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Study of statistical damage constitutive model of layered composite rock under triaxial compression

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Abstract

The layered composite rock was subjected to triaxial compression tests under constant confining pressure and the stress-strain curves under different confining pressures were obtained. Based on the continuous damage theory and statistical strength theory, it is assumed that the strength of rock microelements obeys Weibull distribution by taking the defects such as random micro-cracks in the rock into account. The statistical constitutive model of layered composite rock with damage correction is established by taking the axial strain of rock as a random distribution variable of microelement strength. The model parameters were determined by the curve fitting method and referring to some test parameters. By comparing the experimental data and the constitutive model curve, the rationality and feasibility of the model are verified.

Keywords: layered composite rock, Weibull distribution, damage variable correction factor, damage constitutive model

1 Introduction

With the continuous development of economic construction and national defence construction, and the continuous expansion of underground space development, the study of deep rock mechanics problems has been closely linked to my country's economic construction and national defence construction. Energy mining, water conservancy, hydropower, nuclear waste treatment, mine excavation and other projects all involve deep rock mechanics problems [1]. With the continuous enlargement of the engineering depth, the geological conditions have become more complex, and a series of engineering hazards such as severe roadway deformation and instability, rock bursts and surges of low pressure have become more and more serious. In addition, under the conditions of modern high-tech warfare and high-precision reconnaissance technology, precision-guided weapons and small ground-penetrating nuclear weapons continue to develop. Severe challenges are presented to the construction and survival of underground protection projects. The status and role of deep underground protection projects have become more important and prominent.

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Layered composite rock is one of the most common rock masses in various types of geotechnical engineering. Because composite rock is a natural material composed of many different properties, different thicknesses, different components and different combinations in a certain order, its characteristics are significantly different from that of a single rock [2, 3]. Various rock-related projects are affected by the strength, deformation and destruction of composite rocks, which often cause instability disasters such as tunnel collapse, mine pressure manifestation, edge wave slip, ground subsidence and building cracking (influenced by rock mass foundation). In recent years, important research results have been obtained by using damage mechanics to study the properties of rock materials and explore the laws of deformation and failure of rocks. Kachanov [4] first introduced the concept of damage, and then he proposed the concept of the ‘damage factor’. Later scholar Lemaitre [5] combined various aspects of mechanical knowledge (such as effective stress, strain and continuum mechanics) and established ‘damage mechanics’ based on the principle of irreversible thermodynamics. Bazant [6] proposed distributed fracture mechanics and discussed that geotechnical materials have some special characteristics, such as the scale effect of mesomechanical model failure, strain localisation or instability, and the sensitivity of the finite element network caused by distributed fractures [7–10]. Wengui and Sheng [11] started from the Mohr-Coulomb criterion, based on the representation method of the microelement strength of the rock that obeys the Weibull distribution, and established a damage-softening statistical constitutive model for the whole process of rock deformation and fracture. A large number of experiments have verified its rationality and correctness, and it has been applied in engineering practice. However, the Mohr-Coulomb criterion does not consider the effect of the intermediate principal stress on the strength of the microelement. Tao et al. [12] assumed that the strength of rock microelements obeyed a normal distribution and proposed the influence factors of the relationship between damaged materials and micro-defects that change due to material damage. The damage mechanics theory is used to analyse the change of rock strength with confining pressure, and a rock damage mechanics model under the new damage definition is established. The constitutive relation of rocks under low confining pressures is well described by this model, but it is not accurate for rocks under high confining pressures. Xiaofeng [13] proposed a new attenuation function – Harris function on the basis of previous studies. Assuming that the probability density of rock microelement strength obeys a new distribution function – the improved Harris function, a new constitutive model is established based on this, which better reflects the stress–strain relationship and the whole failure process of the rock under the three-dimensional stress state. The theoretical curve of the model has a high coincidence with the experimental curve at the stage before rock failure, but the theoretical curve of the model after rock failure does not have a good coincidence with the experimental curve. Due to the fact that the study of random damage of the under layered composite rock under load conditions is relatively rare, an effective statistical damage model is rarely proposed. Based on the Weibull distribution characteristic of rock microelement strength, the damage variable is modified and the damage variable correction coefficient is introduced in this study. A new statistical constitutive model of layered composite rock damage is established by defining the random distribution variable of the strength of rock microelement more concisely. MATLAB software was used to fit the experimental data with the constitutive model to determine the relevant model parameters and verify the correctness of the constitutive model of layered composite rock damage. Finally, the influence of relevant parameters on the accuracy of the model is analysed.

