

# Applied Mathematics and Nonlinear Sciences

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## The art design of industrialised manufacturing furniture products based on the simulation of mathematical curves

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### Abstract

After a lot of literature reading and practical research, the paper uses the parametric modelling method of curve simulation characteristic line to establish the mathematical model of the furniture product leg shape. This article first uses the surface construction method to analyse the composition of the leg size of the industrialised production and manufacture of furniture products, and makes the solid modelling according to the construction method of non-uniform rational spline mathematical curve and surface simulation. At the same time, the parameter function of furniture leg type is set on SolidWorks, and the research results obtained in the paper are found by simulation, which opens up a new research path for the application of mathematical curve model in furniture design.

**Keywords:** Non-Uniform Rational B-Splines furniture product leg shape, mathematical curve simulation, industrialised manufacturing, computer-aided geometric design

## 1 Introduction

At present, the focus of research on Ming and Qing furniture at home and abroad still lies in collecting antiques and drawings. In the design and manufacture of antique furniture, the handicraft industry is still the

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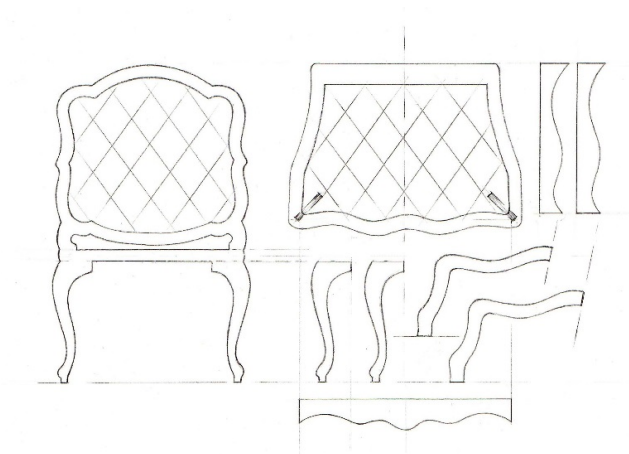
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mainstay. After my country joined the WTO, with the further opening of the Chinese market, computer-aided furniture design and manufacturing technology are the general trends [1]. From the perspective of integrating theoretical research and engineering application, the author studied the critical technical issues of the non-uniform rational B-splines mathematical curve simulation method in the leg shape design. Further, it is realised in the developed CAD system of Ming and Qing furniture surface modelling.

## 2 Analysis of curve modelling of furniture parts

### 2.1 Part curve modelling

Furniture can be categorised into beds, cabinets, tables, chairs, etc., according to varieties. Among them, chair and bed are the more representative types of furniture [2]. Chair is composed of chair brain, front foot, back foot, armrest, backrest, seat frame and other parts. The more typical curved parts are the chair brain, rear legs, armrests and front legs, as shown in Figure 1. The chair brain is the top crossbar on the back of the chair, and its curve forms are mainly curved, corrugated and arched. The back foot has generally three stages, and there are two main types of curves: one is a trapezoid, the other is based on the backrest and the upper and lower sides are arcs with a certain angle to the vertical. The armrests are two crossbars parallel to the seat surface on both sides of the chair. Most of the curves are wavy, and some are similar to ‘S’-shaped curves. The front legs are two vertical planks of wood connected to the chair frame. There are three main types of curves: (1) The upper part of the front foot is rectangular, and the lower part is an ‘S’ curve; (2) the upper part of the front foot is rectangular, and the lower part is sloping outward. Saber shape; (3) Improved saber leg.

