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Application of B-theory for numerical method of functional differential equations in the analysis of fair value in financial accounting

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Abstract

Financial accounting, the use of historical cost of assets, is an important basic principle of historical cost, which is to become the dominant mode of accounting measurement. Background analyses, as well as the historical cost basis and fair value, result from the development of the theory of historical cost and fair value. Historical cost and fair value measurement model has its own advantages and problems. Based on this background, the paper applies B- theoretical numerical methods to differential equations pan function analysis for calculation of fair value accounting and conducts theoretical analysis of their stability and convergence. Finally, numerical examples with different methods of calculating an approximate solution are provided and a comparison of the various methods is done based on the results obtained. The results show fair value accounting better meets the needs of the target –decision-making availability, compared to historical cost or fair value, more in line with the requirements of Accounting Information Quality.

Keywords: fair value, universal function, differential equation, B-theory, financial accounting
AMS 2010 codes: 12H99

1 Introduction

The 2008 financial crisis in the fair value almost quit the stage of history, showing that fair value can be more reliable than historical cost financial information owing a series of empirical tests by international scholars on fair value. With the economic recovery after the crisis, the measurement attribute is gradually being recognized by the market, generally the role of enterprises, especially in the financial sector listed companies. In addition, as China's market economy continues to mature, users of financial information on corporate accounting standards have proposed new requirements; they hope to get more reliable accounting information to help them make

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investment decisions more effectively. Moreover, driven by international accounting standards, in 2014, the Ministry of Finance issued document number CAS39, which contains the latest requirements for fair value measurement of property, in which the major response is to the international call. The document adds the international facet of the current widespread use of Value hierarchy theory. The Ministry of Finance, in July 2014, began to stratify the fair value disclosures in all walks of life, to enhance the reliability of financial accounting information and user information in order to achieve its new requirements [1].

Currently, one of the key accounting difficulties facing the industry is the fair value; it is important to explore the contents of the development of the accounting profession, which is also the objective of the application of accounting practices. The fair value of the research can promote the application of corporate accounting standards and practice of continuous improvement. Therefore, the performance of the main meaning and purpose of this thesis are as follows [2]: (1) a reasonable application of fair value measurement can be obtained through the hierarchical listing of the Company's assets and liabilities more accurately, to provide more accurate data for business users of accounting information to help investors better understand the financial reports of listed companies, to guide investors to make the right investment decisions; (2) through the application of the stratification of the measured fair value of listed companies , we can more clearly understand the extent of the use of fair value measurement stratification in our country, to propose guidance to standardize the application of fair value measurement stratification, and on this basis, the financial report of listed companies, and promote the healthy development of China's accounting theory [3].

Many phenomena in nature and engineering technology, such as the operation of automatic control systems, the operation of power systems, the movement of aircraft, the process of chemical reactions, and some problems of ecological balance, can be abstracted into an initial value problem for ordinary differential equations (ODEs). Its true solution is usually difficult to obtain by analytical methods. Up to now, there are many types of differential equations that cannot give analytical expressions for solutions. Generally, they can only be calculated by numerical methods. Research on this problem dates to the 18th century and can be seen especially in the general application of computers. Many differential equation problems have yielded numerical solutions, so that people can understand the properties of the solutions and their numerical characteristics, and these provide a quantitative basis for practical problems such as in engineering technology. The calculation of fair value is no exception. The numerical algorithms for ODEs have developed to the present day with the linear multistep method, the Runge-Kutta method [4], the single-branch method, block method, loop method, extrapolation method, hybrid method, order derivative method, and various commonly used estimation and correction algorithms. Among the commonly used linear multistep method formulas are B-theory, Heun formula, midpoint formula, Milne formula, Adams formula, Simpson formula, Hamming formula, Gear method, Adams prediction-correction method, and Milne-prediction–Hamming correction formula, etc. Borage once compared the linear multistep method and the Runge-Kutta method to two small islands in the sea. In the vast ocean, there are many new methods that have not been discovered until now.

