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Calculating university education model based on finite element fractional differential equations and macro-control analysis

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Abstract

Firstly, based on the charging theory of ‘education cost-sharing,’ under appropriate assumptions, two basic differential equation models are proposed to describe the problem of college education charges; secondly, through qualitative analysis of the basic model, it is concluded that colleges and universities maintain or impose several conditions for stabilising its education fees; finally, through the analysis of two basic models in three unique models under three situations, some new conclusions and suggestions on the macro-control of college education fees and enrolment scale are given. Also, three extended differential equation models are proposed.

Keywords: higher education fees, differential equation model, education cost-sharing, macro-control

AMS 2010 codes: 34A34

1 Introduction

Relative to demand, resources are always scarce, so people need to integrate and efficiently use the resources so that they can somehow meet demand. The development of higher education and the school-running activities of universities also conform to this law. As public institution, colleges and universities took care of teaching and research activities, and they do not directly produce material products with independent legal status like companies. However, according to the process of resource input, resource use and outcome output, it can still be regarded as an educational enterprise that uses resources to obtain learning outcomes. Therefore, colleges and universities also face cost problems similar to those in companies [1]. Here we try to find out the ways and

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means to control unnecessary costs in the process of running colleges and universities while ensuring the quality of talent training, improving the efficiency of resource use, and improving the efficiency of running schools, which are important issues facing the development of colleges and universities [2]. The current situation of college education and the cost in the process of running a school have highlighted the necessity and urgency of scientific cost control, which are mainly manifested in the following three aspects:

Firstly, the total amount of educational resources has increased, but education funding continues to be in short supply. With the rapid development of China's economy, the society's demand for higher education is increasing and the requirements are getting higher and higher, prompting the rapid expansion of the scale of higher education.

Secondly, we have to deal with the financial crisis of colleges and universities. According to the 'Year: Analysis and Forecast of China's Social Situation' issued by the Chinese Academy of Social Sciences, the number of students in China's regular colleges and universities has doubled during the year. Colleges and universities want to achieve leap-forward development by expanding enrolment and the scale of running schools. Many colleges and universities have successfully upgraded to universities with new houses, old teachers and a large number of students, blindly increasing the subject categories, and there are both engineering and science colleges [3]. The strange phenomenon of journalism and advertising has also come into play. Local governments are more motivated. Officials in many regions apply urban management and market operations to the management of colleges and universities, and they do not hesitate to invest huge sums of money to develop college parks and college towns, hoping to use the development of higher education to stimulate the economic development of a region, and even promote the comprehensive development of a city's society, technology and education. As a result, the popularisation of higher education [4], the gross enrolment rate, and universities with 10,000 people have become the boasting achievements of local governments. The bank has found a group of large customers without major risks and can provide loans to colleges and universities with confidence. After the extreme expansion, the sequelae of college loans subsequently appeared. College funds were limited, the loan period was short, and short-term loan repayments were impossible. The scale of some college loans greatly exceeded the economic capacity and colleges were unable to repay loans. As the country raised bank interest rates, some schools found it difficult to even repay the interest. Tight funding has affected the quality of higher education [5]. To repay loans, some colleges and universities have to reduce student funding and office funding. Many school students could not take experimental courses, had no money for internships and the quality of education could not be guaranteed.

2 Higher education cost and its control

2.1 The connotation of higher education costs

Cost refers to the sum of all actual resources consumed to engage in an investment plan. Under the conditions of a market economy [6], the cost is a part of the value of commodities, materialised labour and living labour consumed in the production of commodities. With the transformation of the social-economic system and the transformation of people's educational concepts, the cost of education began to be involved when studying the economic benefits of educational investment, so we introduced the cost of economics into the field of education to use the concept of education cost [7]. Education cost refers to all the expenses paid by each student, including the direct and indirect expenses borne by the government, society, schools and educated individuals. The cost of higher education refers to the total cost of training each college student, that is, the sum of direct and indirect living labour and materialised labour consumed by students of various types of colleges and universities during their studies. Specifically, the cost of higher education includes the direct and indirect expenses paid by the government, society, schools and educated individuals to train students [8]. Therefore, the cost of higher education is the subordinate concept of the cost of higher education, that is, the total value of materialized labour and living labour consumed by each student in the stage of receiving higher education [9], and the value con-

sumed to maintain personal life needs. The cost of higher education is the direct social cost of higher education, and it is the actual cost.