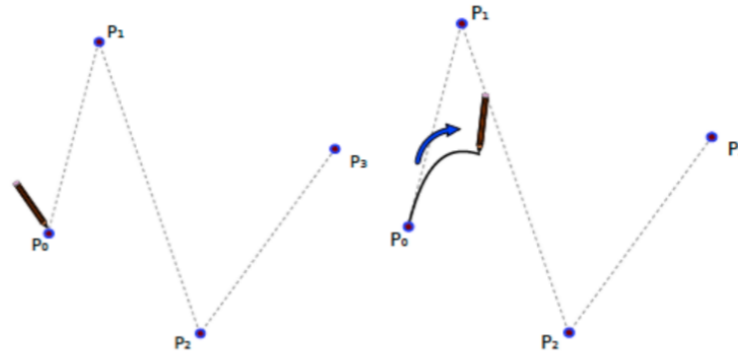


**Fig. 1** Curved shape of chair furniture parts.

Bed furniture is mainly composed of four parts: headboard, footboard, bed support and mattress. The curved parts mainly include the head (tail) top line and the head (bottom) foot. The curve of the full line of the head (bottom) of the bed is symmetrical, mainly including arc, arch, wave etc. The curvilinear shape of the head (tail) foot of the bed is an extension of the sleigh bed's head (tail) foot style.

### 2.2 Curve modelling simulation

The curve shapes of furniture parts mainly include arcs, waves, ‘S’ shapes, saber shapes etc. These curve shapes are similar to some mathematical curves. By analysing the correlation that the curve shapes of furniture parts bear with mathematical curves, it is possible to simulate these parts with mathematical curves. The curvilinear modelling shown in Figure 2 is partly related.



**Fig. 2** The correlation between the mathematical curve and the part curve.

It can be seen that these mathematical curves are similar to the curves of certain parts of furniture. These curve shapes can be simulated through the setting of mathematical curve variables and intervals. For example, a cosine curve can be modified by determining its period and amplitude and using symmetric intervals [3]. We adopt the method of mathematical curve simulation and use its calculation formula to generate the curve characteristics of the required part quickly, thereby further generating the three-dimensional model of the position. Since the mathematical curves are relatively rigid, different angles can only be obtained by changing the variables and value ranges. However, designers often have many random variations when drawing these part curves; moreover, it is not easy to achieve a complete match in the simulation process. We can use the approximation method to fit to find a closer angle continuously.

### 3 Non-uniform rational B-splines technology

Free-form curve and surface design is often a difficult point in CAD. Recently, the methods that have been more successful and widely used in the design of free-form curves and surfaces include the Bezier method, the 13-spline method and the non-uniform rational B-splines method [4]. Because of the outstanding advantages of the non-uniform rational B-splines method. In 1991, the International Organisation for Standardisation (ISO) officially promulgated the STEP standard for the geometric definition of industrial products as an international standard for product data exchange. In the STEP standard, free-form curves and surfaces are uniquely represented by non-uniform rational B-splines. This technology has been applied to professional software in many industries such as machinery manufacturing, automobiles, aviation and shipbuilding.

#### 3.1 Non-uniform rational B-splines curve

We assume that the control points  $V_0, V_1, \dots, V_m$  must reach the corresponding weight mountain  $\omega_0, \omega_1, \dots, \omega_m$  and the node vector  $u_0, u_1, \dots, u_{m-k+1}$ ; then the  $k$ th non-uniform rational B-splines curve is defined as:

$$C(u) = \frac{\sum_{i=C}^n \omega_i V_i N_{i,p}(u)}{\sum_{t=c}^n \omega_t V_t N_{t,k}(u)} \quad (1)$$

The primary objective is to find a unified mathematical method with other forms through which free-form curves and surfaces can be described and accurately represent elementary analytical curves and surfaces. From Eq. (1), it can be seen that non-uniform rational B-splines has not only more denominators but also more weight factors than non-rational B-spline curves. We can use it to adjust the fullness of the curve, provide sufficient

flexibility for geometric calculations and make it an additional modelling parameter.

