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Mathematical simulation analysis of optimal detection of shot-putters' best path

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Abstract

In order to consider many uncertain factors in the process of shot-put, a fuzzy optimisation model of shot-put is proposed. With the help of fuzzy anthropometric data and strength data, the model calculates the fuzzy solution set of the athlete's best throwing mode and throwing distance with a known probability distribution, which reflects the actual process of shot throwing better than the non-fuzzy optimisation model. Then, using MATLAB6 software, the program design of the model solving and the user interface of optimisation software are developed, which realises fast calculation and good user interaction function. Finally, the actual measurement data of university shot-putters are used to verify the feasibility and effectiveness of the fuzzy optimisation model.

Keywords: sports training, shot-put, shot angle, model
AMS 2010 codes: 62J05

1 Introduction

With the successful bid for the 2008 Olympic Games, greatly improving the performance of various sports in my country has become the focus of attention of sports management departments, coaches and athletes. At the end of the 1980s, my country's shot-put athletes had achieved good results. However, the overall situation of the current level of shot-put in my country is in a downward trend, and there is still a certain gap compared with the world's first-class level. In order to narrow the gap, improve the performance of our shot-put athletes faster and win gold and silver medals in the 2008 Olympic Games, it is necessary to accurately establish a mathematical model of shot-put throwing. Using this mathematical model, coaches and athletes can analyse the best angle of shot throwing and obtain the functional relationship between the thrust size, the height of the shot, the speed of the shot and other parameters and the throwing distance so as to achieve the athlete's high performance through scientific training methods and improving them quickly.

In the past few decades, people have launched a lot of research on the mathematical model of shot-put. The establishment and use of these mathematical models have played a very good role in the development of the

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theoretical level of shot-put [1]. In these mathematical models, all numbers are accurate, that is, the mathematical model is based on precise and definite traditional mathematics. However, in the actual process, it is impossible to accurately obtain the value of the athlete's thrust applied to the shot-put and the height of the shot in the model, and the athlete's throwing state will not remain unchanged. All these will cause uncertainty in the model parameters. Obviously, the mathematical model based on certain parameters cannot accurately describe this uncertain shot-throwing process. Therefore, the optimal throwing mode and throwing distance calculated by these mathematical models will have corresponding errors. Based on the above analysis, in order to consider uncertain factors in the process of shot-putting, this paper introduces fuzzy mathematics and proposes a fuzzy optimization model for shot-putting. This model overcomes the shortcomings of the existing shot-putting model and can describe the fuzzy factors in the shot-putting process, thereby obtaining a more reasonable throwing mode and throwing distance [2].

2 Mathematical model of shot-put

As we all know, the shot throwing process can be roughly divided into the sliding phase and the force phase. (1) In the sliding phase, the shot will produce an initial speed v_0 with the human body, that is, the sliding speed. (2) In the exertion stage, the shot-put under the action of thrust F , following the linear motion of the human arm, obtains the speed v , that is, the shot speed θ . In the exertion stage, the relationship between each physical quantity is shown in Figure 1. Suppose that the shot height h , thrust F and shot angle θ are not completely independent, then they satisfy the following functional relationship:

$$\begin{aligned} h(\theta) &= h_1 + l \cdot \sin \theta - r_{\text{shot}} \\ F(\theta) &= F_0 - a \cdot \theta \end{aligned} \quad (1)$$

In the formula, h_1 is the shoulder height of the human body; l is the arm length; r_{shot} is the radius of the shot put; F_0 is the horizontal thrust when the firing angle is 0° and a is the rate at which the thrust decreases as the firing angle increases (it is the rate of decline), which is related to the athlete's muscle strength and throwing technique. Different athletes have different values [3].

Fig. 1 Schematic diagram of the relationship between physical quantities in the exertion phase.

On the basis of the above assumptions, according to the law of conservation of energy, the following equations are derived:

$$W = Fl \cos(\varphi - \vartheta) = Fl \left[1 - \left(\frac{mg}{F} \right)^2 \sin^2(90^\circ + \theta) \right] = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgl \sin \theta \quad (2)$$

where m is the quality of the shot. When the shot-put is released, the shot-put can be regarded as a projectile.