2.2 Classification of the cost of college education

To conduct an in-depth analysis of the cost of a college education, many scholars have classified the cost of a college education. Since it is a classification, there must be a certain classification standard [10]. For example, according to the burden of the education cost or the source of education funding (social, personal), the nature or function of the education resource (recurring, capital), the use of the education resource, the material), the technical characteristics of the education resource (fixed, variable) And other standards for classification. Concerning the merits of various education cost classification methods, this research proposes the classification of college education costs. This classification conforms to the essential connotation of education cost and reflects the composition and its function. It should be noted that the opportunity cost here includes not only the income loss of capital assets but also the income loss of non-capital public funds and personal funds. In most of the literature, the educational opportunity cost does not include the above-mentioned non-capital public funds and personal capital loss. This research adopts the ideas of Han Zongli and Gao Jianmin and incorporates them into opportunity cost. It is because they are of the same nature as the loss of capital assets. Since the income loss of capital assets is included in the opportunity cost, they should also be included in the opportunity cost. You cannot simply add up school costs and personal costs to get the total financial cost, because there are also several duplication problems. They are mainly the duplication of tuition and miscellaneous fees and school costs in personal costs. The actual total financial cost is the addition of tuition and miscellaneous fees and the two [11].

2.3 Analysis of the necessity of cost control of higher education

How to reduce the waste of resources and improve the efficiency of resource utilisation based on controlling the cost in the process of running a university and ensuring the quality of talent training are the major problems facing the development of universities. However, the many problems in cost control in the process of running colleges and universities now highlight the necessity of scientific cost control.

2.3.1 Education cost is an important indicator to measure the level of school management

Education cost is a comprehensive reflection of resource consumption in educational economic activities of colleges and universities. It reflects school teaching! The comprehensive index of work quality also directly reflects the level of management and school-running efficiency of a school. If colleges and universities can use the same amount of cost input to produce more benefits, it should be said that this school has a better management level and higher school-running benefits. Source, distribution, use and supervision of funds are important factors in the function of school management. The quality of teaching, the level of teaching management and the strength of scientific research capabilities can be directly or indirectly reflected through costs. It can be said that education cost is an important indicator to measure the level of school management.

2.3.2 Education cost control is an important means to improve the efficiency of running a school

In a century, western economists applied the investment-cost-benefit analysis to education. Cost-benefit analysis is also the most effective way to improve the efficiency of school management. Through the cost analysis of school teaching and storage, we can find problems in school management and management, find out the weak links in management, improve all aspects of work, and ensure that the limited teaching and storage resources can be used to maximise their functions, thereby effectively improving the school-running benefits.

2.3.3 Education costs can provide a basis for education cost investment decisions

Education cost investment is not only a consumer investment, but it also has the same problem of investment efficiency as enterprise investment. When making decisions, college leaders must consider many aspects and analyse various influencing factors. Especially when funds are short, it is necessary to focus on analysing the

investment benefits of various decision-making programmes. The benefit is related to the cost. Only when the output is greater than the cost of the input, can the university obtain the benefit of running a school. At the same time, for the entire school to obtain good school-running benefits, the limited educational resources must be tilted towards high-efficiency sections, to realise the optimal allocation of resources and ensure the high-efficiency of educational resources input. It can be seen that education cost is an important basis for the allocation of educational resources, and it can provide a scientific basis for educational cost investment decisions. Therefore, it is very important to choose decision-making schemes carefully, evaluate them reasonably, and effectively improve them.

From the perspective of educational economics, resources are scarce, and colleges and universities face the problem of striving for more educational resources and use them effectively in the development process. The result of this problem is that the total amount of educational resources has increased, but the education funding is still in short supply, and the efficiency of the use of educational resources in colleges and universities is low. All these affect the healthy, sustained and stable development of our country's higher education. Education cost is an important indicator to measure the management level of a university. Education cost control is an important means to improve the efficiency of running a school. Education cost is an important basis for formulating tuition fees. Education cost is also an important basis for educational cost investment decisions. Therefore, the solution to the education cost problem is of great significance to the improvement of the management level and the efficiency of the university.

