's socialist market economy. Since the implementation of the reform of multi-financing

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joint-stock enterprises in China, China Securities Regulatory Commission (CSRC), has promoted the trial of non-tradable shares, standardised the listing and refinancing procedures of companies, promulgated a large number of regulations to strengthen the information disclosure of listed companies, and strengthened the investigation and punishment of fraud, the securities market has achieved rapid and standardised development. With the emergence of listed companies and the rapid development of the capital market, the main role of enterprise accounting disclosure has shifted from providing useful accounting information to enterprise owners – government macroeconomic managers to providing decision-making information to investors, creditors, managers and other information users of enterprises. At the same time, fair value can make up for the deficiency of historical cost because its measurement attribute is the objective measurement of current assets and liabilities. From this fair value accounting concept into accounting standards. However, the application of fair value should conform to its specific economic environment conditions. In theory, the fair value of the perfect theory needs to be on the basis of microeconomics in a perfect competition market, even if the fair value accounting in reality also should set in mature markets, so that it can be observed in the active market quotation trading assets or liabilities, transfer price, to reflect the fair value of the objective, fair, easy access, etc.

Considering China's unique market economic environment, its accounting standards committee carefully selected the fair value: only financial assets, investments in real estate, debt restructuring and other a few projects under strict conditions to implement the fair value principle. Since the implementation of accounting standards on 1 January 2007, this paper has obtained a certain amount of fair value data information to conduct empirical research on fair value, reflecting whether the application of fair value by Chinese listed companies has improved the quality and value correlation of accounting information, and testing the application effect of fair value in China. Therefore, it is of great theoretical significance and practical value for the development of fair value accounting in China to have a deeper understanding of the economic consequences of prudent selection of fair value. In China, since the establishment of the concept of growing concern, the historical cost has occupied a dominant position in accounting measurement. Combined with the characteristics of accounting and reliability, the historical cost has become the most basic measurement attribute. Although historical cost still has many defects, it is difficult to replace other measurement attributes in view of the current situation.

On the value correlation of fair value accounting information, domestic and foreign scholars have made a large number of studies. The conclusions of the normative studies made by Oyewo B and Irwin T C basically confirm the fair value measurement, and they believe that the measurement attribute of fair value should be widely used [1,2]. Domestic scholars, such as Keshk W, also believe that fair value measurement can improve the value correlation of accounting information, thus providing investors with more useful accounting information [3].

In terms of empirical research, Lilien and Miah believe that fair value accounting information is value-related [4,5]. McDonough found that the accounting environment would significantly affect the relationship between fair value accounting information and stock price [6]. When conducting empirical research on fair value correlation, some scholars mostly use the Ohlson model or make some modifications on the basis of the Ohlson model, in order to test the explanatory power of accounting data on stock prices and earnings. Ohlson model mainly includes the price model and income model. The price model is used to test the explanatory power of accounting data on stock price, and the main variables used are stock price, net assets per share and earnings per share. The earnings model is used to test the explanatory power of accounting data to stock return rate. The variables used are mainly stock return rate, earnings per share and the change amount of earnings per share.

When using the Ohlson model, some scholars modify the variables in the model according to their own research characteristics, so as to improve the accuracy of empirical research. Anren and Lindner replaced the earnings per share in the Ohlson model with the profit and loss with and without profit and loss with the fair value change, respectively, so as to investigate whether the fair value change has the ability of incremental value interpretation [7,8]. On this basis, Moretti divides net assets per share into financial assets with fair value measurement and net assets without financial assets, which are used as variables of the price model [9].

In the model design, some scholars use the incremental information content method to test whether the

fair value measurement of financial instruments in China's new accounting standards has an incremental information content and whether the measurement in the fair value statement has more information content than that disclosed off the balance sheet. After studying the samples of net assets per share, earnings per share and changes per share of financial assets measured at fair value on and off the balance sheet, Xu Hong conducted the research on the full sample.