This article details some of the numerical methods for ODE initial value problem, which are derived using several numerical methods. The theoretical papers applied B-numerical methods to the differential equations on functional analysis calculating fair value accounting, as well as conducted theoretical analysis of stability and convergence. Finally, a numerical example is provided, using different methods to calculate the approximate solution and comparison of various methods is provided with the results obtained. The results find that fair value accounting better meets the needs of the target – decision-making availability, compared to historical cost or fair value, more in line with the requirements of Accounting Information Quality [5].

2 Numerical solution of initial value problems for differential equations of functional functions

2.1 Basic ideas of numerical methods

Consider the initial value problem of first-order ODEs

$$\frac{dx}{dy} = f(x,y), x \in [a,b] \qquad (1)$$
$$y(x_0) = y_0$$

where f is a known function of x and y and y_0 is a given initial value.

1. One-step method – The value of $y(x)$ at $x = x_{n+1}$ calculated depends only on the amount of strain at x_n and its derivative.

2. Multistep method – To calculate the value of $y(x)$ at $x = x_{n+1}$ requires the value of the strain variable and its derivatives at multiple nodes before x_{n+1} [6].

2.2 Functional differential equations

2.2.1 B-theory

Let the derivative $y'(x_n)$ of function $y(x)$ at point x_n be replaced by a two-point expression, i.e.,

$$y'(x_n) \approx \frac{y(x_{n+1}) - y(x_n)}{h} \qquad (2)$$

Then use y_n to replace $y(x_n)$ approximately, then the initial value problem in Eq. (2) becomes

$$\begin{cases} y_{n+1} = y_n + hf(x_n, y_n) \\ y_0 = y(x_0), n = 0, 1, 2, \cdots \end{cases} \qquad (3)$$

Equation (2) is the famous B-theoretical formula. Figure 1 shows the B-theoretical image representation.

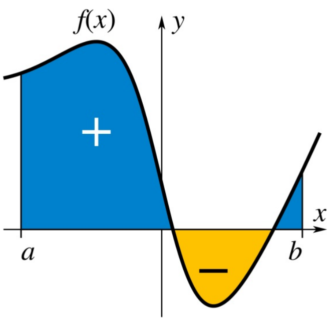

Fig. 1 B-theoretical image representation.

2.2.2 Ladder formula

The B-theoretical method has a simple form and low accuracy. In order to improve the accuracy, the two ends of the equation $y' = f(x,y)$ are integrated over the interval $[x_n, x_{n+1}]$.

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} f[x, y(x)]dx \qquad (4)$$

Use the trapezoid method to calculate its integral term, i.e.,

$$\int_{x_n}^{x_{n+1}} f[x,y(x)]dx \approx \frac{x_{n+1}-x_n}{2}\left[f(x_n,y(x_n))+f(x_{n+1},y(x_{n+1}))\right] \tag{5}$$

Substituting into Eq. (3) and replacing y_n in the equation with $y(x_n)$, we obtain the following trapezoidal formula;

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n,y_n)+f(x_{n+1},y_{n+1})\right] \tag{6}$$

Because the trapezoidal formula of numerical integration is more accurate than the rectangular formula, the trapezoidal formula in Eq. (6), which is a numerical method, is more accurate than the B-theory (3). The right end of Eq. (6) contains the unknown factor y_{n+1}, which is a function equation about y_{n+1}. This type of method is called an implicit method. Figure 2 shows a schematic diagram of a trapezoidal formula [7].

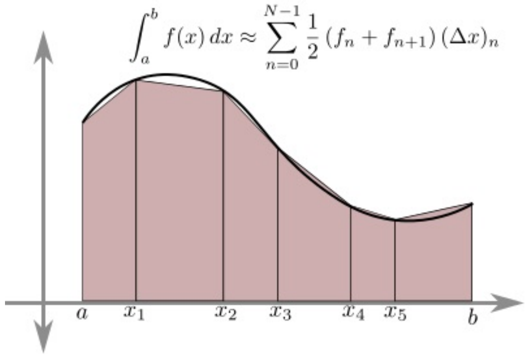

Fig. 2 Ladder formula.

