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Information technology of preschool education reform of fine arts based on fractional differential equation

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Abstract

Objective: To study the information reform of preschool education of fine arts under fractional differential equation. Methods: In this paper, the fractional Navier-Stokes equations are compared with the traditional models through some numerical experiments, which provides a certain basis for future physical application. In order to improve students' comprehensive ability and quality, we should strengthen education and teaching in all aspects, and art education is a more important aspect. The results show that the numerical solution of the integral equation with different values is close to a point at $x = 1$, that is, $u(1) = 0.5$.

Conclusion: In the current preschool education professional art education teaching process, the use of information technology means has become a necessary teaching requirement and need to improve the teaching effect, and the quality is of great significance and value, thus ensuring better able to carry out education of fine arts teaching, which enables students for better development, guarantees for a better future that is better for preschool education.

Keywords: fractional order, differential equation, art preschool education**AMS 2010 codes:** 97B10

1 Introduction

In the process of fine arts education of preschool education speciality, the backward teaching method is also a common problem and a key influencing factor. To be specific, in actual education and teaching, many teachers still choose traditional teaching methods, namely instilling and lecturing teaching methods. This teaching method is boring, and students' interest and enthusiasm in learning are low. They often learn passively and the

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learning effect is poor, which affect the teaching effect. Therefore, it is necessary to study the informatisation reform of fine arts at preschool education [1]. Based on fractional calculus, fractional partial differential equation (PDE) is an extension of the traditional model and is being applied in more and more different fields. From the definition of the fractional derivative of time, the fractional derivative of a function at a certain time depends on the value of the function at all times before that time, and so the fractional PDE has more advantages than the integer order equation when studying some materials with memory process, genetic properties and heterogeneous materials [2].

In fact, such equations are increasingly being used for modelling in a variety of fields, such as describing the electrical properties of materials, boundary layer effects in transport pipes, control theory in dynamical systems, electrolyte polarisation in viscoelastic materials, fractal dynamics and anomalous ion diffusion processes in nerve cells [3]. There are many other fractional models, such as NMR diffusion in disordered materials, the dynamics of beads in polymer networks and the propagation of mechanical diffusion waves in viscoelastic media [4].

In the current preschool education professional art education teaching process, the application of informatisation technology has become a very important teaching task and teaching requirement and plays an important role in the promotion of teaching quality; and the information technology teaching means in the process of application should pay attention to follow certain principles not related to basic teaching principles and teaching methods. At the same time, it can also realise that the application of information teaching means cannot completely solve the problems existing in art education [5].

Specifically speaking, the application of information means to art education should follow the following principles. First, in the process of fine arts education and teaching, for the application of information means, we should ensure that students and teachers can master the relevant information means. Use of related software effectively masters, for example, the network search, word processing, and image editing and other related basic skills, in terms of a certain extent, students and teachers for proficiency in computer operation of the teaching of fine arts education informationisation means using effect can play a decisive role, therefore for this principle on the one hand, should strengthen the attention.

Second, in the fine arts education in the teaching process, the application of informatisation technology can make students to have more freedom in learning, but the effective use of this kind of teaching method is based on students' interest in fine arts learning. The lack of premise may result in laziness in the students using information technology means and network cheating happens [6]. Not only can the information means play the ideal effect but will also lead to a negative effect and so we need to pay attention to its use.

Again, in the process of fine arts education in preschool education, it is necessary not only to enable students to master the basic painting skills and encourage students to actively carry out painting skills training but also to enable students to have a certain art education and teaching ability. So, the fine arts education in the teaching process, in the process of informatisation technology application, not only should pay attention to the cultivation of the students' art skills and the art level to enhance the students ability of fine arts and literacy but also should be through the information means to better cultivate students' art education teaching ability to ensure that the future can be better to carry on the fine arts teaching and better meet the demand of preschool education professional [7].

The application of fractional-order calculus equation model in physics, biology and other fields makes the study of fractional-order calculus equation widely concerned. In terms of theoretical research, Schneider, Li X et al. discussed the analytical solutions of some specific fractional differential equations. Podlubny M and Diethelm M summarise the existence and uniqueness of solutions for initial value problems of fractional-order ordinary differential equations. In addition, the research on weak solution theory has also been developed rapidly [8].

Compared with theoretical results, numerical methods are more abundant, for example, for the time fractional differential equation of linear multi-step method solution, extrapolation method, predictive correction method, Adams method, difference method and spectral method. The offset difference method, spline method

for spatial fractional differential equation, the difference scheme of fractional derivative defined by Riesz, the spectral collocation method based on weak form, the finite element method/spectral method for spatial and time fractional Fokker-Planck equation [124], K alternating direction implicit method [99], the fast difference algorithm, etc. With the wide application of fractional differential equations, theoretical research and numerical methods are important research topics.