### 3.2 Non-uniform rational B-splines surface

Similar to the non-uniform rational B-splines curve, a non-uniform rational B-splines surface of degree  $k \times l$  can be expressed as

$$\sum_{i=0}^m \sum_{j=0}^n \omega_i d_i N_{i,k}(u) N_{j,l}(v) \quad (2)$$

The control vertex  $d_i (i = 0, 1, \dots, m, j = 0, 1, \dots, n)$  presents a topological rectangular array and then forms a control network;  $\omega_i$  is the weighting factor connected with the vertex. We specify the positive weighting factor  $\omega_{1,m}, \omega_{2,m}, \omega_{3,m}, \omega_{4,m} > 0$  at the vertices of the four corners and the remaining  $\omega_i \geq 0, N_{i,k}(u) (i = 0, 1, \dots, m)$  and  $N_{j,l}(v) (j = 0, 1, \dots, n)$  are the canonical B-spline bases of  $k$  times in  $u$  direction and  $l$  times in the  $v$  direction, respectively [5]. They are, respectively, determined by the node vector  $U = [u_0, u_1, \dots, u_{m+k-1}]$ ,  $V = [v_0, v_1, \dots, v_{n+l-1}]$  of the  $u$  and  $v$  directions, according to the De Boer recurrence formula.

## 4 Parametric design of leg shape

The aggravated leg shape mentioned in the thesis is a kind of leg shape widely used in Ming and Qing furniture, and its lower end is like a horse hoof. The middle future hangs upright, and the upper end turns inward to connect with the teeth. Traditionally, this kind of leg is called a horseshoe leg. The structure and shape of horseshoe legs are relatively finalised, so we use a parametric design method based on shape feature lines to construct a mathematical model of the leg shape [6]. The typical size relationship of the two legs (flat legs and drum legs) is shown in Figure 3 and Table 1. Based on this, we can use a set of parameters to agree on the size relationship of the horseshoe legs.

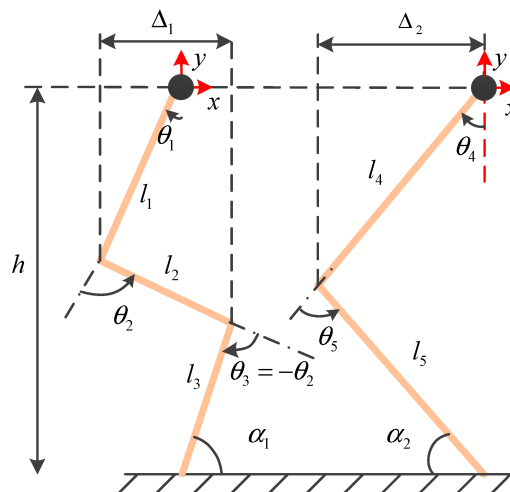


Fig. 3 The configuration parameters of the leg shape.

Most of the parameters for constructing the leg shape are related to the plane curve. Although the curve segment is on the one-dimensional form of the leg shape rather than on the two-dimensional outline, the relevant parts of the three-dimensional body are projected from front to back. Still, the curve change directly affects the construction of the model. Further, from the user's point of view, when designers in the furniture industry design leg shapes, after determining the main dimensions of the furniture (length, width, height, leg height etc.),

the primary consideration is also the change of this line. Therefore, based on the above reviews, we use the mathematical curve as the characteristic line for constructing the leg shape.

**Table 1** Furniture leg size relationship.

Size	Name	Size relationship	
		Flat leg (m)	Oblique leg (m)
$u$	Tip width	1/15–1/20	
$b$	Outer leg width	0.5–1.0	
$t$	The angle of inclination of the foot	0.15–0.65	
$d$	Inner crotch width	0.1–0.8	
$h0$	Foot height	0.9–1.5	0.9–1.1
$h1$	Inner crotch height	1.1–1.5	1.1a
$h2$	Tip height	2.0–5.0	
$h3$	Foot length	0.7–1.5	0.23

The curve non-uniform rational B-splines is expressed as:

$$C(u) = \frac{\sum_{i=1}^{10} V_i \omega_i N_{i,3}}{\sum_{i=1}^{10} \omega_i N_{i,3}(u)} \quad (3)$$

Among the 10 model value points on the distinctive line, the points I, II, V and X correspond to the characteristic points A, D, B and C. The two points III and IV are the uniform interpolation points of the DB segment, and the four points VI, VII, VIII and IX are the consistent interpolation points of the BC segment. After defining the characteristic line, the cross-sectional shape of the height of a particular point of the leg shape can be determined, thereby forming the three-dimensional shape of the leg shape [7].