Under the premise of ignoring the air resistance, the throwing distance s of the shot-put is given as follows:

$$s = \frac{v^2 \sin 2\theta}{2g} + \sqrt{\left(\frac{v^2 \sin 2\theta}{2g}\right)^2 + 2h \frac{v^2 \cos^2 \theta}{g}} \quad (3)$$

In the sport of shot-putting, the goal pursued is to obtain the farthest throwing distance, even if s is the largest. Therefore, the mathematical model of the shot-putting process is as follows:

$$\max s = \frac{v^2 \sin 2\theta}{2g} + \sqrt{\left(\frac{v^2 \sin 2\theta}{2g}\right)^2 + 2h \frac{v^2 \cos^2 \theta}{g}} \quad (4)$$

$$s.t. Fl \left[1 - \left(\frac{mg}{F}\right)^2 \sin^2 (90^\circ + \theta) \right] = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgl \sin \theta \quad (5)$$

$$h(\theta) = h_1 + l \cdot \sin \theta - r_{shot}$$

$$F(\theta) = F_0 - a \cdot \theta$$

$$0 \leq \theta \leq 90^\circ$$

$$F_0 = F'_0, \quad a = a', \quad h_1 = h'_1, \quad l = l', \quad v_0 = v'_0$$

Among them, F'_0, a', h'_1, l', v'_0 are the athlete's horizontal thrust F_0 , descent rate a , shoulder height h_0 , arm length l and sliding speed v , respectively. Therefore, under the premise of knowing these anthropometric data and strength data, by solving the nonlinear optimisation problem (5), the best shot angle and the farthest throwing distance of the shot putter can be obtained [4].

3 Fuzzy optimisation model of shot-put

In the mathematical model of shot throwing established above, all parameters are determined, and the relationship between equations and inequalities is clear. However, in the actual shot throwing process, there are various ambiguities, such as the measurement errors of data parameters such as F_0, a, h_0 and l and the fluctuation of the above parameters caused by the change of the athlete's throwing state. Therefore, a mathematical model based on certain and precise mathematical concepts cannot describe these ambiguities in shot-puts, and the best angle of shot-put is not really optimal. For the calculated throw, there will inevitably be a corresponding error between the distance and the actual situation [5].

In order to describe the fuzzy factors in the process of shot-putting and establish a more accurate mathematical model of shot-putting, this paper introduces fuzzy mathematics and proposes a fuzzy optimisation model of shot-putting. Its mathematical expression is as follows:

$$\max s = \frac{v^2 \sin 2\theta}{2g} + \sqrt{\left(\frac{v^2 \sin 2\theta}{2g}\right)^2 + 2h \frac{v^2 \cos^2 \theta}{g}} \quad (6)$$

$$s.t. Fl \left[1 - \left(\frac{mg}{F}\right)^2 \sin^2 (90^\circ + \theta) \right] = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgl \sin \theta \quad (7)$$

$$h(\theta) = h_1 + l \cdot \sin \theta - r_{shot}$$

$$F(\theta) = F_0 - a \cdot \theta$$

$$0 \leq \theta \leq 90^\circ$$

$$F_0 \approx F'_0, \quad a \approx a', \quad h_1 \approx h'_1, \quad l \approx l', \quad v_0 \approx v'_0$$

Since the horizontal thrust, descent rate, shoulder height, arm length and sliding speed are all inaccurate quantities, in model (2), their corresponding equation constraints take the form of fuzzy constraints. The fuzzy optimisation model describes that under the condition that hard constraints (i.e. precise equality and inequality constraints) and soft constraints (i.e. fuzzy equality constraints) are satisfied as much as possible, the throwing distance as far as possible can be obtained. The model better captures the fuzzy characteristics of shot throwing and can get a more reasonable optimal throwing mode and throwing distance [6]. The following uses triangle membership to describe the fuzzy factors in the model. Taking horizontal thrust as an example, its value range may appear between $[F'_0 - d_{F-}, F'_0, F'_0 + d_{F+}]$ and most likely to appear at F'_0 . The uncertainty of the horizontal thrust can be represented by the triangular fuzzy number $F_0 = [F'_0 - d_{F-}, F'_0, F'_0 + d_{F+}]$ as shown in Figure 2. Its membership function (that is, the membership function of the fuzzy constraint $F_0 \approx F'_0$) E is given as follows:

$$\mu_F = \begin{cases} 0, F'_0 + d_{F+} \leq F_0 \\ \frac{F'_0 + d_{F+} - F_0}{d_{F+}}, F'_0 + d_{F+} > F_0 > F'_0 \\ 1, F'_0 \leq F_0 \leq F'_0 + d_{F+} \\ \frac{F_0 - F'_0}{d_{F-}}, F'_0 < F_0 < F'_0 + d_{F+} \\ 0, F'_0 - d_{F-} \leq F_0 \leq F'_0 \end{cases} \quad (8)$$

Among them, d_{F+} , d_{F-} is the maximum range of positive or negative horizontal thrust that may fluctuate.