3 Two basic differential equation models of college education fees

According to the theory of 'education cost sharing' put forward by New York University President D. Bruce Johnstone in 1986, the educational cost of colleges and universities should be shared by the government, society and educated individuals. In fact, the educational fees of colleges and universities are only shared by educated individuals. The cost of education is related to the cost of education. According to the principle of supplementation and affordability of education fees, we believe that the main influencing factors of college education fees are the charging standard (which is reflected by the education cost shared by the educated) and the inability of poor students to pay up tuition fees arrears). Now, based on this, we set the number of enrolment of colleges and universities N as a continuous variable, and the number of enrolment of colleges and universities is $R(N)$, the education cost shared by per capita is $\gamma(N)$, and the unpaid expenses of poor students are $p(N)$, they are all functions of enrolment. Because $\gamma(N)$ and $p(N)$ meet the following conditions: when $\gamma(N)$ increases, $R(N)$ increases; when $p(N)$ increases, $R(N)$ decreases, so we can assume that the rate of change of $R(N)$ is positive with $\gamma(N)$ Linearly related, and negatively linearly related to $p(N)$, so we get the following differential equation model describing the issue of college education fees (1)

$$R' = \delta\gamma(N) - \sigma p(N) \quad (1)$$

Among them, $\delta > 0, \sigma > 0$ is the proportional coefficient. Since the per capita unpaid expenses of poor college students $p(N)$ will increase with the increase in the per capita education cost $\gamma(N)$ shared by the individual educated, we also have $p(N) = g(\gamma(N))$, where $g(\gamma)$ is a monotonically increasing function, and $g'(\gamma) \geq 0$ represents Growth rate. At this time, we further obtain the following differential equation model for the issue of college education fees (2)

$$R' = \delta\gamma(N) - \sigma g(\gamma(N)) \quad (2)$$

Further, if it is assumed that $\gamma(N)$ and $p(N)$ are determined by $R(N)$, then we have $\gamma(N) = f(R(N))$ and $p(N) = h(R(N))$ respectively, where $f(R)$ and $h(R)$ are both monotonically increasing functions, and $f'(R) \geq 0$ and $h'(R) \geq 0$ can be set. At this time, we get the following college education Differential equation model of charging problem (3)

$$R' = \delta f(R) - \delta h(R) \quad (3)$$

In the same way, we can also get the differential equation model (4) corresponding to (2) describing the issue of college education charges

$$R' = \delta f(R) - \sigma g(f(R)) \quad (4)$$

If $N, R, f(R), h(R)$ in model (3) is regarded as time, population number, absolute reproduction rate and mortality rate, then (3) describes the differential equation model of the free development of the population, and different functional forms represent different populations. Development status. Because in the population development model, mortality and reproduction rate are not directly related, for the issue of college education charges here, (2) and (4) are the main governing equation models proposed in this article.

4 Qualitative analysis of the basic model of higher education fees

In this section, we mainly use qualitative analysis (2) and (4) to find a solution to the problem of college education fees. From model (2), it can be seen that the sufficient and necessary conditions for college education fees to be constant are

$$\delta\gamma(N) = \sigma g(\gamma(N)) \quad (5)$$

This shows that the necessary and sufficient condition for universities to keep their education fees unchanged is to balance the per capita education costs shared by individuals and the per capita unpaid expenses of poor students $g(\gamma(N))$. In order to achieve this balance, the existence of Eq. (5) must be required. The root is N_0 so that the number of college enrolment N_0 and the per capita education cost $\gamma_0 = \gamma(N_0)$ shared by individuals can be determined.