2 Establishment of damage constitutive model of layered composite rock

Assuming that no defects in the rock under ideal conditions, the constitutive relationship of layered composite rock under three-dimensional stress are given as:

$$\begin{cases} \sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3 \\ \sigma_2 = (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_1 + \lambda\varepsilon_3 \\ \sigma_3 = (\lambda + 2G)\varepsilon_3 + \lambda\varepsilon_1 + \lambda\varepsilon_2 \end{cases} \quad (1)$$

where σ_1 , σ_2 and σ_3 are the three principal stresses of the rock; ε_1 , ε_2 and ε_3 the principal strain of the rock

in the three principal stress directions; G is the rock shear modulus; and λ is the Lamé constant.

Under the condition of constant confining pressure, the axial stress–strain relationship of the non-destructive rock material in triaxial compression is:

$$\sigma_1 = E\varepsilon_1 + 2\nu\sigma_3 \quad (2)$$

where E is the rock elastic modulus; and ν is the Poisson's ratio.

Due to the distribution of a variety of micro-cracks and structural planes in the rock, the rock may have many weak links with different strengths, and the strength of each element is not the same. Assuming the damage of rock material in the loading process is a continuous process, the following bases are made:

- (1) The rock material is isotropic in the macroscopic view;
- (2) Before the failure of the rock element, it obeys Hooke's law. The element has linear elastic properties and loses its bearing capacity after failure;
- (3) The intensity of each microelement F obeys the Weibull distribution, and its probability density function is:

$$P(F) = \frac{m}{F_0} \left(\frac{F}{F_0} \right)^{m-1} \exp \left[- \left(\frac{F}{F_0} \right)^m \right] \quad (3)$$

where F is the strength of the rock's infinitesimal element; m and F_0 are the Weibull distribution parameters, which reflect the mechanical properties of the rock.

Assuming that the number of damaged microelements under a certain load is n , and the total number of microelements is N , the damage variable D can be defined as the ratio of the number of damaged microelements to the total. Then the statistical damage variable can be expressed as:

$$D = \frac{n}{N} \quad (4)$$

The value of D reflects the degree of damage inside the rock material. When $D = 0$, the rock is in a non-destructive state, and when $D = 1$, the rock is in a completely damaged state.

When reaching a certain load level F , the total number of damaged microelements can be expressed as:

$$n = \int_0^F NP(F)dF = N \left\{ 1 - \exp \left[- \left(\frac{F}{F_0} \right)^m \right] \right\} \quad (5)$$

Thus the calculated damage variable is:

$$D = 1 - \exp \left[- \left(\frac{F}{F_0} \right)^m \right] \quad (6)$$

Under uniaxial compression, from the continuous damage theory, we can get:

$$\sigma_1 = E\varepsilon_1(1 - D) \quad (7)$$

It can be deduced that the basic relationship of the damaged rock under triaxial compression is:

$$\sigma_1 = E\varepsilon_1(1 - D) + 2\nu\sigma_3 \quad (8)$$

Heping [14] presented that the damage variable under three-dimensional conditions is the ratio of the damage equivalent area in a representative volume element to the total area of the section. If the rock is isotropic, the ratio has nothing to do with the section orientation, and the damage degree of each stress component is identical, which is the same damage. However, this is only satisfied by ideal rock materials. Therefore, it is not appropriate to assume that the damage situation satisfies the Weibull distribution. This paper attempts to introduce a correction factor $\delta \in (0, 1)$ to modify the damage variable D so that it can reflect the residual

strength and the characteristics of the rock after rupture. It is corrected by multiplying the correction coefficient and the damage variable D so that the resulting rock damage statistical constitutive model can reflect the residual strength and the characteristics of the anisotropy of the rock material. Therefore, the obtained model is closer to the actual situation and can better simulate the stress–strain characteristics of the rock after reaching the peak point. The new statistical constitutive model of rock with the introduction of damage variable correction coefficient D is given as:

$$\sigma_1 = E\varepsilon_1(1 - \delta D) + 2\nu\sigma_3 \quad (9)$$

The current studies on rock damage constitutive models have introduced different rock strength criteria (Mohr-Coulomb criterion [15], Hoek-Brown criterion [16, 17] and Druckre-Prager criterion [18], etc.) as the rock microelement strength random distribution variables. It is found that if the rock yield criterion is used as the random distribution variable of the microelement strength, the derived expression is more complicated by researching. However, it is much simpler to take the axial strain of the rock as the random distribution variable of the microelement strength. Therefore, this paper adopts this simplified method and takes the axial strain of the rock as the random-distribution variable of the microelement strength with $x = 1$.