Fig. 2 μ_F graph.

It can be seen from Figure 2 that when $F_0 = F'_0$, the membership function is 1, that is, the equality constraint is absolutely satisfied; when $F'_0 < F_0 < F'_0 + d_{F+}$ or $F'_0 - d_{F-} < F_0 < F'_0$, the membership function of the constraint is between (0, 1), and the degree of satisfaction of the equality constraint varies with the change changes and away from F . The farther the value, the lower the degree of satisfaction of the constraint; when $F'_0 + d_{F+} < F_0$ or $F_0 \leq F'_0 - d_{F-}$, the membership function of the constraint is 0, indicating that the equality constraint is absolutely not satisfied. Similar to the horizontal thrust fuzzy number, the triangular fuzzy number is still used to characterise the fuzzy characteristics of descent rate, shoulder height, arm length and sliding speed, namely:

$$\begin{aligned} \tilde{a} &= (a' - d_{a-}, a', a' + d_{a+}) \\ \tilde{h}_1 &= (h' - d_{h-}, h'_1, h'_1 + d_{h+}) \\ \tilde{l} &= (l' - d_{l-}, l', l' + d_{l+}) \\ \tilde{v}_0 &= (v'_0 - d_{v-}, v'_0, v'_0 + d_{v+}) \end{aligned} \quad (9)$$

Then, the membership function μ_a , μ_h , μ_l , μ_v of their respective fuzzy constraints can be obtained. In the training of athletes, statistical data of horizontal thrust, descent rate, shoulder height, arm length and sliding speed can be obtained after multiple measurements. According to the frequency of the data, the respective membership functions can be determined [7].

4 Solution of fuzzy optimisation model

4.1 Solution process

All fuzzy constraints form a fuzzy set, and the membership function of the set is given as follows:

$$\mu_D = \mu_F \wedge \mu_a \wedge \mu_h \wedge \mu_l \wedge \mu_v \quad (10)$$

Define the A-cut set of the fuzzy set as follows:

$$\begin{aligned} X_\lambda &= \{x\}x \in R^n, \mu_F(x) \geq \lambda, \\ \mu_a(x) &\geq \lambda, \quad \mu_h(x) \geq \lambda, \\ \mu_l(x) &\geq \lambda, \quad \mu_v(x) \geq \lambda \end{aligned}$$

For $\lambda \in [0, 1]$, the fuzzy optimisation problem (6) is transformed into a deterministic parameter problem (11):

$$\max s = \frac{v^2 \sin 2\theta}{2g} + \sqrt{\left(\frac{v^2 \sin 2\theta}{2g}\right)^2 + 2h \frac{v^2 \cos^2 \theta}{g}} \quad (11)$$

$$\begin{aligned} s.t. Fl \left[1 - \left(\frac{mg}{F}\right)^2 \sin^2 (90^\circ + \theta) \right] &= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgl \sin \theta \\ h(\theta) &= h_1 + l \cdot \sin \theta - r_{shot} \\ F(\theta) &= F_0 - a \cdot \theta \\ 0 \leq \theta &\leq 90^\circ \\ \mu_F \geq \lambda, \quad \mu_a \geq \lambda, \quad \mu_h \geq \lambda, \quad \mu_l \geq \lambda, \quad \mu_v \geq \lambda \\ 0 \leq \lambda &\leq 1 \end{aligned} \quad (12)$$

Given the specific value of A, by solving the nonlinear optimisation problem, the best throwing mode (including the best shooting angle, shooting speed and shooting height) and throwing distance can be calculated. A corresponds to a set of A values, and a set of optimal throwing modes and throwing distances can be obtained, which form the fuzzy optimal solution set. In the fuzzy optimal solution set, the membership degree of each optimal solution is given by the corresponding A value, which represents the possibility of the optimal solution [8].

4.2 Program design

In order to solve the nonlinear optimisation problem (11), MATLAB software is used to realise the program design. The block diagram of program design is shown in Figure 3. In the MATLAB 6 environment, run the script file to obtain the optimal solution set of the fuzzy optimisation model.