At present, China limits the upper limit of the proportion of education costs shared by individuals to 25%. In fact, the proportion of poor students in colleges and universities has exceeded 30%. This is an imbalance. The objective reason is that the cost of higher education is difficult to calculate and easy to charge. The standard exceeds the upper limit of the proportion of education costs shared by individuals, or part of the government investment is compensated through fees. Therefore, under the condition of expanding the enrolment scale of colleges and universities, increase the government's education funding investment from policy measures to preferentially encourage and attract social donations to run schools. Solving the ability of poor students to pay in the form of student loans, scholarships, work-study, etc. are all solutions to achieve equilibrium and solve the problem. Further, according to the function extreme value judgement method, we still have colleges and universities that keep the education fees unchanged as follows with sufficient conditions:

1. When $\gamma'(N_0) > 0, 0 < g'(\gamma_0) < \frac{\delta}{\sigma}$, there is $R'(N_0) = 0, R''(N_0) > 0$, and the education fees of colleges and universities have reached a minimum. This shows that under the condition that the per capita education cost $\gamma(N)$ shared by individuals increases, as long as the growth rate $g'(\gamma)$ of the unpaid expenses per poor student is controlled at a certain level Within the limit, the minimum fee $R(N_0)$ of universities can be kept unchanged.
2. When $\gamma'(N_0) < 0, g'(\gamma_0) > \frac{\delta}{\sigma}$, there is $R'(N_0) = 0, R''(N_0) > 0$, and college education fees reach a minimum. This shows that if the growth rate of unpaid expenses per capita of poor students exceeds a degree $\frac{\delta}{\sigma}$, then only by reducing the per capita education cost $\gamma(N)$ shared by individuals, To keep the minimum fee $R(N_0)$ of universities unchanged.
3. When $\gamma'(N_0) < 0, 0 < g'(\gamma_0) < \frac{\delta}{\sigma}$, there is $R'(N_0) = 0, R''(N_0) < 0$, and the education fees of colleges and universities reach a maximum value. This shows that when the per capita education cost $\gamma(N)$ shared by individuals decreases, and the growth rate of unpaid expenses per capita of poor students is within a certain limit $\frac{\delta}{\sigma}$, Universities can keep their extremely large fees $R(N_0)$ unchanged.
4. Under condition $\gamma'(N_0) > 0, g'(\gamma_0) > \frac{\delta}{\sigma}$, $\gamma(N)$ increases monotonously, $g'(\gamma)$ is also larger, and the educational fees of colleges and universities reach a maximum value. This situation is equivalent to high

fees, and some poor students will be in arrears with more tuition due to their inability to pay. Or the phenomenon of being out of school, so we should try our best to avoid it when solving the actual problem of education fees.

From model (4), it can be seen that the necessary and sufficient condition for the education charge $R(N)$ of colleges and universities to be constant is

$$\delta f(R) = \sigma f(f(R)) \quad (6)$$

This is another kind of equilibrium. It requires (6) that there is a positive root R_0 , so there is $\gamma_0 = f(R_0)$. Because R_0 is the equilibrium point of (4), according to the stability judgement method, we have the following stability of education fees R_0 in conclusion:

5. When $0 < g'(\gamma_0) < \frac{\delta}{\sigma}$, $f'(R_0) > 0$, R_0 is the stable equilibrium point of model (4). This shows that under the condition that the per capita education cost $\gamma = f(R)$ shared by individuals increases, only the growth rate of unpaid expenses per poor student is within a certain limit $\frac{\delta}{\sigma}$, Will keep the education charge R_0 of colleges and universities stable.
6. When $g'(\gamma_0) > \frac{\delta}{\sigma}$, $f'(R_0) < 0$, R_0 is also the stable equilibrium point of model (4). This shows that if the growth rate of the unpaid expenses of poor students exceeds a degree $\frac{\delta}{\sigma}$, then only by reducing the per capita education cost shared by individuals can it be Keep the education fees R_0 of colleges and universities stable.
7. Under other conditions, the educational fees R_0 of colleges and universities are unstable.

5 Several special models and macro-control analysis

In this section, we turn to the analysis of the basic models (2) and (4) of college education fees under several special functional forms, and give some new conclusions and suggestions on the macro-control issues of education fees and enrolment scale. In the case of free education ($\gamma = 0$), the number of enrolled students is n ; while in the case of fee-based education, the per capita education cost shared by the government stipulated by the individual is γ_M , and the number of college enrolment is $M > n$. We will discuss in three situations.