2.2.3 Improved B-theory

The trapezoidal formula requires multiple iterations in actual calculation, so the calculation amount is large. Practically, the trapezoidal formula in Eq. (6) is only iterated once, i.e., the estimated value B of y_{n+1} is first calculated using $\bar{y}_{n+1}$,-theory, and then using the trapezoidal formula Eq. (6) to perform an iteration to obtain the correction value y_{n+1}, i.e.,

$$\begin{cases} \bar{y}_{n+1} = y_n + hf(x_n,y_n) \\ y_{n+1} = y_n + \frac{h}{2}\left[f(x_n,y_n)+f(x_{n+1},\bar{y}_{n+1})\right] \end{cases} \tag{7}$$

2.2.4 B-theoretical local truncation error

A major criterion for measuring the quality of a solving formula is the accuracy of the solving formula, so the concepts of local truncation error and order are introduced.

For the B-theory, assuming $y_n = y(x_n)$, then

$$y_{n+1} = y(x_n) + h\left[f(x_n,y(x_n))\right] = y(x_n) + hy'(x_n) \tag{8}$$

And the true solution $y(x)$ according to the second-order Taylor expansion at x_n, has

$$y_{n+1} = y(x_n) + hy'(x_n) + \frac{h^2}{2!}y'''(\xi), \quad \xi \in (x_n,x_{n+1}) \tag{9}$$

So, there is

$$y(x_{n+1}) - y_{n+1} = \frac{h^2}{2!}y''(\xi) \tag{10}$$

If the local truncation error of the numerical method is $O(h^{p+1})$, then the order of this numerical method is called P. The smaller the step size (h <1), the higher is the P, the smaller is the local truncation error, and the higher is the calculation accuracy [8].

2.3 Runge-Kutta method

2.3.1 The basic idea of Runge-Kutta method

B-theory can be rewritten as follows:

$$\begin{cases} y_{n+1} = y_n + hK_1 \\ K_1 = f(x_n, y_n) \end{cases} \tag{11}$$

Then the expression of y_{n+1} is exactly the same as the first two terms of Taylor's expansion of $y(x_{n+1})$, i.e., the local truncation error is $O(h^2)$. The improved B-theory can be rewritten as follows:

$$\begin{cases} y_{n+1} = y_n + \frac{h}{2}(K_1 + K_2) \\ K_1 = f(x_n, y_n) \\ K_2 = f(x_{n+1}, y_n + hk_1) \end{cases} \tag{12}$$

The above two sets of formulas have one thing in common in their form: they both use the linear combination of the values of $f(x,y)$ at some points to obtain the approximate value y_{n+1} of $y(x_{n+1})$, and increase the number of calculations of $f(x,y)$, which can improve the order of truncation error. In the B-theory, the value of $f(x,y)$ is calculated once per step, which is a first-order method. The improved B-theory needs to calculate the value of $f(x,y)$ twice, which is a second-order method. Its local truncation error is $O(h^3)$.

2.3.2 Runge-Kutta method construction

Generally, Runge-Kutta method sets the approximate formula as

$$\begin{cases} y_{n+1} = y_n + h \sum\limits_{i=1}^{p} c_i K_i \\ K_1 = f(x_n, y_n) & (i = 2, 3, \cdots, p) \\ K_i = f(x_n + a_i h, y_n + h \sum\limits_{j=1}^{i-1} b_{ij} K_j) \end{cases} \tag{13}$$

Among them, a_i, b_{ij}, and c_i are parameters. The principle to determine them is to make the Taylor expansion of the approximate formula at y_m and the Taylor expansion of $y(x)$ at x_n coincide as much as possible, so that the approximate formula has as much high accuracy as possible. With this, we can get the commonly used third- and fourth-order Runge-Kutta formulas through a complicated calculation process, as shown in Figure 3.

$$\begin{cases} y_{n+1} = y_n + \frac{h}{6}(K_1 + 4K_2 + K_3) \\ K_1 = f(x_n, y_n) \\ K_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_1) \\ K_3 = f(x_n + h, y_n - hK_1 + 2hK_2) \end{cases} \tag{14}$$

With

$$\begin{cases} y_{n+1} = y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\ K_1 = f(x_n, y_n) \\ K_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_1) \\ K_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_2) \\ K_4 = f(x_n + h, y_n + hK_3) \end{cases} \tag{15}$$

Equation (15) is called the classic Runge-Kutta method.