4.3 The user interface of shot-put optimisation software

Using the above fuzzy optimisation model, MATLAB program can quickly find the best shot angle, shot speed, shot height and the throwing distance in this throwing mode. However, as a piece of practical shot-put optimisation software, a good user interface needs to be designed to realise the simple interaction between the user and the program. To this end, the following uses the fuzzy optimisation model solver as the core of the software, with the help of the graphical user interface function of MATLAB6, to develop the user interface of the shot-put optimisation software [9].

The user interface consists of a series of graphic objects, such as windows, menus, text, images, etc. The user performs corresponding operations by selecting and activating some graphical objects to complete the interactive

Fig. 3 Block diagram of program design.

process. The main menu of this interface includes four functions: raw data, membership degree, optimisation result and exit. Among them, the original data menu mainly realises the modification, saving and printing of the athlete's body measurement data and strength data; the membership degree menu completes the selection of the membership curve; the optimisation result menu realises the display of the fuzzy optimisation results of the athlete's throwing process [10]. The structure diagram of the menu is shown in Figure 4.

Fig. 4 Schematic diagram of menu structure.

In order to cooperate with the realisation of the menu function, it is necessary to design a series of windows to truly complete the interactive function with the user. Figure 4 shows the window corresponding to the optimisation results menu, showing the athlete's anthropometric data and strength data membership (i.e. changes), as well as the athlete's best throwing mode and the possibility of throwing distance under this fuzzy data distribution, and the range of changes in the flight trajectory of the shot-put in the air (where the thicker the line, the greater the probability of the flight trajectory, and the thinner the line, the less likely the flight trajectory is) [10].

5 Example analysis

Data measured by Maheras of college male shot-putters in 1995 as an example to illustrate the feasibility and effectiveness of the fuzzy optimisation model proposed in this paper are taken. After many measurements, the anthropometric data and strength data of a college male shot-put athlete are shown in Table 1. (Because the sliding speed has little effect on the shot throwing distance, Maheras did not measure the data, and it slipped during the following calculations. The speed is 2.5 ms, as shown in Figure 5). The Pulnix high-speed camera (frequency 120 Hz) was used to shoot 10 groups of athletes in the best throwing state of the two-dimensional film [11], through analysis to obtain the athlete's shooting angle, shooting speed, shooting height and throwing distance measurement data (Table 2).

Table 1 List of human test data and strength data of athletes.

Anthropometric data		Power data	
Shoulder height (m)	Arm length (m)	Horizontal thrust (N)	Decline rate (N·degree ⁻¹)
1.68 ± 0.05	0.87 ± 0.08	462 ± 11	3.2 ± 0.3

Table 2 List of athletes throwing data.

Throwing distance (m)	Shot speed (m·s ⁻¹)	Shot angle (°)	Shot height (m)
15.9 ± 0.8	11.9 ± 0.3	34.1 ± 1.5	2.11 ± 0.05

Fig. 5 Optimisation result display window.

On the premise that the anthropometric data and strength data of the athletes are known, the fuzzy optimisation model proposed in this paper is used to obtain the fuzzy solution set of the best throwing state. From the measured values of anthropometric data and strength data (Table 1), their membership degrees can be obtained. In the triangle membership function, the values of the three key data are shown in Table 3. The fuzzy optimi-

sation model is solved by the nonlinear programming method. The fuzzy solution set of the best shot angle, shot speed, shot height and throw distance is shown in Table 4. Corresponding to different throwing modes, the trajectory of the shot-put in the air is shown in Figure 4.

Table 3 List of membership functions.

	d–	Most likely value	d+
Shoulder height (m)	0.05	1.68	0.05
Arm length (m)	0.08	0.87	0.08
Horizontal thrust (N)	11	462	11
Decline rate ($\text{N} \cdot \text{degree}^{-1}$)	0.3	3.2	0.3

It can be seen from Table 4 that the athlete's throwing mode and throwing distance are also ambiguous. They are not given by a certain value but displayed by a set of values with known probability distribution. For example, the best shot angle varies between 31.94 and 33.36, where 31.94° is the most likely best shot angle. Corresponding to the throwing mode, the throwing distance will be between 13.39 m and 17.27 m, and the throwing distance of 15.21 m is most likely to occur [12]. This fuzzy representation of the throwing mode and throwing distance is basically consistent with the actual measured data of the throwing mode and throwing distance preferred by the athletes (Table 2), thus verifying the feasibility and effectiveness of the fuzzy optimisation model proposed in this paper.