5.1 The per capita education cost shared by individuals increases with the increase in enrolment

For the sake of simplicity, under the condition that the per capita education cost shared by individuals increases with the increase in enrolment, we set $\gamma(N) = a + bN$ and $g(\gamma) = g_0\gamma$ as linear functions in model (2). So, we have $\gamma(N) = b(-n + N)$, where $b = \frac{\gamma_M}{M-n} > 0$. After substituting (2), We get the following education fee model

$$R'(N) = b(\delta - \sigma g_0)(-n + N) \quad (7)$$

It can be seen that when $N > n$, the equilibrium condition to keep the university's education fees unchanged is $\delta = \sigma g_0$; again, $0 < \gamma(N) \leq \gamma_M$ knows, $n < N \leq M$, so the size of the enrolment of colleges and universities can be expanded within the range specified by the government. Under unbalanced conditions, when $g_0 < \frac{\delta}{\sigma}$, $N > n$, There is $R' > 0$. This shows that when colleges and universities expand the scale of enrolment, only when the growth rate of unpaid fees per poor student is small (or large), can we consider raising (or lowering) education fees. Points (7), we the education charge of some colleges and universities is a function of the number of enrolments

$$R(N) = \frac{1}{2}b(\delta - \sigma g_0)(-n + N)^2 \quad (8)$$

It can be seen that the adjustment of education fees according to (8) will be too high. If $g(\gamma) = g_0\gamma^2$ is changed to a quadratic function in model (2), then it is easy to know that the scale of enrolment of colleges and universities

whose education fees are positive is

$$n < N \leq n + \frac{3\delta(M-n)}{2g_0\gamma_M} \quad (9)$$

It can be seen that when the government stipulates that the per capita education cost is lower ($\gamma_M \rightarrow 0$), it will help universities to expand the scale of autonomous enrolment. Below, we will conduct a qualitative analysis from model (4). Assume that $\lambda = f(R) = c + dR$ is a linear function and $g(\gamma) = g_0\gamma^{1+T}$ is a power function, where $T > 0$. Because in free education, there is $R = 0, \gamma = 0$; and in charging education, if the government-specified education fee is R_0 , the per capita education cost shared by individuals is γ_0 , then we have $\gamma = dR, g(\gamma) = g_0d^{1+T}R^{1+T}$, where $d = \frac{\gamma_0}{R_0} > 0$. After substituting (4), we Get the following education fee model

$$R' = \delta dR - \sigma g_0 d^{1+T} R^{1+T} \quad (10)$$

Model (10) has an unstable equilibrium point $R_1 = 0$ (which corresponds to free) and a stable equilibrium point $R_2 = \frac{1}{d} \sqrt[T]{\frac{\delta}{\sigma g_0}}$ (which corresponds to stable charges). In particular, when $T = 1$ is taken, (10) is a Logistic model, and there is $\frac{R_2}{R_0} = \frac{\delta}{\sigma g_0 \gamma_0}$. Therefore, when $0 < \gamma_0 < \frac{\delta}{\sigma g_0}$, there is $R_2 > R_0$, which shows that when the per capita education cost shared by individuals is within a certain limit, the stable education fees of colleges and universities will increase; when $\gamma_0 > \frac{\delta}{\sigma g_0}$, there is $R_2 < R_0$, which shows that when the per capita share of the individual is When the cost of education exceeds one degree, the stable education fees of universities will be reduced due to the phenomenon of some students in arrears.

5.2 The per capita education cost shared by individuals decreases with the increase in the number of students enrolled

Under the condition that the per capita education cost shared by individuals decreases with the increase in the number of enrolled students, we assume $\gamma(N) = \frac{b}{N}$ and $g(\gamma) = g_0\gamma$ in model (2). From the previous assumption, we have $\gamma(N) = \gamma_M \frac{M}{n}$. After substituting (2), the following education charge model is obtained

$$R'(N) = (\delta - \sigma g_0) \frac{M\gamma_M}{N} \quad (11)$$

Points (11) have a logarithmic relationship between education fees and enrolment

$$R(N) = (\delta - \sigma g_0) M\gamma_M \ln \frac{N}{n} \quad (12)$$

Because of $\ln \frac{N}{n} = o(N) = o(N^2)$ ($N \rightarrow \infty$), the fee determined according to (12) is lower than that of (8), and the education fees of colleges and universities can increase slowly with the expansion of enrolment scale, and then from $0 < \gamma(N) \leq \gamma_M, N \geq M$. It can be seen that the enrolment scale of colleges and universities can exceed the government's Regulations, and fees increase slowly, this is an education fee and expansion model worth promoting.

