Applied Mathematics and Nonlinear Sciences

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Health status diagnosis of the bridges based on multi-fractal de-trend fluctuation analysis

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Submission Info

Communicated by Juan Luis García Guirao

Received March 19th 2021

Accepted May 10th 2021

Available online October 19th 2021

Abstract

More and more attention has been paid to the research on bridge structure health monitoring and state analysis technology, especially the theory and technology of time series analysis based on bridge deflection information. In this paper, multi-fractal de-trend analysis (MF-DFA) is used to analysis the deflection information of bridges. The deformation characteristic parameters can be obtained by the multi-fractal de-trend fluctuation method, which can effectively identify the deformation degree of bridges. The analysis result shows that the multi-fractal de-trend fluctuation analysis method provides the theoretical basis and implementation approach for the health monitoring of the bridge.

Keywords: deflection signal, Multi-fractal, De-trend analysis, health monitoring of bridge

1 Introduction

The bridge is an important component in the traffic network, its quality being crucial to people's lives. As of 2016, one-quarter of China's bridges have potential problems [1] according to statistics. These defective bridges need to be repaired regularly, which is an expensive task. If regular repairs are not undertaken, it will seriously affect the safety and stability of bridges. Bridge health inspection is an important measure to prevent and reduce accidents. Bridge deflection refers to the linear displacement of the centre of the bridge cross-section along the vertical direction of the axis when the bridge is stressed or the non-uniform temperature changes. In bridge health inspection, the bridge deflection information is often used to detect the deformation of the bridge.

The deflection signal will change due to the damage or mutation of the bridge [2]. Currently, the time-frequency analysis method is the main process of handling the non-stationary signal. For example, In [3] Wavelet

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transform is proposed to separate the transient and slow information of deflection signal. Hu et al. [4] verified the practicability of principal component analysis (PCA) in fault diagnosis, but the result also shows that PCA is not very sensitive to small faults. Due to the influence of nonlinear factors (impact load, damping, friction etc.), the deflection information of the bridge is nonlinear, non-stationary and non-periodic. Therefore, the structure state of the bridge cannot be fully reflected by the physical model of the bridge deflection data based on the traditional method [5].

Now, the method that is widely used to describe the object of study is fractal dimension [6]. As the bridge damage is affected by many factors, using the multi-fractal can depict the signal from overall irregularity and expresses the local behaviour finely [7, 8]. Therefore, this paper analyses the bridge deflection time-domain signal, and then uses the multi-fractal de-trend fluctuation analysis method to carry out the multi-fractal spectrum analysis of the bridge deflection signals, and summarises the connection between the parameters of the multi-fractal spectrum and the health state of the bridge, which provides the theoretical basis and the way of implementation for bridge health monitoring.

2 Multi-fractal de-trend analysis (MF-DFA) theory

Bridge deflection signal is a non-stationary signal with small amplitude variation, and its inherent non-stationary characteristics are difficult to estimate. In this paper, the trend component of the bridge deflection signal is eliminated by the de-trended fluctuation analysis, and the deformation characteristics are described by the multi-fractal spectrum.

2.1 Generalised Hurst exponent

For deflection signal time series $x(k)$, $k = 1, 2, \dots, N$, MF-DFA calculation steps are as follows:

Step 1: Determine the ‘profile’

$$Y(i) \equiv \sum_{k=1}^i |x_k - \bar{x}|, \quad i = 1, \dots, N \quad (1)$$

Subtraction of the mean $\bar{x}$ is not compulsory, since it would be eliminated by the later de-trending in the third step.

Step 2: Divide the profile $Y(i)$ into $N_s \equiv \text{int}(N/s)$ non-overlapping segments of equal length s . Since the length N of the series is often not a multiple of the considered time scale s , a short part at the end of the profile may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether.

Step 3: Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the series. Then determine the variance

$$F^2(s, v) \equiv \frac{1}{s} \sum_{i=1}^s \{Y[(v-1)s+i] - y_v(i)\}^2 \quad (2)$$

for each segment $v, v = 1, \dots, N_s$ and

$$F^2(s, v) \equiv \frac{1}{s} \sum_{i=1}^s \{Y[N-(v-N_s)s+i] - y_v(i)\}^2 \quad (3)$$

for each segment $v = N_s + 1, \dots, 2N_s$. Here, $y_v(i)$ is the fitting polynomial in segment v . Linear, quadratic, cubic, or higher-order polynomials can be used in the fitting procedure (conventionally called DFA1, DFA2, DFA3...) [9]. Since the de-trending of the time series is done by the subtraction of the polynomial fits from the profile, different order DFA differ in their capability of eliminating trends in the series. In (MF-)DFA m (m th order (MF-) DFA) trends of order m in the profile (or, equivalently, of order $m-1$ in the original series) are eliminated. Thus a comparison of the results for different orders of DFA allows one to estimate the type of the polynomial trend in the time series [9].