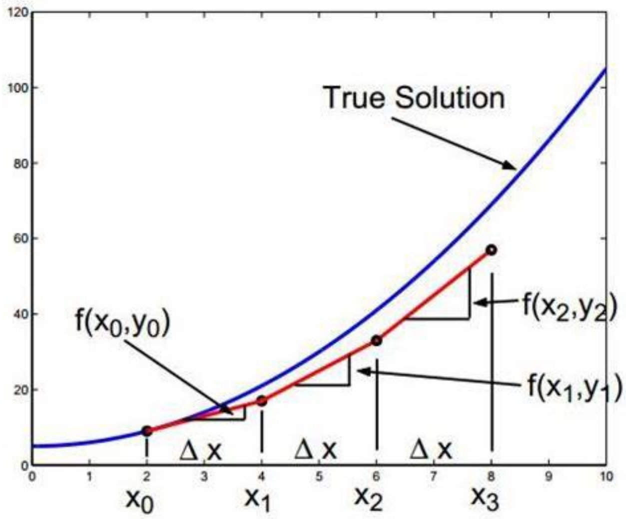

Fig. 3 Runge-Kutta method.

2.4 Linear multistep method

The general formula of the linear k-step method is

$$y_{n+1} = \sum_{j=0}^{k-1} a_j y_{n-j} + h \sum_{j=-1}^{k-1} b_j f(x_{n-j}, y_{n-j}) \tag{16}$$

$$R_{n+1} = y(x_{n+1}) - \left[y_{n+1} = \sum_{j=0}^{k-1} a_j y_{n-j} + h \sum_{j=-1}^{k-1} b_j f(x_{n-j}, y_{n-j}) \right] \tag{17}$$

Local truncation error of k-step formula is Eq. (17) at x_{n+1} when

$$R_{n+1} = O(h^{p+1}) \tag{18}$$

Equation (17) is said to be of order p.

Using equation $y'(x) = f(x, y(x))$, we know that the local truncation error can also be written as

$$R_{n+1} = y(x_{n+1}) - \left[y_{n+1} = \sum_{j=0}^{k-1} a_j y_{n-j} + h \sum_{j=-1}^{k-1} b_j y'(x_{n-j}) \right] \tag{19}$$

2.4.1 Adams extrapolation

Differential equation:

$$\frac{dx}{dy} = f(x,y), x \in [a,b] \tag{20}$$

Integrating both ends from x_n to x_{n+1}, we get

$$y(x_{n+1}) = y(x_i) + \int_{x_n}^{x_{n+1}} f(x, y(x))dx \tag{21}$$

We replace the integrand on the right with an interpolation polynomial, as shown in Figure 4.

Adams extrapolation selects k points $x_n, x_{n-1,=}, \cdots, x_{n-k+1}$ The $k-1$ order polynomial in Adams extrapolation method used as the interpolation base point to construct $f(x,y)$ is calculated as follows [9]:

$$y_{n+1} = y_n + h \sum_{j=0}^{k-1} a_j \nabla^j f_m \tag{22}$$

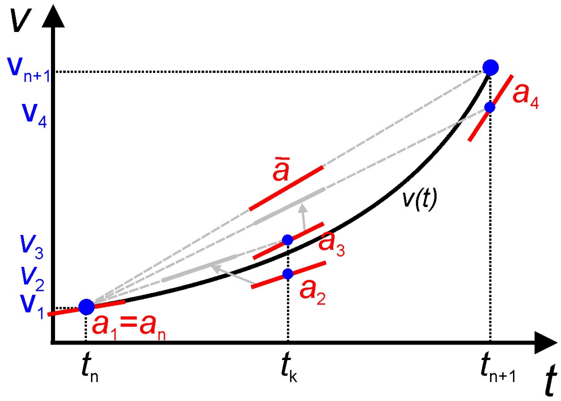

Fig. 4 Adams extrapolation.

where a_j satisfies the following algebraic recursion:

$$\mathrm{a_j}+\frac{1}{2}a_{j-1}+\frac{1}{3}a_{j-2}+\cdots+\frac{1}{j+1}a_0=1 \quad j=0,1,\cdots \tag{23}$$

According to this recursive formula, $a_j\,(j=0,1,\cdots)$ can be calculated one by one. Table 1 gives some values of a_j.