If $g(\gamma) = g_0\gamma^2$ is changed in model (2), then the number of enrolled students in universities that maintains the increase in education fees is

$$N \geq \frac{\sigma g_0 M \gamma_M}{\delta} \quad (13)$$

And the education fees of universities are

$$R(N) = \delta M \gamma_M \ln \frac{N}{n} - \sigma g_0 M^2 \gamma_M^2 \left(\frac{1}{n} - \frac{1}{N} \right) \quad (14)$$

When $N \rightarrow \infty$, the approximate enrolment scale for which the college's education fees are positive is

$$N \geq n \exp \left[\frac{\sigma g_0}{\delta n} M \gamma_M \right] \quad (15)$$

It is shown that the enrolment of colleges and universities must reach a certain scale to avoid the result of school losses.

5.3 The case where the per capita education cost shared by the individual is a bounded function of the number of enrolments

Under the condition that the per capita education cost shared is a bounded function of the number of enrolment, considering the previous two situations, we assume $\gamma(N) = \frac{bN+c}{1+N}$ and $g(\gamma) = g_0\gamma$ in model (2). Similarly, from the previous assumption, we have a monotonic bounded function $\gamma(N) = \frac{b(-n+N)}{1+N}$, where $b = \frac{\gamma_M(1+M)}{M-n} > 0$. After substituting (2), the following educational charging model is obtained

$$R'(N) = \frac{\delta b(-n+N)}{1+N} - \frac{\sigma g_0 b(-n+N)}{1+N} \quad (16)$$

Points (16) have a logarithmic relationship between education fees and enrolment

$$R(N) = (\delta - \sigma g_0) b \left[N - n - (n+1) \ln \frac{N+1}{n+1} \right] \quad (17)$$

If $g(\gamma) = g_0\gamma^2$ is changed in model (2), then it is easy to know that when $\delta < \sigma g_0 b$, the enrolment scale for universities to maintain their educational fee increase

$$n < N \leq \frac{\sigma g_0 b + \delta}{\sigma g_0 b - \delta} \quad (18)$$

Therefore, under condition $\delta < \sigma g_0 b$, colleges and universities should limit the scale of enrolment. Finally, we assume $\gamma = f(R) = \frac{dR}{1+R}$, $g(\gamma) = g_0\gamma^2$ in model (4). If the government-specified education fee is R_0 , the per capita education cost shared by individuals is γ_0 , then we have $\gamma = d_1 R$, where $d_1 = \frac{1+R_0}{R_0} \gamma_0$. Substituting (4), we get the following Logistic model of education charges

$$R' = \delta d_1 R - \sigma g_0 d_1^2 R^2 \quad (19)$$

It is easy to know that when $\delta < \sigma g_0 d_1$, Eq. (19) has unstable education charges

$$R = \frac{\delta R_0}{\delta R_0(1+R_0) - \sigma g_0} \quad (20)$$

6 Extension of the model

In the previous section, the several college education charging models in the form of special functions we gave each have their own advantages and disadvantages. If you maximise your strengths and avoid weaknesses, you can consider appropriately combining them, then you can get many extended models.

6.1 Mixed differential equation model

Imagine that colleges and universities consider adopting corresponding education charging models according to different enrolment numbers. Then, we can mix and connect two or more models to adjust the college education charging methods according to the number of people, such as

$$\begin{cases} R'_1(N) = b(\delta - \sigma g_0)(N - n) \\ R'_2(N) = (\delta - \sigma g_0) \frac{M\gamma_M}{N} \\ R'_3(N) = 0, R_2(N_1) = R_1(N_1) \end{cases} \quad (21)$$

6.2 Two-dimensional coupled differential equation model

Imagine that colleges and universities in different (regions) all adopt the same charging model, and there is a certain mutual influence between them, then we can integrate more than two models, such as the Logistic model that mainly considers the second-order nonlinear coupling. Study the synchronisation of the scale of enrolment and charging standards in different (regions) universities.