The comprehensive elastic modulus E of the layered composite rock is calculated according to the equivalent elastic modulus of the layered composite rock in the literature [19]. As shown in Figure 1, the equivalent elastic modulus of the layered composite rock is:

$$\frac{\sigma}{E} L = \sum_{i=1}^n \frac{L_i}{E_i} \sigma \quad (10)$$

$$\frac{1}{E} = \sum_{i=1}^n \frac{L_i}{E_i L} = \frac{L_1}{E_1 L} + \frac{L_2}{E_2 L} \cdots + \frac{L_i}{E_i L} + \cdots + \frac{L_n}{E_n L} \quad (11)$$

Therefore, the equivalent elastic modulus of the layered composite rock is:

$$\frac{1}{E} = \frac{L_1}{E_1 L} + \frac{L_2}{E_2 L} + \frac{L_3}{E_3 L} \quad (12)$$

$$E = \frac{2E_1 E_2 E_3}{9E_1 E_2 + 2E_1 E_3 + 9E_2 E_3} \quad (13)$$

In the formula, L_1 , L_2 and L_3 are the heights of the upper, middle and lower three-layer rock samples, respectively; E_1 , E_2 and E_3 are the elastic moduli of the upper, middle and lower three-layer rock samples, respectively, where $L_1 = L_3 = 4.5L_2$.

Therefore, the constitutive model of the layered composite rock in this paper can be obtained:

$$\sigma_1 = E\varepsilon_1 \left(1 - \delta \left\{ 1 - \exp \left[- \left(\frac{\varepsilon_1}{\varepsilon_0} \right)^m \right] \right\} \right) + 2\nu\sigma_3 \quad (14)$$

3 Model test verification

In order to verify the derived constitutive model, the test sample is selected as a cylindrical shape with a diameter of and a length of 100 mm without obvious cracks and uniform texture. The materials are three kinds of base rocks: blue sandstone, red sandstone and white sandstone, which are the upper, middle and lower layers of the sample (see Figure 2). A microcomputer-controlled rock triaxial test system (see Figure 3) is used to perform triaxial compression experiments on layered composite rocks. The stress–strain curves of layered composite rock under four different confining pressures of 5 MPa, 10 MPa, 15 MPa and 20 MPa are obtained.

Fig. 1 Action diagram of layered composite rock under transverse stress σ .

Fig. 2 Layered composite rock sample.

The values of model parameters ε_0 , m and correction coefficient δ are shown in Table 1 using the curve fitting method and MATLAB software. Finally, the statistical constitutive model of rock damage is obtained, and the model curve is fitted with the experimental data (see Figure 4). As shown in Figure 4, it can be found that the statistical damage constitutive model curves of layered composite rocks derived can well reflect the stress–strain process of rocks under different confining pressures by comparing the coincidence of model curves and experimental data. The experimental data are closely consistent with the model curve. The coincidence is good at not only the pre-peak stage of the rock but also the post-peak failure stage of the rock due to the introduction of the correction factor δ . It can be known from the experiment that as the confining pressure increases, the brittleness of the rock decreases and the corresponding ductility increases. The rock statistical damage constitutive model derived in this study has a high fitting accuracy for the post-peak stage of the rock, which is superior to the traditional linear fitting method.

Fig. 3 Microcomputer controlled rock triaxial test system.

Table 1 Fitting parameters under different confining pressures.