2 Economic analysis of fair value

The marginal utility theory of value is the cornerstone of western economics. Although it does not reveal the true meaning of value like Marx's labour theory of value, it depicts the objective law of Commodity Exchange in the market economy for us after all. The description of fair value and marginal utility value to exchange value is based on the subjective value of people's feelings and evaluation of assets. Therefore, the formation of fair value can be understood from the perspective of utility economics.

Just as consumers consume goods, firms are 'consumers' of assets. If the utility of commodities to consumers is to meet their needs, then, for enterprises, the utility of assets is to help enterprises realise the purpose of the operation.

2.1 Model and Assumption of Fair Value

(1) Hypothesis

Assume that there are N possible assets. Let the asset to be measured is NTH. For a particular seller (firm), let its asset utility function on the balance sheet date be $u = U_S(x_1, x_2, \dots, x_N, m)$, where U represents the total utility of maintaining the use or holding of these assets in the current state, represents the number of non-monetary assets and m is the number of monetary assets

Assumption: The quantity of non-monetary assets and monetary assets of the seller before the transaction is $s_1, s_2, \dots, s_N, m_s$, respectively. After exchanging the NTH asset of unit Q with the price P , the seller's total asset utility changed as follows:

$$f_S(q, p) = U_S(s_1, s_2, \dots, s_N - q, m_s + qp) - U_S(s_1, s_2, \dots, s_N, m_s)$$

For a particular buyer, let the asset utility function on the balance sheet date be $u = U_B(x_1, x_2, \dots, x_N, m)$, and assume that the number of non-monetary assets and monetary assets of the buyer before the transaction is $b_1, b_2, \dots, b_N, m_b$; after buying the NTH asset of q unit at the price p , the change in the total utility of the asset is

$$f_B(q, p) = U_B(b_1, b_2, \dots, b_N + q, m_b - qp) - U_B(b_1, b_2, \dots, b_N, m_b).$$

Using the asset utility function, the concept of fair value can be re-expressed as:

For a specific amount of asset ($\bar{q} (\bar{q} > 0)$ the NTH asset), the exchange price $\bar{p}$ can be called the fair value corresponding to the quantity supplied $\bar{q}$ if:

(I) For the buyer, when the purchase quantity is $\bar{q}$, $f_B(\bar{q}, \bar{p}) \geq 0$, and $\frac{\partial U_B}{\partial x_N} \geq \bar{p} \frac{\partial U_B}{\partial m}$ and at any exchange price $p < \bar{p}$, $\bar{q}$ assets cannot be bought.

(II) For the seller, when the selling amount is $\bar{q}$, there is $f_S(\bar{q}, \bar{p}) \geq 0$, and $\frac{\partial U_S}{\partial x_N} \leq \bar{p} \frac{\partial U_S}{\partial m}$ and at any exchange price $p > \bar{p}$, they cannot sell all of $\bar{q}$ the assets.

Conditions (I) are fair conditions of the purchaser; conditions (II) are fair conditions of the seller. The constraint condition of the increase of total utility is the basis of both parties' willingness to exchange. The marginal utility constraint represents the rationality of the exchange parties in terms of the quantity exchanged, that is, the exchange of the $\bar{q}$ asset is cost-effective, so the exchange of all $\bar{q}$ assets can be completed. The constraint condition of exchange price represents the rationality of both parties in exchange price, that is, the buyer cannot lower the price and the seller cannot raise the price [10].

Model

For a given price p , build the following buyer optimisation model:

$$\begin{aligned} \max_{0 \leq q} f_B(q, p) \\ s.t. f_B(q, p) \geq 0. \end{aligned}$$

The corresponding relationship between the price determined by the buyer's optimisation model and the buyer's optimal purchase quantity is called the buyer's optimal purchase quantity function, denoted as $q = B(p)$. When there are multiple buyers in the market, the optimal buying quantity function can be obtained by summing their optimal buying quantity function. The optimal buying volume function decreases as the price rises. In a coordinate system with price as the vertical axis and trading volume as the horizontal axis, the curve of the optimal buying volume function is called the optimal buying volume curve (see Figure 1). The inverse function of the optimal buying quantity function is denoted as $p = P_B(q)$.