Table 1 Some values of a_j

j	0	1	2	3	4	5	$\cdots$
a_j	1	1/2	5/12	3/8	251/720	95/288	$\cdots$

2.4.2 Adams interpolation

According to the interpolation theory, the choice of the interpolation node directly affects the accuracy of the interpolation formula, and the accuracy of the interpolation formula of the same degree is higher than that of the extrapolation formula. The Adams interpolation formula for the initial value problem in Eq. (1) can be derived as follows:

$$y(x_{n+1})=y(x_n)+\int_{x_n}^{x_{n+1}} L^{(p)}_{n,k-1}(x)dx \tag{24}$$

The above formula degenerates into the interpolation formula at $p=0$, as shown in Figure 5.

In addition to the known value of f at $x_{n-k+1},\cdots,x_n$, the interpolation formula in Eq. (24) also contains the unknown value at point $x_n,\cdots,x_{n+p}$, so the interpolation Eq. (9) only gives the relationship of the unknown quantity $y_n,\cdots,y_{n+p}$. Actual calculation is still necessary to solve the simultaneous equations. The interpolation formula of $p=1$ is the most applicable. $L^{(1)}_{n,k-1}(x)$ uses the Newton backward interpolation formula to obtain the Adams interpolation formula.

$$y_{n+1}=y_n+h\sum_{j=0}^{k}a^{*}{}_j\nabla^j f_m \tag{25}$$

where the coefficient $a^{*}{}_j$ is defined as

$$a^{*}{}_j=(-1)^j\int_{-1}^{0}\binom{-\tau}{j}d\tau \quad j=0,1,\cdots \tag{26}$$

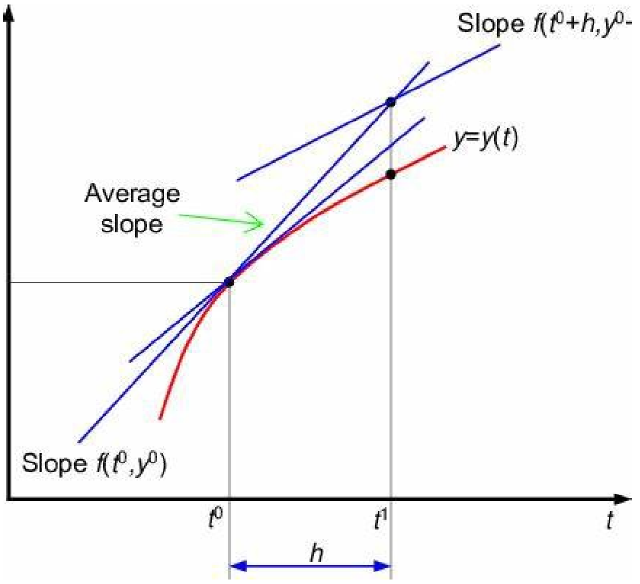

Fig. 5 Adams interpolation.

Here, $a^*{}_j$ satisfies the following algebraic recursion:

$$a^*{}_j + \frac{1}{2}a^*{}_{j-1} + \frac{1}{3}a^*{}_{j-2} + \cdots + \frac{1}{j+1}a^*{}_0 = \begin{cases} 1, j = 0 \\ 0, j > 0 \end{cases} \tag{27}$$

According to this recursive formula, $a^*{}_j\,(j = 0, 1, \cdots)$ can be calculated one by one. Table 2 gives some values of $a^*{}_j$:

Table 2 Some values of $a^*{}_j$

j	**0**	**1**	**2**	**3**	**4**	**5**	$\cdots$
$a^*{}_j$	1	$-1/2$	$-1/12$	$-1/24$	$-19/720$	$-3/160$	$\cdots$

3 Case analysis

Solve the initial value problem with Euler's method, improved Euler's method, and Runge-Kutta:

$$\begin{cases} y' = -2xy^2 (0 \le x \le 1.2) \\ y(0) = 1 \end{cases} \tag{28}$$

For the numerical solution, take $h = 0.1$, $N = 10$, and compare with the real solution.

Since this simple equation can be obtained mathematically by its exact description $y = \frac{1}{1+x^2}$, it can be used to check the error between the approximate numerical solution and the true solution. The method of implementing the numerical solution for other ODEs with complex structures is the same [10].