## 5 Realisation technology of Non-uniform rational B-splines method in leg shape

Since the CAD platform system involves geometric configuration and is inseparable from the database and the graphical interface, the database and the graphical interface are substantial software packages. To develop these low-level frameworks at the same time or on the original basis requires a lot of time and energy from developers, which is often difficult to adapt to the rapid development of the software industry. At present, the development of software component technology and object-oriented technology provides an excellent way to solve the problem, making full use of advanced software platforms, and researching and developing their superior products at a higher starting point should be a more feasible technical approach.

### 5.1 Principles of secondary development

The system is operated under the Windows environment using powerful, object-oriented Visual Basic visual-programming software. The system uses SolidWorks as the primary platform for its secondary development and realises the parametric design of the curve of American furniture parts. We use mathematical curves to simulate part curves to achieve a part curve parametric design system that integrates part curve selection, geometric curve design, parametric modelling and engineering drawings.

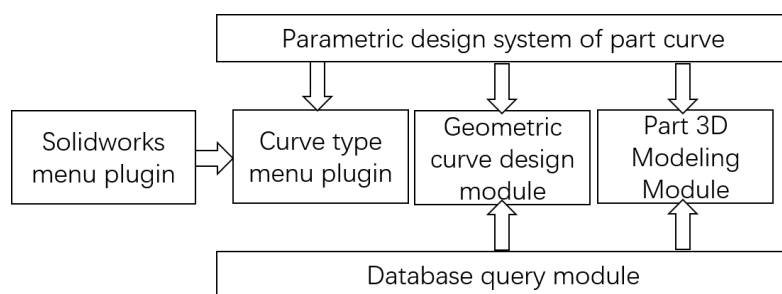
SolidWorks is a feature-based solid modelling software. The system can give different design schemes to avoid or reduce errors in the design process. It uses intuitive design technology, Windows OLE technology, Parasolid kernel and excellent technology integration with third-party software. SolidWorks is currently a commonly

used 3D CAD software.

SolidWorks has an open interface, and provides convenient user-development tools and rich data-exchange methods [8]. The secondary development of SolidWorks is mainly to use its embedded Application Programming Interface (API) objects. Solid Works API provides users with free, open and fully functional development tools. Reliable Works API provides users with many functions that Visual Basic can call, such as Delphi, VBA and Visual C++ through OLE technology. These functions are related to the methods and properties of the object, which improve the programmer's ability to develop SolidWorks directly. By calling these methods and setting object properties, users can control various operations on SolidWorks in their applications.

## 5.2 System overall planning

According to the characteristics and goals of the system, the system we designed is mainly composed of five parts: curve-type selection module, geometric curve design module, part 3D modelling module, database query module and menu plug-in. Simultaneously, we have established a SolidWorks menu plug-in, from which it is more convenient for users to query and call the required part curves. The overall plan of the system is shown in Figure 4.



**Fig. 4** System master plan.

### 5.2.1 Curve type selection module

This module is mainly used for curve selection. After entering the system interface, we select the corresponding mathematical curve type according to user needs and ensure that these are equipped with complementary graphics and text descriptions [9]. When you click the 'OK' button, you will enter the following design interface. When you click the 'Cancel' button, it will return to the program's main interface.

### 5.2.2 Geometric curve design module

This module mainly performs geometric curve drawing. Query the parameters of the required part curve in the database. After we input the geometric parameters and complete the design calculation, click the 'Geometric Curve' button to draw the geometric curve of the part and save it to the corresponding folder.