Now the more common fractional models are derived from R-L or Caputo fractional calculus. In the part of theoretical research, integral boundary conditions are usually proposed for R-L differential equations and Dirichlet boundary conditions are usually proposed for Caputo differential equations [9]. Under homogeneous boundary conditions, R-L fractional derivative is equivalent to Caputo fractional derivative, so Dirichlet boundary conditions are sometimes proposed for R-L differential equations. However, for R-L differential equations, not all Dirichlet boundary condition problems are well-posed. In this paper, we discuss the existence of strong solutions and the derivation of weak forms for R-L or Caputo type fractional differential problems with order $0 < \alpha < 1$ and $1 < \alpha < 2$ under two different boundary conditions. We also give some proof of existence and uniqueness of strong solutions for mixed R-L and Caputo type two-point fractional boundary value problems. The aim is to supplement the original theory and provide theoretical basis for the discussion in the following sections [10].

The innovation of this paper is that a fractional Stokes equation derived from the symmetric fractional derivative is considered, the well-posed problem is discussed and the error estimate of the spectral approach is given. We also consider a class of fractional Navier-Stokes equations, and through some numerical experiments and comparison with the traditional model, we provide a certain basis for the future physical application.

2 Research methods

In this chapter, the algorithm combining the Haar wavelet method and the Galerkin method is used to solve the fractional differential equation. Through theoretical analysis and algorithm design, the error analysis of the algorithm is given and the applicability and robustness of the algorithm are verified by numerical examples, which provides a theoretical basis for the popularisation of the algorithm.

2.1 Theoretical basis

There are many types of research available on solving equations using wavelet algorithm. The calculus equation is designed using the wavelet analysis method.

The solving algorithm has been developed for a long time. It was used to solve the calculus equation in the early 1990s. At present, this method has been developed and is mature. However, the simple wavelet method has some problems; so many related scholars have applied the multi-wavelet theory to the solution of the equation.

2.1.1 Basic principle of Haar wavelet analysis

Haar wavelet is a family of functions. The function family is generated from a parent wavelet function $H_1(t)$ by stretching and shifting. The definition of Haar scale function is shown in Eq. (1). The definition of the parent wavelet function is shown in Eq. (2), and the stretching and shifting of the parent wavelet function generate the wavelet function family shown in Eq. (3). It is easy to verify that Haar wavelet $\{h_n(t)\}$ constitutes a set of orthonormal bases on the space $L_2([0,1])$. The verification formula is shown in Eq. (3). For any function $u(t) \in L_2([0,1])$, it can be expanded by Haar wavelet in the form shown in Eq. (4).

The series shown in Eq. (4) contains an infinite number of terms. If $u(t)$ is truncated into piecewise functions, then Eq. (4) can be truncated into a finite term as shown in Eq. (4), which is the process of transformation from infinity to finitude.

The definition of square error is shown in Eq. (4):

$$h_0 = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{others} \end{cases} \quad (1)$$

$$h_1 = \begin{cases} 1 & 0 \leq t < 1.5 \\ -1 & 0.5 \leq t < 1 \\ 0 & \text{others} \end{cases} \quad (2)$$

$$h_n(t) = h_t(2^j t - k), \quad n = 2^j + k, \quad j \geq 0, \quad 0 \leq k \leq 2^j, \quad n, j, k \in \mathbb{Z} \quad (3)$$

$$\int_0^1 h_i(t) h_i(t) dt = \begin{cases} 2^{-i} & i = 1 \\ 0 & i \neq 1 \end{cases} \quad (4)$$

2.2 Solving fractional differential equations with wavelet Galerkin method

2.2.1 Algorithm design

If I call an integral operator K , let K be a compact operator from Banach null V to V_n . The equation can be written in the operator format of Eq. (5).

$$Ku = f \quad (5)$$

The algorithm design steps are as follows:

Step 1. Select a sequence V_n of finite-dimensional subspace, n is greater than or equal to 1, and the dimension of V_n is n .

Step 2. Select the Haar wavelet basis in V_n and assume that $U_n \in V_n$ is the numerical solution of the objective equation, then U_n is used to approximate U .

According to Eq. (6), $U_n(x)$ can be expanded into the form shown in Eq. (6), and in Eq. (6) is unknown.

$$u(x) \approx u_n(x) = \sum_{i=1}^N c_i h_i(x) \quad (6)$$

Step 3. Use the traditional Galerkin method. Substitute Eq. (5) into the objective equation to obtain Eq. (6). R_n in Eq. (6) represents the remainder term. When it is zero, we can get $U_n(x) = U(x)$. In order to obtain a more accurate numerical solution conveniently, we need to make $R_n(x)$ as close to zero as possible.

$$r_n(x) = \sum_{i=1}^N c_i \frac{1}{\Gamma(\alpha)} \int_0^1 k(x,t) (x-t)^{\alpha-1} h_t(t) dt - f(x) = Ku_n(x) - f(x) \quad (7)$$

Step 4. Set $V = L^2[0,1]$ and $N = 2J$ as the wavelet resolution. The application of Galerkin method needs $I = 1, 2, \dots, N$. Combining with Eq. (6), the linear equations shown in Eq. (7) can be derived.