Step 4: Average over all segments to obtain the q th order fluctuation function

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s, v)]^{q/2} \right\}^{1/q} \quad (4)$$

where, in general, the index variable q can take any real value except zero [9]. For $q = 2$, the standard DFA procedure is retrieved. We are interested in how the generalised q dependent fluctuation functions $F_q(s)$ depend on the time scale s for different values of q . Hence, we must repeat steps 2-4 for several time scales s . It is apparent that $F_q(s)$ will increase with increasing s . Of course, $F_q(s)$ depends on the DFA order m . By construction, $F_q(s)$ is only defined for $s \geq m + 2$.

Step 5: Determine the scaling behaviour of the fluctuation functions by analysing log-log plots $F_q(s)$ versus s for each value of q . If the series x_i are long-range power-law correlated, $F_q(s)$ increases, for large values of s , as a power-law

$$F_q(s) \propto s^{h(q)} \quad (5)$$

Among solutions of $h(q)$ there is one that is known as the generalised Hurst index, which represents the correlation of the original sequence. When $0.5 < h(q) < 1$, it indicates that the signal has a long-range correlation and that $h(q)$ is closer to 1; and so, the correlation is stronger. When $h(q) = 0.5$, there is no correlation between the time series. When $h(q) < 0.5$, the signal only has a negative correlation.

2.2 Multi-fractal spectrum

The generalised Hurst exponent $h(q)$ is one way to describe time series with multi-fractal properties. Multi-fractal spectrum [10] completely presents the fractal singular probability distribution form of a signal, which improves the degree of the fine description of signal geometry and local scale behaviour. In the de-trending multi-fractal method $h(q)$ is converted into mass exponent $\tau(q)$, singularity exponent α_0 and multi-fractal spectrum $f(\alpha)$. The transformation relationship is as follows:

$$\tau(q) = qh(q) - 1 \quad (6)$$

$$\alpha = \tau'(q) = h(q) + qh'(q) \quad (7)$$

$$f(\alpha) = q\alpha - \tau(q) = q[\alpha - h(q)] + 1 \quad (8)$$

The multi-fractal singularity spectrum obtained by the MF-DFA method is a set of parameters that can accurately describe the multi-fractal dynamic behaviour characteristics of deflection signals.

2.3 Multi-fractal spectral parameters

The singularity exponent α_0 corresponding to the maximum value of the spectral function $f(\alpha)$ describes the irregularity of the deflection signal. The larger value of α_0 represents the more irregular deflection signal. Δf is the proportion of large and small deflection signal peak; its value is $\Delta f = f(\alpha_{\max}) - f(\alpha_{\min})$. Multi-fractal spectrum width $\Delta\alpha$ is bigger, the deflection signal wave is more irregular, and the value of $\Delta\alpha$ can be obtained by $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$. When Δf and $\Delta\alpha$ are larger and α_0 is smaller, and the deflection signal fluctuation is more intense, the multi-fractal features are stronger [11]. Therefore, in this paper, we have selected α_0 , Δf and $\Delta\alpha$ as the bridge deformation characteristics.

3 Deflection signal analysis of the experimental data

The deflection signals of bridges in a normal state, fatigue deformation and sudden deformation were collected from the detection signals of Chongqing xxx Bridge. The time-domain diagram is shown in Figure 1.

Fig. 1 Deflection signals in the time domain.

It can be seen from Figure 1, that the frequency and amplitude of bridge deflection signals are very small, which makes it difficult to obtain deformation characteristics by traditional spectrum analysis methods. Therefore, it is difficult to diagnose and predict bridge health state directly by amplitude spectrum.

By the theory of multi-fractal, the characteristics of deflection signals can be effectively described. Using this characteristic, trend analysis of deflection signals can be carried out by DFA to eliminate the non-stationary trend influence of time series multi-fractal features, and then the multi-fractal spectrum can be calculated for feature extraction.

