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Human Body Movement Coupling Model in Physical Education Class in the Educational Mathematical Equation of Reasonable Exercise Course

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Abstract

In physical education class, the movement of the human body requires multiple joints to cooperate, and a multi-link system coupling is presented. In the teaching of physical education curriculum, the impact of the force received by students jumping up and down shows the characteristics of the non-linear system of physics and mathematics. Aiming at the movement process of jumping up and down, we established a joint mathematical equation model of the motion state of the human lower limb joints. We use a non-linear system to solve the mathematical model of the joint force coupling problem of the human body jumping up and down.

Keywords: Physical education; exercise coupling model; lower limb system; landing moment; mathematical model
AMS 2010 codes: 70H20.

1 Introduction

Analyzing human movement from biomechanical characteristics is a multi-link chain system characteristic. Different joints play different roles in this multi-link limb chain. At present, there is not much research on the multi-joint motion chain of the lower limbs, and the mechanism of motion control is not precise. The kinematic characteristics of the knee joint when the human body falls from a height are important factors that affect the ground reaction force. The impact at the moment of landing is a complex physical process, and the human body is viscoelastic [1]. Therefore, the human body has a series of dynamic responses to the impact at the moment of landing. This article attempts to use the dominant joint decomposition theory to establish a collaborative mathematical model of the lower limb system at the moment of landing. This article hopes that the conclusions drawn in the thesis will provide a theoretical basis for analyzing the complex motion mechanism of lower limbs.

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2 Mathematical model of the foot and ankle joint at the moment of human landing

The thesis defines the moment when the foot and ankle joints are in two states during the moment when the human body lands from a height to land (using a single foot as the research object). The first state is when the toe touches the ground, and the second state is when the foot is flat [2]. The moment the toe touches the environment comes from the muscle Achilles tendon force. For the ankle system, it is the longitudinal internal force F , and on the vertical plane $F_1 + F_G = mg + F_G$ and $F_1 = mg$.

$$\begin{cases} F_1 = mg \\ F_1 l_1 \sin \alpha_1 = F l_1 \cos \alpha_1 \\ F_G l_2 \sin \alpha_2 = F l_2 \cos \alpha_2 \end{cases} \quad (1)$$

We put together formula (1) to get a mathematical model of the moment the toe touches the ground

$$\begin{cases} F_1 = mg \\ F = F_1 \tan \alpha_1 \\ F_G = F / \tan \alpha_2 \end{cases} \quad (2)$$

Fig. 1 Schematic diagram of the human body landing from the toe touching the ground to the foot lying flat.

For the second state, the foot is always flat, assuming that the human body maintains a balanced and static condition $F_1 + F_2 = mg$. Decomposing from the moment of the ankle joint we get:

$$\begin{cases} F_1 l_1 \sin \alpha_1 = F l_1 \cos \alpha_1 \\ F_2 l_2 \sin \alpha_2 = F l_2 \cos \alpha_2 \end{cases} \quad (3)$$

Organize formula (2) to get

$$\begin{cases} F_1 = F / \tan \alpha_1 \\ F_2 = F / \tan \alpha_2 \end{cases} \quad (4)$$

From this we get the mathematical model of the human foot and ankle joint

$$\begin{cases} F_1 + F_2 = mg \\ F_1 = F / \tan \alpha_1 \\ F_2 = F / \tan \alpha_2 \end{cases} \quad (5)$$

From model (2) and formula (5), the joint mathematical model of the ankle joint of the human body at the moment of landing impact is obtained. F_1 is the ground reaction force at the forefoot (forefoot). F_2 is the ground

reaction force at the back of the foot (heel). m is the weight of the human body. g is the acceleration due to gravity. F is the resultant force of the longitudinal structure. l_1 The distance from the sole to the center of the ankle joint. l_1 The length from the sole of the back of the foot to the center of the ankle joint. α_1 is the angle between the line between the sole of the forefoot and the center of the ankle common and the vertical (y) direction. α_2 is the angle between the line from the sole of the back of the foot to the center of the ankle joint and the straight movement (y).