### 5.2.3 Part 3D modelling module

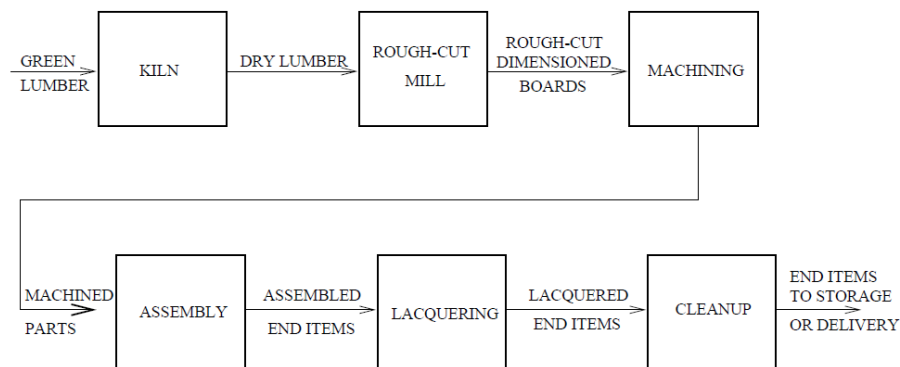
This module is mainly used for drawing the three-dimensional model of parts and engineering drawings. Query and enter the corresponding part design parameters in the database. Click the 'Part Model' and 'Export Engineering Drawing' buttons to enter the SolidWorks interface of the part 3D model and engineering drawing. We save the drawn engineering drawings to the corresponding folder.

### 5.2.4 Database query module

This module mainly provides users with a convenient platform for querying part curve geometric parameters and part design parameters [10]. Multiple data tables are used to record different types of part curve data, which can be continuously expanded, modified and updated. The Access database management system completes this module.







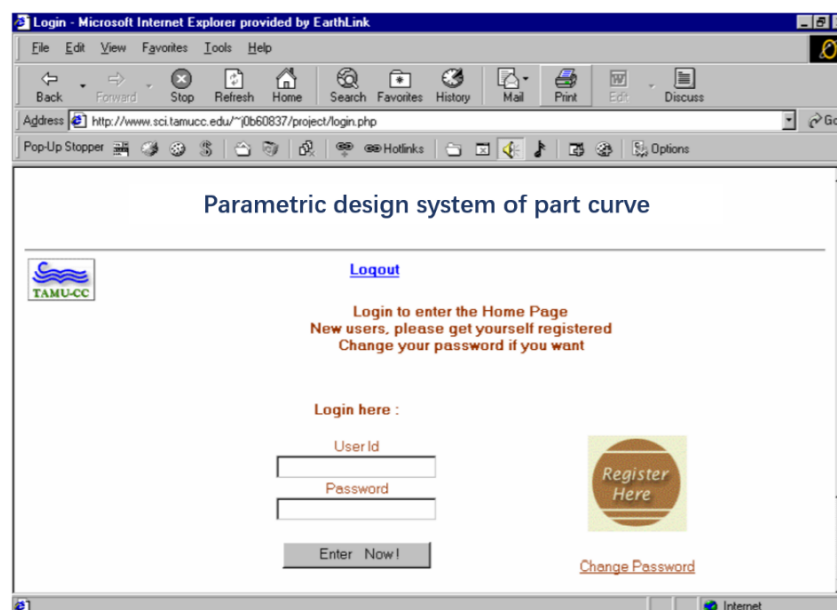
**Fig. 6** The flow chart of leg shape parameter modelling.

## 5.4 Application examples

We use the example of a cosine curve to simulate the angle of furniture parts to illustrate the operation method of the whole system, the operation process and the programming code to realise the process.

### 5.4.1 System start-up interface

In the Windows environment, we open the executable program of this system and start the login interface of the furniture part curve parametric design system. We enter the corresponding account and password to enter the following window interface, as shown in Figure 7.