$$\begin{cases} R'_1 = \delta d_1 R_1 - \sigma g_0 d_1^2 R_2^2 \\ R'_2 = \delta d_1 R_2 - \sigma g_0 d_1^2 R_1^2 \\ R_1(N_0) = R_{10}, R_2(N_0) = R_{20}, \end{cases} \quad (22)$$

6.3 Three-dimensional differential equation model

Due to the imbalance of economic and social development, different colleges and universities adopt different charging models, which has led to the unfair phenomenon of education fees and the scale of enrolment. To promote education fairness and build a harmonious society, the government needs to be. If we introduce the control variable $a(N)$ that changes with the number of students enrolled, and consider the linear feedback control between two different models, then we can get a three-dimensional differential equation model. For example,

$$\begin{cases} R'_1 = \delta d_1 R_1 - \sigma g_0 a \\ R'_2 = \delta d_2 R_2 - \sigma g_0 d_2^2 R_1^2 \\ a' = e_1 R_1 + e_2 R_2 \\ R_1(N_0) = R_{10}, R_2(N_0) = R_{20}, a(N_0) = a_0 \end{cases} \quad (23)$$

In turn, we can in-depth study the dynamics of the bifurcation and chaos of different (regional) colleges and universities' education fees and enrolment scale.

7 Conclusion

This article mainly proposes two basic differential equation models (2) and (4) to describe the issue of university education fees. Through qualitative analysis of these two basic models, several conditions for universities to maintain or stabilise their education fees are obtained; through analysing several special models of models (2) and (4) in three situations, we arrive at some new conclusions and suggestions on the macro-control of university education fees and enrolment scale. This article also proposes three extended differentials equation models, and further research results on these models will be published in a separate article. In addition, we will also consider using the new results of the literature to study the issue of higher education fees.

Conflict of interest.

The authors declare no conflicts of interest.

References

- [1] Parga, C., Yu, W., & Li, X. A low-cost ball and plate system for advanced control education. *International Journal of Electrical Engineering Education*. 2015. 52(4):pp. 370-384.
- [2] Couceiro, M. S., Fernandes, A., Rocha, R. P., & Ferreira, N. M. F. Understanding the communication complexity of the robotic darwinian pso. *Robotica*. 2015. 33(1):pp. 157-180.
- [3] Uyanik, I., & Catalbas, B. A low-cost feedback control systems laboratory setup via arduino-simulink interface. *Computer Applications in Engineering Education*. 2018. 26(3):pp. 718-726.

- [4] Zhang, Y., & Zhai, X. H. Noether symmetries and conserved quantities for fractional birkhoffian systems. *Nonlinear Dynamics*. 2015. 81(1-2):pp. 469-480.
- [5] Wipf, D., & Nagarajan, S. Iterative reweighted and methods for finding sparse solutions. *IEEE Journal of Selected Topics in Signal Processing*. 2016. 4(2):pp. 317-329.
- [6] Mahajan, D., Ramkrishnan, K., Jariwala, R., Yazdanbakhsh, A., Park, J., & Thwaites, B., et al. Axilog: abstractions for approximate hardware design and reuse. *IEEE Micro*. 2015. 35(5):pp. 1-1.
- [7] Masuda, N., & Rocha, L. E. C. A gillespie algorithm for non-markovian stochastic processes: laplace transform approach. *Siam Review*. 2017. 60(1):pp. 95-115.
- [8] Li, W., Lu, W., & Qian, J. A level-set method for imaging salt structures using gravity data. *Geophysics*. 2016. 81(2):pp. 23-36.
- [9] Lee, D., Shibahara, K., Kobayashi, T., Mizuno, T., Takara, H., & Sano, A., et al. A sparsity managed adaptive mimo equalization for few-mode fiber transmission with various differential mode delays. *Journal of Lightwave Technology*. 2016. 34(8):pp. 1754-1761.
- [10] L. Bunimovich, D. Smith, B. Webb, Finding Hidden Structures, Hierarchies, and Cores in Networks via Isospectral Reduction. *Applied Mathematics and Nonlinear Sciences*. 2019. 4(1):pp. 231-254.
- [11] T. Ugür, Fan-Gottesman Compactification and Scattered Space Ceren Sultan Elmali, *Applied Mathematics and Nonlinear Sciences*. 2020. 5(1):pp. 475-478.