3.1 Analysis of multi-fractal characteristics based on DFA

Using DFA for fault prediction, the main operation is a de-trending operation, and the use of different order polynomial fitting means the removal of different types of a trend in the signal. The scale range of choice within the scale index can reflect the intrinsic characteristics of the signal. Only an appropriate scale can obtain reliable statistics, with the time sequence changing fast and with the maximum recommended being $n/20$ [12]. The DFA method is used to analyse the deflection signal, and the relationship between the scaling exponent $\tau(q)$ and the order q of the wave function was obtained, as shown in Figure 2.

Due to the non-periodic and non-stationary characteristics of the bridge deflection signals, the scaling exponents $\tau(q)$ and q have an obvious nonlinear relationship. From Figure 2, we observe that the distribution in the

Fig. 2 Deflection signal scale index.

figure is a convex function, which indicates that the bridge deflection signals have multi-fractal characteristics and the time series has a long-range correlation. Therefore, the MF-DFA method can directly distinguish the state of the bridge.

Figure 3 shows the variation relationship between the generalised Hurst index h_q and q of the deflection signals.

Fig. 3 Generalized Hurst index for deflection signals.

From Figure 3, we observe that the dynamic mechanism of deflection signal generated by the bridge under different deformation is different, which leads to great differences in the generalised Hurst index under different states. Moreover, the generalised Hurst index h_q decreases with the increase of q , which further proves that the

bridge deflection signal has multi-fractal characteristics.

3.2 Multi-fractal spectrum parameter analysis

Figure 4 shows the variation rule of the multi-fractal spectrum wave function of the bridge deflection signals with the singularity index in different states.

Fig. 4 Multi-fractal spectrum of bridge deflection signal.

From Figure 4, it can be seen that the multi-fractal spectrum of the deflection signals of bridges varies with the degree of deformation. For example, the width of the singularity index $\Delta\alpha$ is different, and the distribution of the singularity index α_0 is also regular. On selection of the multi-fractal spectrum parameter, we ascertain α_0 , Δf and $\Delta\alpha$ as bridge deformation characteristics; the results are shown in Table 1.

Table 1 Feature parameters of the bridge deflection signal.

Parameter	Mutation	Fatigue Signal	Normal_1	Normal_2
α_0	2.072	1.873	1.835	1.780
Δf	0.0254	-0.2887	0.1633	0.04698
$\Delta\alpha$	2.55	1.31	0.775	0.576

Table 1 shows that when the bridge is in a healthy state, the deflection of the multi-fractal spectrum parameter $\Delta\alpha$ value is far less than the fault state, and with the increase of bridge deformation, the spectral width $\Delta\alpha$ and singularity index α_0 gradually increase. Δf has obvious variation in high-frequency mechanical signals [13], but it is not suitable for bridge health diagnosis due to the small amplitude and frequency of deflection signals in bridge health detection.

In conclusion, the multi-fractal spectrum parameter α_0 and $\Delta\alpha$ can preliminarily distinguish the health status of the bridge, and the spectrum width $\Delta\alpha$ can effectively distinguish the deformation degree of the bridge. Therefore, the singularity exponent α_0 and multi-fractal spectrum width $\Delta\alpha$ can be used as characteristic parameters for bridge health diagnosis.

4 Conclusions

For bridge deflection signal, it is difficult to obtain effective fault characteristic quantity by the traditional spectrum analysis method. Therefore, the MF-DFA method is used to analyse the multi-fractal spectrum of the deflection signals of bridges under different states. The conclusions are as follows:

1) Deflection signals have obvious multi-fractal characteristics, and the multi-fractal characteristics of bridge deformation state are stronger than those corresponding to the normal state.

2) The MF-DFA method can effectively identify the fault state of the bridge deflection signals. The multi-fractal spectrum parameter α_0 and $\Delta\alpha$ can preliminarily determine the deformation state of the bridge, and the degree of bridge deformation can be distinguished effectively by the spectrum width $\Delta\alpha$ of the multi-fractal spectrum, which can quantitatively represent the fatigue degree of the bridge to check the health status of the bridge.

Acknowledgments

This work is supported by Fund of State Key Laboratory of Bridge Engineering Structural Dynamics (2019-01); National Nature Science Foundation of China under Grant 61663008; and 2018 Team Building Project for Graduate Tutors in Chongqing under Grant JDDSTD2018001.

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