**Fig. 7** Login screen.

### 5.4.2 Curve-type selection interface

After we enter the login interface, an interface window—shown in Figure 8—will pop up, and we can use it to select the desired curve type. Select the cosine curve here, and then click the button to the right to pop up a legend and text description of the cosine curve. If you are sure of the choice made, click the 'OK' button to enter the design interface of the geometric cosine curve.



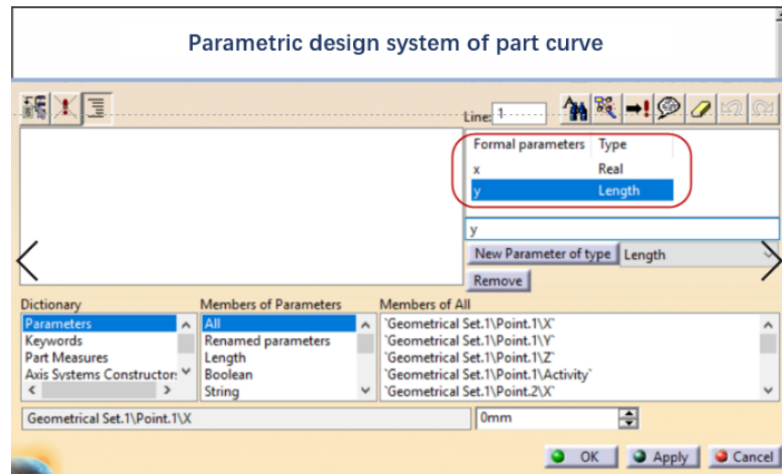


Fig. 8 Curve-type selection interface.

### 5.4.3 Geometric curve design interface

After viewing the cosine curve data table, enter the name of the required curve part, and we obtain the geometric parameters and enter them in the corresponding positions. The relevant parameters cover four parameters: amplitude, period, left interval and suitable interval. We can calculate the value of through four parameters [12]. In this way, a geometric curve model can be built in SolidWorks and saved to the corresponding folder. After completion, click the 'Enter Part 3D Modelling Interface' button to enter the following interface, as shown in Figure 9 [13].

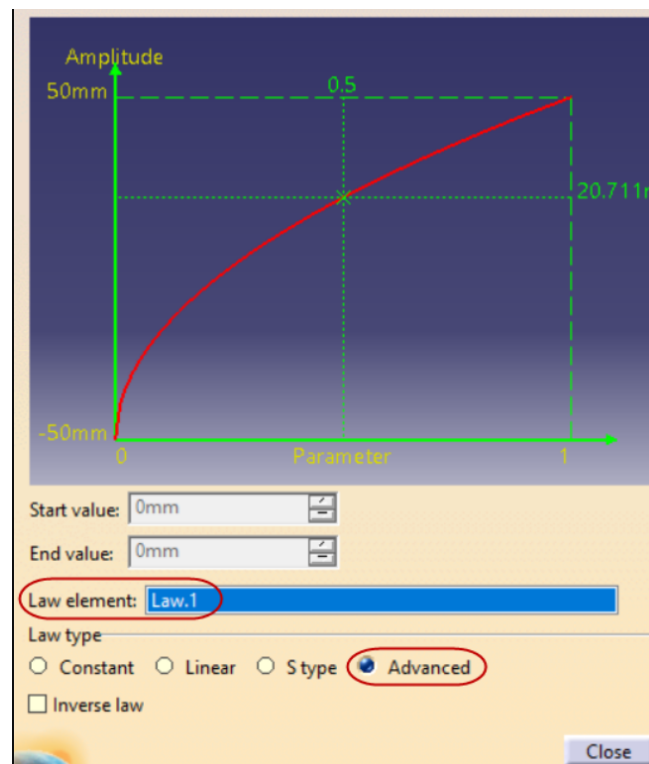
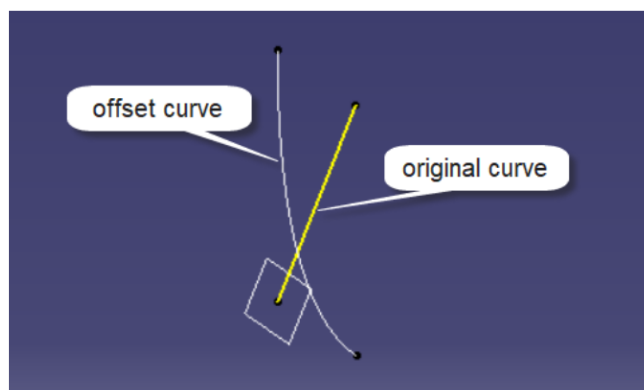