Hypothesis: There is a non-zero optimal solution in the buyer's optimisation model. According to the necessary condition $\frac{\partial f_B(q, p)}{\partial q} = 0$ of the optimal solution, it can be known that the non-zero optimal solution satisfies $\frac{\partial U_B}{\partial x_N} = p \frac{\partial U_B}{\partial m}$.

Similarly, for a given price p , the following seller optimisation model is established as follows:

$$\begin{aligned} \max_{0 \leq q} f_S(q, p) \\ s.t. f_S(q, p) \geq 0. \end{aligned}$$

The corresponding relationship between the price determined by the seller's optimisation model and the seller's optimal selling quantity is called the seller's optimal selling quantity function, denoted as $q = S(p)$. When there are many sellers in the market, the optimal selling quantity function of the market can be obtained by summing their optimal selling quantity function. The optimal selling function obviously increases as the price increases. In a coordinate system with price as the vertical axis and volume as the horizontal axis, the curve of the optimal selling volume function is called the optimal selling volume curve (see Figure 1). Consider the inverse function of the optimal selling quantity function as $p = P_S(q)$.

Hypothesis: The seller optimisation model has a non-zero optimal solution Q , and the non-zero optimal solution satisfies $\frac{\partial U_S}{\partial x_N} = p \frac{\partial U_S}{\partial m}$:

Fig. 1 Optimal selling curve

It can be seen that no matter for the seller or the buyer, the quantity willing to exchange is different at different exchange prices. If there is an intersection point between the optimal selling volume curve and the

optimal buying volume curve, the volume corresponding to this point is called the market equilibrium volume, and the exchange price corresponding to this point is the market equilibrium price (see Figure 1). If the market is in a state of equilibrium volume and price, then the market is called a market of equilibrium supply and demand or in a state of equilibrium supply and demand [11].

Fair value determination model: it is assumed that on the balance sheet date, the total supply of assets is Q ($0 > 0$). In the market environment of free competition, the market optimal selling volume curve of the asset is $q = B(p)$ and the anti-rain number is $p = P_S(q)$. The optimal buying volume curve of the market is $q = B(p)$ and the inverse function is $p = P_B(q)$; there is an intersection point between the optimal selling volume curve and the optimal buying volume curve; the equilibrium trading volume is $\bar{q}$ and the equilibrium price is $\bar{p}$. So (q

If $Q \leq \bar{q}$, then the fair value; $= P_B(Q)$.

If $Q > \bar{q}$, then there is no fair value for selling all of Q .

The fair value determination model is proved as follows:

Assumption $Q \leq \bar{q}$.

According to the definition of the optimal buying volume curve, there are one or more buyers, and the technical price $P_B(Q)$ is willing to buy an asset quantity of Q , after buying, meet: $f_B(q, p) \geq 0$ and $\frac{\partial U_B}{\partial x_N} = P_B(Q) \frac{\partial U_B}{\partial m}$.

According to the definition of the optimal selling curve, there are one or more sellers, so $P_S(Q)$ is willing to sell a total of Q assets. After selling, meet $f_S(q, p) \geq 0$ and $\frac{\partial U_S}{\partial x_N} = P_S(Q) \frac{\partial U_S}{\partial m}$.

Because of the monotonically selective and decreasing properties of $Q \leq \bar{q}$, the monotonically increasing properties of the optimal buying function and the optimal selling function, then $P_S(Q) \leq P_B(Q)$ (see Figure 1). So $\frac{\partial U_S}{\partial x_N} \leq P_B(Q) \frac{\partial U_S}{\partial m}$.

In addition, the marginal utility of the asset to the buyer has reached $\frac{\partial U_B}{\partial x_N} = P_B(Q) \frac{\partial U_B}{\partial m}$, obviously, if the price is higher than $P_B(Q)$, the buyer will be reluctant to buy the whole Q . This means that if the seller raises the price and makes it higher than $P_B(Q)$, then they cannot sell all Q . The sellers have the information about the optimal buying volume curve and take advantage of the buyer's competition; if the price $P_B(Q)$ sells all the Q , the price will not be lowered. So, the buyer cannot buy Q for less than $P_B(Q)$.