Table 4 List of athletes' best throwing mode and throwing distance.

Possibility	Throwing distance (m)	Shot speed ($\text{m} \cdot \text{s}^{-1}$)	Shot angle ($^\circ$)	Shot height (m)
0	17.27	12.38	33.36	2.31
0.25	16.75	12.15	33.01	2.28
0.5	16.24	11.99	33.66	2.26
0.75	15.75	11.80	32.30	2.23
1	15.21	11.61	31.94	2.20
0.75	14.86	11.41	32.04	2.18
0.5	14.23	11.33	32.16	2.15
0.25	14.11	11.19	32.27	2.13
0	13.93	11.01	32.38	2.11

6 Conclusion

In order to consider various fuzzy factors in the actual shot-putting process, this paper proposes a fuzzy optimisation model for shot-putting. It calculates the athlete's best throwing mode and the fuzzy solution set of the throwing distance based on the measured anthropometric data and the fuzzy characteristics of the strength data. It describes the athlete's shot throwing process more accurately than the non-fuzzy optimisation model. The training provided a more scientific theoretical basis. The idea of establishing a fuzzy optimisation model in this paper can also be applied to other sports and establish a fuzzy optimisation model of other sports considering fuzzy factors.

References

- [1] Zhang, T., Liu, C., Guo, K., Liu, H., & Wang, Z. Analysis of flow field in optimal cyclone separators with hexagonal structure using mathematical models and computational fluid dynamics simulation. *Industrial & Engineering Chemistry Research*, 2016.55(1):pp. 351-365.
- [2] Shaofei Wu. Research on the application of spatial partial differential equation in user oriented information mining, *Alexandria Engineering Journal*, 2020. 59(4):pp. 2193–2199.
- [3] Zhang, H., & Yang, F. Simulation analysis on optimal capital structure model of Chinese real estate enterprises. *Systems Engineering-Theory & Practice*, 2015. 35(4):pp. 865-871.
- [4] Jiang, H., Ebner, A. D., & Ritter, J. A. Importance of incorporating a vacuum pump performance curve in dynamic adsorption process simulation. *Industrial and Engineering Chemistry Research*, 2020. 59(2):pp. 856-873.
- [5] Seddik, R., Ben Sghaier, R., Atig, A., & Fathallah, R. Fatigue reliability prediction of metallic shot peened-parts based on whler curve. *Journal of Constructional Steel Research*, 2017. 130(3):pp. 222-233.
- [6] Shaofei Wu. AN USER INTENTION MINING MODEL BASED ON FRACTAL TIME SERIES PATTERN, FRACTALS-COMPLEX GEOMETRY PATTERNS AND SCALING IN NATURE AND SOCIETY, 2020. 28(8):pp. Article Number 2040017.
- [7] Li, S., Zhang, J. M., Xu, W. L., Chen, J. G., Peng, Y., & Li, J. N., et al. Simulation and experiments of aerated flow in curve-connective tunnel with high head and large discharge. *INTERNATIONAL JOURNAL OF CIVIL ENGINEERING*, 2016. 14(1):pp. 23-33.
- [8] Frimann, Søren, Jørgensen, Jes K., & Haugbølle, Troels. Large-scale numerical simulations of star formation put to the test: comparing synthetic images and actual observations for statistical samples of protostars. *Astronomy & Astrophysics*, 2016. 587(39):pp. 32717-32727.
- [9] Atknin, I. I., Marchenkov, N. V., Chukhovskii, F. N., Blagov, A. E., & Kovalchuk, M. V. Double-crystal rocking curve simulation using 2d spectral angular diagrams of x-ray radiation. *Crystallography Reports*, 2018. 63(4):pp. 521-530.
- [10] Fernandez, J. L., Nogueiras, M. R., Pou, M., & Vazquez, C. Multicurve libor market models and drift-free simulation. *International journal of computer mathematics*, 2017. 94(9-12):pp. 2194-2207.
- [11] T. M. M. Sow., New iterative schemes for solving variational inequality and fixed points problems involving demi-contractive and quasi-nonexpansive mappings in Banach spaces. *Applied Mathematics and Nonlinear Sciences*. 2019. 4(2):pp. 559-574.
- [12] Zhihao Cui., Chaobing Yan., Deep Integration of Health Information Service System and Data Mining Analysis Technology. *Applied Mathematics and Nonlinear Sciences*. 2020. 5(2):pp. 443-452.

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