3.1 General solution using B-theoretical algorithm

The B-theoretical algorithm calculates the approximate numerical solution of y_i at each point x_i and the error between the approximate numerical solution and the exact solution, as shown in Table 3 below.

Table 3 B-theoretical algorithm calculation results

$x[i]$ **points**	$y[i]$ **(numerical solution)**	$y(x[i])$ **(exact solution)**	$\lvert y(x[i]) - y[i] \rvert$ **(error)**
0.1	1.000000	0.990099	0.009901
0.2	0.980000	0.961538	0.018462
0.3	0.941584	0.917431	0.024153
0.4	0.888389	0.862069	0.026320
0.5	0.825250	0.800000	0.025250
0.6	0.757147	0.735294	0.021852
0.7	0.688354	0.671141	0.017213
0.8	0.622018	0.609756	0.012262
0.9	0.560113	0.552486	0.007626
1.0	0.503642	0.500000	0.003642
1.1	0.452911	0.452489	0.000422
1.2	0.407783	0.409836	0.002053

From the experimental results, the error is not too small. Moreover, this is only a relatively simple ODE for experiments. For structurally complex equations applied in actual engineering, the error of the solution result is much larger than this. Because there are also local truncation errors and global truncation errors, some measures can be taken to suppress the reduction of errors and make the results as accurate as possible.

3.2 General solution using improved B-theoretical algorithm

The improved B-theoretical algorithm calculates the approximate numerical solution of y_i at each point x_i and the error of the exact numerical solution with each other, as shown in Table 4.

Table 4 Improved B-theoretical algorithm calculation

$x[i]$ **points**	$y[i]$ **(numerical solution)**	$y(x[i])$ **(exact solution)**	$\lvert y(x[i]) - y[i] \rvert$ **(error)**
0.1	0.990000	0.990099	0.000099
0.2	0.961366	0.961538	0.000173
0.3	0.917246	0.917431	0.000185
0.4	0.861954	0.862069	0.000115
0.5	0.800034	0.800000	0.000034
0.6	0.735527	0.735294	0.000233
0.7	0.671587	0.671141	0.000446
0.8	0.610399	0.609756	0.000643
0.9	0.553289	0.552486	0.000803
1.0	0.500919	0.500000	0.000919
1.1	0.453479	0.452489	0.000990
1.2	0.410859	0.409836	0.001023

The error of this improved B-theoretical algorithm has been greatly reduced. The decrease in error is mainly due to the use of B-theory to estimate the value of y_{i+1}, and then the trapezoidal formula for prediction. The estimation is corrected, thereby reducing errors in the estimation-correction process. This algorithm has certain advantages and can be directly applied separately in some practical and simple engineering calculations [11].

4 Theoretical analysis of fair value

As people increasingly demand high information quality, fair value measurement stratification has become a trend. Because fair value measurement enables the tiered accounting information to be more reliable, the information will be of higher quality. The relevance and value of the stock are based on performance, the main method of information quality, while the value of the theoretical basis of relevance is the efficient market theory. Efficient market theory is the theoretical basis for three assumptions, namely: (1) the efficient market theory assumes that all investors are rational economic people, and assessment of the value of the assets is a rational behavior; (2) the effective market theory assumes that even if there is no rational investor, that is a small part, but their trading activity also occasionally occurs randomly, and the impact of irrational transactions made on the valuation of securities due to chance cancel each other out, i.e., valuation of securities will not be affected by irrational transactions; (3) the efficient market theory assumes that if the irrational trading behavior affects the pricing of the securities market, which is not accidental, it will be because rational investors' hedging exists to eliminate this effect. Therefore, we propose that value relevance of fair value measurement of stratification is feasible under the efficient market theory.

5 Conclusion

B-theory, improved B-theory, classic Runge-Kutta method, and so on are integrated into the computer algorithm, making full use of the speed advantage of the computer, greatly reducing the labor intensity of financial personnel and making the calculation results more reliable and accurate. The use of fair value is an irreversible trend, but this economic crisis also reminds us that the use and implementation of fair value must not be rushed. At the current stage of China's market system, legal and regulatory construction, and staffing, rushing is not ideal. It should be developed step by step and continuously improved. Only in this way can we give full play to the advantages of fair value in accounting measurement and better serve economic construction.

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