Therefore, if $P_B(Q)$ meets all the conditions for fair value, it must be fair value.

Assume $Q > \bar{q}$.

According to the definition of the optimal buying volume curve, if there are one or more buyers, at a price $P_B(Q)$, willing to buy a total of Q assets, after purchase, the marginal utility of the asset meets $\frac{\partial U_B}{\partial x_N} = P_B(Q) \frac{\partial U_B}{\partial m}$. So, if the price is higher than $P_B(Q)$, the buyer does not want to buy the Q th asset, and so they do not want to buy all of the Q s.

According to the definition of the optimal selling curve, there are one or more sellers; if $P_S(Q)$ is willing to sell a total of Q assets, after the sale, the marginal utility of the asset satisfies $\frac{\partial U_S}{\partial x_N} = P_S(Q) \frac{\partial U_S}{\partial m}$, and if the price is lower than $P_S(Q)$, the seller will be reluctant to sell the Q th asset, that is, to sell all of Q .

Due to the monotonically decreasing nature of $Q > \bar{q}$, the optimal buying quantity function and the monotonically increasing nature of the optimal selling quantity function, $P_S(Q) > P_B(Q)$ (see Figure 1). Therefore, the seller's lowest selling price $P_S(Q)$ will not be accepted by the buyer (price higher than $P_B(Q)$ will not be accepted).

So, there is no fair value for exchanging all of Q . After the [12].

3 Empirical Analysis

3.1 Mathematical statistical analysis

The results of mathematical-statistical analysis based on the demonstration of the Feltham-Ohlson model show that the P-value of the independent variable is 0.044, and that of the variable is 0.049, both showing

significance ($\alpha=0.05$). In addition, VIF values of each explanatory variable are less than 2, indicating that there is no serious multicollinearity problem that may affect the research conclusion in this paper. However, the test value of the model is 1.509, indicating that there is no serious serial autocorrelation in the model and the regression effect is good. The regression results of the whole model show that there is a significant correlation between the change of fair value per share and the stock price, and Hypothesis (1) is valid.

Based on the mathematical-statistical analysis results of the original formula of Feltham Ohlson model, it is concluded that the Adj R^2 value of Feltham-Ohlson model after introducing the measurement attribute of fair value reaches 0.548 in 2007 and 0.414 in 2008 and they are far higher than Adj R^2 in each of the five years before the introduction of the fair value measurement attribute (its maximum value was 0.343 in 2006). It shows that the accounting information of A-share listed companies in the past two years after the introduction of the fair value measurement attribute can explain the stock price; compared with the five years before the introduction of the fair value measurement attribute, there is a substantial increase. Namely, hypothesis (2) holds. As for the explanatory ability of accounting information in the following year, compared with that in the previous year, the author believes that part of the reason is the economic background in 2008 has changed significantly compared with that in 2007 due to the arrival of the financial crisis sweeping the whole world in 2008.

To sum up, the empirical analysis results confirm the hypothesis put forward in the paper, which reflects the introduction and use of the fair value attribute in the new accounting standards from one side and improves the value correlation of accounting information, that is, the introduction of the fair value measurement attribute after the implementation of the new accounting standards. It delivers new and useful information to the market, improves the explanatory ability of net assets and net profit per share to the stock price, and makes the accounting information dataset more decision-making value than before.

4 Conclusion

Through empirical research, we come to the following conclusions: (1) Based on the functional differential equation, fair value accounting information has value correlation, and at the same time, it has significant explanatory power for the stock price, but this explanatory power varies in different markets. In the market of fluctuation period, the explanatory power of fair value accounting information to stock price change is higher than that in the market of the stable period. (2) There is a certain correlation between the volatility of the numerical value of the functional differential equation and the fair value accounting information, and it is more significant in the period of market volatility than in the period of stability. In the fluctuation period, there is a positive correlation between volatility and fair value accounting information but not necessarily in the stable period. (3) The positive correlation between the volatility of the function value and the fair value accounting information mainly appears in the long period. In short, there is a positive correlation between fair value accounting information and the volatility of the function value, and this relationship is more obvious in the period of market volatility than in the period of stability.

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