2.2.2 Proof of the existence of solutions

In the formula, matrix A is a sparse matrix, and each element of the matrix is composed of an inner product. We know that matrix A is a reciprocal matrix. In addition, matrix A is bounded satisfying the characteristic that the condition number is equal to the universal condition number. To sum up, a system of linear equations always has a solution. Thus, it is concluded that the solution of the equation exists, and the following propositions need to be proved are as follows:

Proposition. Under the condition that the wavelet basis applied in Eq. (7) has the property of multi-resolution decomposition, there is a numerical solution of $U_n(x) \approx U(x)$ based on the wavelet-Galerkin method at N .

Proof. Let's define PNU as a solution to the above, where P_n is the projection operator mapping V to V_n , and this minimisation problem has a solution, because V_n is a finite-dimensional inner product space, and therefore, the solution is unique. The wavelet basis is orthogonal (if not, it can be orthogonalised by the Gram-Schmit process).

$$\|u - P_n u\| = \min_{s \in V_n} \|u - s\| \quad (8)$$

We can get $\langle PNU, PNV \rangle \geq 0$, $u, v \in V$. Therefore, PNU is the unique solution. Set BBB 0 0, according to the multi-resolution analysis properties. In addition, it is known that $U_n V_n$ is dense on the space $L^2 R$.

3 Study of numerical analysis

Example. $\frac{1}{\Gamma(\alpha)} \int_0^1 \frac{e^{t \sin x}}{(x-1)^{1-\alpha}} u(t) dt = -\frac{e^{\sin x(0.54 \sin x + 0.84)} - \sin x}{\cos x^2 - 2} \sin x, x \in [0, 1]$

In this chapter, a formula is used to estimate the numerical solution error of the first Fredholm integral equation of fractional order. The estimated value and the difference between, respectively, represent the exact solution and the approximate solution of the integral equation $u(s_i), \tilde{u}(s_i)$.

Table 1 shows $\alpha = 1$, the error estimation of the exact solution and the numerical solution under different values of m .

Table 1 The error results of numerical solution of m under two different values when $\alpha, m = 1$

m = 16 x values	m = 32 x' values	m = 16 numerical solution $\bar{u}(s'_i)$	m = 32 numerical solution $\bar{u}(s'_i)$	m = 16 absolute error	m = 32 absolute error
0.0312	0.0157	0.8512	0.8458	1.4835E-1	1.2155E-1
0.1563	0.1406	0.8805	0.8782	1.0733E-1	8.7086E-1
0.2813	0.2656	0.8836	0.8847	7.7067E-1	6.3776E-1
0.4063	0.3906	0.8609	0.8651	5.7660E-2	4.9992E-2
0.5313	0.5153	0.8156	0.8224	4.6530E-2	4.1802E-2
0.6562	0.6406	0.7532	0.7617	3.9118E-2	3.3960E-2
0.7814	0.7656	0.6800	0.6895	2.9988E-2	2.1063E-2
0.9062	0.8906	0.6027	0.6124	1.3915E-2	1.4721E-2
Det(A)		4.0422E6	3.2856E13		
Cond(A)		11.0262	13.6051		
$\ A^{-1}\ _\infty$		0.9990	0.9998		
$\ e_m\ $		7.1786E-2	6.2891E-2		

When is 1 in Equation, the exact analytic solution of the equation can be obtained, which is $u(x) = \cos x$. As can be seen from Table 1, the error of the numerical solution of the equation will decrease with the increase in the scale and resolution, and the accuracy of the numerical solution of the corresponding equation will also increase.

Figure 1 shows when $\alpha = 1$, the comparison between the numerical solution and the exact solution of the integral equation under two different values of m .

Figures 2 and 3, respectively, show the numerical solutions of the integral equation with different values, i.e. when $m = 16$ and $m = 32$. According to Figures 2 and 3, the numerical solution of the integral equation with different values is close to a point at $x = 1$, that is, $u(1) = 0.5$.

Fig. 1 Comparison between numerical solutions and exact solutions of the integral equation for m at two different values when $m = 1$.

Fig. 2 Numerical solutions of the integral equation at different values when $m = 16$.

Fig. 3 Numerical solutions of the integral equation at different values when $m = 32$.

4 Conclusion

In the process of current preschool education professional art education, to make education better guarantee the teaching quality and effect, can be applied to reasonable informationisation means, so as to realize the informationisation teaching, on the basis of this to better cultivate students' ability of fine arts and art accomplishment, to promote the better development, for the better in the future for preschool education to lay the good foundation. The fractional order differential equation is solved using the Haar wavelet and Galerkin method. The equation is discretised into linear equations. The existence of the solution is discussed and the error estimate of the solution is studied. The results of numerical examples show that the discrete coefficient matrix is sparse, which can effectively reduce the calculation amount in the process of solving equations. The wavelet-Galerkin method has strong effectiveness, convenience and stability in the process of numerical solving fractional-order calculus equations.

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