From this model, the longitudinal structural force of the foot can be calculated [3]. For the soles of the same size, the lower the arch (flat feet), the greater the angle between α_1 and α_2 . The greater the longitudinal structural stress acting on the sole that maintains the integrity of the turn. This causes excessive pressure on the connective tissue of the plantar, and the clinical response is to cause plantar fascia disease. The moment the toe touches the ground, the force on the longitudinal arch structure increases significantly. At the same time, a significant internal pressure occurs at the Achilles tendon inside the ankle system. At this time, people with plantar fascia pain will adapt to the pathological condition.

3 Mathematical model of knee and hip system at the moment of impact of human landing

According to the leading joint decomposition theory, we disassemble the common knee axis. We assume that both the hip joint and the knee joint rotate shaft structures around the center [4]. The direction of the axis of rotation is perpendicular to the forward direction. The mathematical model of the knee joint is as follows:

$$\begin{cases} F_C \cos \theta_C = m_C a_C \cos \varphi_C \\ F_C \sin \theta_C = m_C a_C \sin \varphi_C + m_C g \end{cases} \quad (6)$$

Fig. 2 Schematic diagram of the force on the lower limb system at the moment of landing.

The mathematical model of the hip joint obtained by inverse dynamics is as follows:

$$\begin{cases} -F_C \cos \theta_C + F_D \cos \theta_D = m_D \alpha_D \cos \varphi_D \\ -F_C \sin \theta_C + F_D \sin \theta_D = m_D \alpha_D \sin \varphi_D + m_D g \end{cases} \quad (7)$$

We can obtain the combined mathematical model of the knee joint and hip joint torque from the angular movement of the calf around the knee joint

$$\begin{cases} M_C - m_C g l_C \cos \beta_C = (I_C + m_C l_C^2) \alpha_C \\ M_D - M_C - m_D g l_D \cos \beta_D = (I_D + m_D l_D^2) \alpha_D \end{cases} \quad (8)$$

F_C, F_D is the resultant force on the knee joint and hip joint, respectively. θ_C, θ_D is the angle between the knee joint and hip joint force and the horizontal direction (x), respectively. M_C, M_D is the resultant moment on the knee joint and hip joint, respectively. m_C, m_D is the mass of calf and thigh, respectively. a_C, a_D is the acceleration on the mass center of the calf and thigh, respectively. φ_C, φ_D is the angle between the force generated by the mass center of the calf and the thigh and the horizontal direction (x). l_C is the distance from the center of the knee joint to the center of mass of the calf. l_D is the distance from the center of the hip joint to the center of mass of the thigh. β_C is the angle between the calf and the horizontal direction (x), β_D and is the angle between the thigh and the horizontal direction (x). α_C, α_D is the angular acceleration of the knee joint and hip joint, respectively. I_C, I_D is the moment of inertia of the calf and thigh, respectively. g is the acceleration of gravity. The combined torque represents the movement of the joint flexor muscles [5]. Since the human body is a complex structure, muscle contraction causes it to produce a resultant moment.

4 Analysis of the mathematical model of the human body's lower limb system motion coupling problem based on the Hamilton system

We assume $\dot{f} = \partial_\theta f$ and $f' = \partial_x f$, considering the dynamic coupling problem, that is, the Lagrange function is:

$$L(\varepsilon) = \frac{F(R-r)}{2r^2} \varepsilon^2 + \frac{G}{2} \left[(\varepsilon'')^2 + (\ddot{\varepsilon})^2 + 2\mu\beta\ddot{\varepsilon} + 2(1-\mu)(\dot{\varepsilon}')^2 \right] + \frac{N_x}{2} (\varepsilon')^2 - N_{x\theta} \varepsilon' \dot{\varepsilon} \quad (9)$$

We use the space coordinate to simulate the time coordinate t , according to the Hamilton variational principle:

$$\delta \iint \left\{ \frac{F(R-r)}{2r^2} \varepsilon^2 + \frac{G}{2} \left[(\varepsilon'')^2 + (\ddot{\varepsilon})^2 + 2\mu\varepsilon'' \ddot{\varepsilon} + 2(1-\mu)(\dot{\varepsilon}')^2 + \frac{N_x}{2} (\varepsilon')^2 - N_{x\theta} \varepsilon' \dot{\varepsilon} \right] \right\} r d\theta dx = 0 \quad (10)$$