Fig. 9 Geometric curve design interface and model.

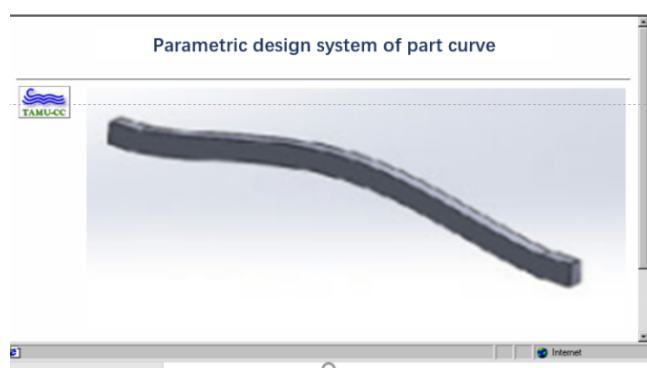
#### 5.4.4 Part 3D modelling interface

The primary purpose of the three-dimensional modelling design is to generate the three-dimensional model of the part and the creation of the engineering drawing interface, as shown in Figure 10.



**Fig. 10** 3D modelling design of curved parts.

After ascertaining the corresponding part design parameters from the data table, we input them into the corresponding position. There are three parameters of straight-line length, part width and stretched thickness [14]. After inputting the parameters, click 'Part Model' to build the necessary curve part in SolidWorks, as shown in Figure 11.

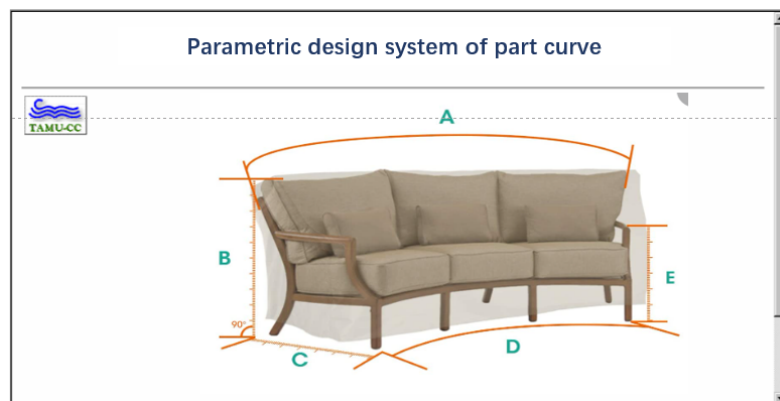


**Fig. 11** 3D model of the curved part.

According to the actual needs, we can click the 'engineering drawing' button to generate the corresponding engineering drawing, as shown in Figure 12.

## 6 Conclusion

The paper analyses the modelling characteristics of the furniture leg curve and the method of mathematical curve simulating the leg curve. We use SolidWorks API technology and VB development tools to realise the parametric design of the furniture leg curve. The paper expounds on the principles of system development, parametric modelling methods and overall system planning and uses cosine curve to simulate furniture leg curve as an example to analyse the operation process of the system and the use of the SolidWorks menu plug-in. The design method is simple to operate. The designer can perform curve design and automatic modelling through simple human–computer interaction, which significantly improves the design efficiency, shortens the product



**Fig. 12** Engineering drawing of a curved part.

development cycle and enables the designer to spend more time and energy in product innovation design.

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