Confining pressure/MPa	E /MPa	δ	ε_0	m	ν	Correlation coefficient
0	20,942	0.7192	0.004159	3.796	0.15	0.9889
5	25,618	0.7615	0.003970	3.626	0.15	0.9826
10	27,790	0.7406	0.004139	3.592	0.15	0.9941
15	22,260	0.7145	0.005430	3.486	0.15	0.9873
20	22,741	0.8990	0.007028	2.340	0.15	0.9856

4 Parameter analysis of constitutive model

The main fitting parameters in this paper are E , ν , ε_0 , m and δ , among which E and ν can be obtained by experiments, and ε_0 , m and δ need to be fitted. In order to study the influence of these parameters on the stress–strain curve and damage variables, the confining pressure of 10 MPa is selected as the basic reference. The influence of the parameters is fitted as follows.

As the two parameters of Weibull distribution, ε_0 and m reflect the physical and mechanical properties of rocks. When the other parameters are fixed and ε_0 is changed, the stress–strain curve changes as shown in Figure 5. It can be seen from Figure 5 that when ε_0 is changed, the elastic phase is extended, the peak point shifts to the right and the peak strength of the rock also increases. It can be concluded that ε_0 mainly reflects the change of peak strength. Therefore, the larger ε_0 is, the stronger the ability of the layered composite rock to resist deformation and failure. When the other parameters are fixed and m is changed, the stress–strain curve changes as shown in Figure 6. For the m parameter, it reflects the concentration of the strength distribution of the rock element. If m is increased, the strength distribution of microelements is more concentrated and the brittleness of the material is higher. When m is increased, the post-peak curve becomes steeper and the brittleness of the material increases. Conversely, if the value of m is decreased, the ductility of the material increases. It can be concluded that m reflects the brittleness of the rock.

It can be seen from Figure 7 that when δ is set to 1, when the rock damage correction coefficient is not introduced, the model curve is obviously different from the actual situation, and it is difficult to reflect the stress–strain situation after the rock is broken. It shows that the introduction of the correction coefficient δ is a good representation of the failure of the layered composite rock after the peak point, and it optimises the shortcomings of the constitutive model in the previous research, which has good practical significance. It can also be seen from Figure 7 that the different values of the correction coefficient δ have little effect on the first half of the model curve, so the effect before the peak point of the rock is small, and the effect after the peak

Fig. 4 Comparison of experimental curves and fitting curves under different confining pressures.

point of the curve is greater. In actual engineering applications, the value of δ will vary with lithology, confining pressure and initial defects. Therefore, the accurate value of δ is very important. It is a good way to use MATLAB software to perform curve fitting to determine the value of δ .

Fig. 5 The influence of the parameter ε_0 on the constitutive model.

Fig. 6 The influence of the parameter m on the constitutive model.

Fig. 7 The influence of the parameter δ on the constitutive model.

5 Conclusions

In this study, based on the theory of continuous damage and the theory of statistical strength, the damage evolution equation of triaxial compression fracture of layered composite rocks under constant confining pressure is derived from the perspective of Weibull distribution of the strength of microelements of rocks, and a three-

dimensional statistical damage constitutive model suitable for layered composite rocks is established. Through model validation and parameter discussion, the following conclusions can be drawn:

(1) Through experimental verification, the established model curve and the measured curve have a good consistency. The results show that the model is reasonable, and the model has fewer calculation parameters, and so the method that use functions of several variables to find extreme values is abandoned. The model parameters are determined by using MATLAB software and the curve fitting method with higher accuracy, which can better describe the constitutive relation of layered composite rocks under the action of three-dimensional stress. It provides theoretical and technical support for rock load damage calculation, anisotropy study, stability analysis of surrounding rock and rock excavation.

(2) The physical significance of Weibull distribution parameters ε_0 and m is studied, and it is concluded that the parameter ε_0 mainly reflects the change of peak strength of layered composite rocks. The larger the parameter ε_0 is, the higher the yield strength and peak strength of layered composite rocks are, and the stronger the resistance to deformation and failure is. The parameter m can reflect the brittleness of layered composite rock materials. When m is increased, the brittleness of the material increases and the yield strength also increases.

(3) In this paper, the rock damage correction factor δ is introduced to further improve the fitting accuracy of the model curve in the post-peak stage of the rock, which well reflects the failure characteristics of the layered composite rock under the three-dimensional stress state. Through the influence of the different values of δ under the same conditions on the fitting accuracy of the model curve and the experimental data, it shows the necessity of introducing δ and provides a reference for theoretical research and practical engineering, which has good practical significance.

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