We double-integrate the formula (2) to get:

$$\delta \iint \left\{ \frac{F(R-r)}{2r^2} \varepsilon^2 + \frac{G}{2} \left[(\varepsilon'')^2 + (\ddot{\varepsilon})^2 + 2(\varepsilon'')^2 \right] + \frac{N_x}{2} (\varepsilon')^2 - N_{x\theta} \varepsilon' \dot{\varepsilon} \right\} r d\theta dx = 0 \quad (11)$$

We set the initial variable $q = \{\varepsilon, -\dot{\varepsilon}\}$ and the dual variable $p = \{p_1, p_2\}$. $p_1 = Q_\theta = -D(\dot{\varepsilon}'' + \varepsilon)$, $p_2 = (M_x + M_\theta) / [1 + (R-r)] = -D(\varepsilon'' + \ddot{\varepsilon})$, p_1 is the equivalent shear force in the θ direction and p_2 is the comparable torsional moment in the θ order [6]. The Hamilton function that we use the initial variable and the dual variable to express is:

$$M(q, p) = p^T q - L(q, p) = -\frac{F(R-r)}{2r^2} \varepsilon^2 + \frac{1}{2G} p_2^2 + p_1 \varepsilon + p_2 \varepsilon'' - \frac{N_x}{2} (\varepsilon')^2 + N_{x\theta} \varepsilon' \dot{\varepsilon} \quad (12)$$

The Hammon equation is described as follows:

$$\begin{pmatrix} \dot{\varepsilon} \\ -\dot{\varepsilon} \\ \dot{p}_1 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} \delta M / \delta p_1 \\ \delta M / \delta p_1 \\ -\delta M / \delta \varepsilon \\ \delta M / \delta \dot{\varepsilon} \end{pmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ \partial_x^2 \varepsilon & 0 & 0 & 1/D \\ \frac{F(R-r)}{r^2} + N_x \partial_x^2 \varepsilon & -N_{x\theta} \partial_x \varepsilon & 0 & -\partial_x^2 \varepsilon \\ N_{x\theta} \partial_x \eta & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \varepsilon \\ -p_2 \\ p_1 \\ p_2 \end{pmatrix} \quad (13)$$

We abbreviate formula (13) as:

$$\dot{\psi} = H\psi \quad (14)$$

The entire state vector:

$$\psi = \{q^T, p^T\} = \{\varepsilon, -\dot{\varepsilon}, p_1, p_2\} \quad (15)$$

The operator matrix is:

$$M = \begin{bmatrix} 0 & -1 & 0 & 0 \\ \partial_x^2 \varepsilon & 0 & 0 & 1/G \\ \frac{E(R-r)}{r^2} + N_x \partial_x^2 \varepsilon - N_{x\theta} \partial_x \varepsilon & 0 & -\partial_x^2 \varepsilon \\ N_{x\theta} \partial_x \varepsilon & 0 & 1 & 0 \end{bmatrix} \quad (16)$$

The identity matrix is expressed as

$$J = \begin{bmatrix} 0 & I_2 \\ -I_2 & 0 \end{bmatrix} \quad (17)$$

We have

$$\langle \psi_1, \psi_2 \rangle = \int_0^{x_e} \psi_1 J \psi_2 dx \quad (18)$$

$\psi = \{\varepsilon, -\varepsilon, p_1, p_2\}$ forms a symplectic space vector. In the linear Hamilton system, we use the generalized variable separation method to make

$$\psi(x, \theta) = \phi(x) e^{\lambda \theta} \quad (19)$$

$\phi(x)$ is the intrinsic solution, which is a function of x . The Eigen equation is expressed as follows:

$$H\phi = \lambda \phi \quad (20)$$

$$\phi(x) = \phi(x) e^{2\lambda \pi} \quad (21)$$

Among them, when $\lambda = in (n = 0, \pm 1, \dots)$, $n = 0$ is the eigenvalue $\lambda = 0$, the formula (12) becomes $H\phi = 0$. At this time, the characteristic equation is

$$\eta^4 - \frac{2n^2}{r^2} \eta^2 + \frac{n^4}{r^4} + \frac{E(R-r)}{Dr^3} = 0 \quad (22)$$

$\phi(x)$ has nothing to do with $N_{x\theta}$. When $n \neq 0$, the eigenvalue $\lambda_n \neq 0$, $\phi_n(x)$ is a non-zero eigensolution. If $\lambda_i + \lambda_j \neq 0$, then

$$\langle \phi_i, \phi_j \rangle = \int_0^{x_e} \phi_i^T J \phi_j dx = 0 \quad (23)$$

Solutions ϕ_i and ϕ_j have a symplectic orthogonal relationship.

$$\langle \phi_i, \phi_j \rangle = \int_0^{x_e} \phi_i^T J \phi_j dx \neq 0 \quad (24)$$

The relationship between ϕ_j and ϕ_j is symplectic conjugate [7]. The vector space is complete, and any eigenstate vector can be represented by a combination of function eigenvectors.

$$\psi(x, \theta) = \sum_{n=0}^{\infty} \left[a_n \phi_n(x) e^{in\theta} + b_n \phi - n(x) e^{-in\theta} \right] \quad (25)$$

We can get this by solving the Eigen equation:

$$\phi_n(x) = \sum_{k=1}^4 C_k x_k^n A_k^n(x) = \sum_{k=1}^4 C_k x_k^n e^{\eta_k x} \sum_{k=1}^4 C_k \left\{ \frac{1}{D \left(in^3/r^3 + \eta_k n^2/r^2 \right)} \right\} e^{\eta_k x} \quad (26)$$

$C_k, (k = 1, 2, 3, 4)$ is the undetermined Eigen coefficient [8]. The eigenvalue equation of symplectic space is shown below. $\eta_k, (k = 1, 2, 3, 4)$ is the solution of the Eigen-characteristic equation in symplectic space.

$$\eta^4 + \left(\frac{N_x}{Dr^2} - \frac{2n^2}{r^2} \right) \eta^2 + \frac{2niN_{xy}}{Dr^2} \eta + \frac{n^4}{r^4} + \frac{E(R-r)}{Dr^3} = 0 \quad (27)$$

5 Simulation of calculation results of lower limb system at the moment of human landing impact

We simulate the action of the human body jumping from a height of 66 cm. The load is the mechanical parameter that occurs in the dynamic experiment [9]. When the subject is landing from a height, the angle between the extension line of the calf.

Fig. 3 Schematic diagram of the simulation of a human body landing from the toe touching the ground to the foot lying flat.

From the simulation results, it can be seen that the knee joint has an excellent cushioning effect on the vertical ground reaction force, and the angular velocity of the hip joint has an advantage in cushioning the horizontal backward reaction force.

We set the unit properties of the material of the parts and the contact between the components. The model setting boundary conditions and loads have been developed. During the simulation, a rotational load was applied to the femur during buckling [10]. We simulate the internal and external rotation of the femur, and the angle of internal and external rotation is gradually increased from 5° to 30° , as shown in Figure 4. We set the measurement parameters and then solve them. The result is shown in Figure 5 [11].

The landing method that reduces the maximum vertical reaction force has a greater degree of joint flexion than the proper landing method [12]. Therefore, the human body delays the cushioning of the landing process by increasing the degree of flexion of the joints and reduces the reaction force of the ground under the state of falling motion-fixed.

The skeletal muscle system of the lower limb system of the human body immediately reaches a large load during the landing. Therefore, the method used to control the reaction force must be activated before touching

Fig. 4 Schematic diagram of the momentary load of the human lower limbs landing.

Fig. 5 The stress cloud surrounding the lower limbs of the human body at the moment of landing.

the ground. Thus, the muscles of the lower limbs of the human body enter the contraction state before the end of the link chain touches the ground, and the joint chain should be in a flexion state before touching the ground [13].

6 Conclusion

According to the decomposition theory of dominant joints, a joint mathematical model of the lower limb system of the human body is established at the moment of landing. Based on the Hamiltonian system, the mathematical model of the coupling problem of the lower limbs of the human body is analyzed. From the simulation results, it can be known that since the skeletal muscle system reaches a large load immediately when it touches the ground, the method used to control the reaction force must be activated before the ground touches.

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