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Energy-saving technology of BIM green buildings using fractional differential equation

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Abstract

In order to solve the problem of the traditional gray prediction model (GM) during determination of the accuracy of buildings' energy savings and its poor fitting of data, the idea of a fractional model based on the traditional first-order one-variable GM(1,1) model is applied. We use the GM-backpropagation (GM-BP) neural network to solve the optimal fractional order and establish a fractional GM(1,1) model based on the GM-BP neural network. Example calculation shows that the fractional GM(1,1) model can improve the prediction accuracy of buildings' energy savings, and selecting the optimal order can further improve the prediction accuracy and decrease the error level when using the GM-BP neural network. This work shows that the fractional GM(1,1) model based on the GM-BP neural network has an important guiding role in the energy savings of buildings.

Keywords: buildings' energy savings prediction, GM(1,1) model, fractional GM(1,1) model**AMS 2010 codes:** 53Z99

1 Introduction

At present, most of my country's existing buildings and most of the new buildings are still high-energy-consuming buildings, and the building energy environment is not optimistic. As green buildings have gradually increased, more and more new buildings incorporate energy-saving design concepts, giving rise to passive low-energy buildings, zero-energy buildings, etc. In general, in addition to paying attention to the buildings and

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building styles, the main goal of green buildings is to reduce buildings' energy consumption; in other words, energy saving has become the primary starting point of architectural design [1].

From an economic point of view, energy conservation in buildings brings certain economic benefits, social benefits, and environmental benefits; hence, the process of designing buildings' energy-saving feature is essential for the process of pursuing energy-saving benefits. With the rapid development of Building Information Modeling (BIM) technology for buildings, BIM technology-based architectural design, simulation analysis of buildings' energy consumption, comfort simulation analysis, and research on energy-saving design applications have become research hotspots in the field of building energy conservation [2].

In the early days of the development of green buildings in my country, many green buildings have added new technologies to reduce buildings' energy consumption, using new materials. Many green energy-saving buildings use very complicated designs, and the cost is very high. My country is still in the process of building a well-off society; very high construction costs and housing prices are not the development path that green buildings should follow. Therefore, green architectural design should consider the full life cycle capital investment in energy-saving technology and give full play to energy saving and energy efficiency of energy-saving technology. The highest and lowest levels of residential buildings' energy consumption depend on multiple factors, such as time-dependent degradation, complexity, randomness, and geographical characteristics [3].

Determining the factors influencing regional residential buildings' energy consumption, systematically collecting the basic data on residential buildings' energy consumption, and establishing the corresponding energy consumption factor forecasting model form the basis and premise for arriving at the development strategy and planning in the industry, which are important for promoting buildings' energy savings.

1.1 Realistic meaning

Based on the first-order one-variable gray model [GM(1,1)] model, this paper proposes the fractional main model idea, and the optimal order of the model is obtained by the GM-backpropagation (GM-BP) neural network. Finally, the model is applied for the buildings' energy-saving prediction. By adopting some improvement measures, the energy-saving potential can be estimated by using some of the model parameters to predict the energy consumption results, then comparing with the base energy value given by the country's authorities.

1.2 Expected energy performance model

At present, the research on improvement of buildings' energy-saving prediction model and calculation methods mainly include the following.

BIM is a data-based tool for engineering design and construction management. The relevant data based on the construction project is integrated through the parameters of the model. During building life-cycle management involving project planning, operation, maintenance, sharing, and delivery, engineers and technicians should have a correct understanding of information on various buildings and efficient response design teams. Building construction units, including architectural pairs, are involved in architectural collaboration work, thereby increasing production efficiency, optimizing costs, and shortening duration, thus play important roles.

In accordance with a priority design of current office buildings, the office buildings' energy-saving design has been studied by Li, with an in-depth study of the case in architectural design technology, with a comparison of various cases. The application of BIM technology guides the energy-saving design of office buildings and related practical methods. Finally, with the author's related design practice, an example of the design of the office building using BIM technology is provided, demonstrating how to achieve the goal of buildings' energy efficiency in architectural design, as well as reduce energy consumption during construction and operation. Green building energy contributes to the design goals of social sustainable development [4].

Yong et al. reviewed the most advanced technology that can be semiautomated or automatically created, analyzing the geometry and thermal models for buildings' energy modeling and transformation assessment purposes. They outline the main algorithm used by these methods to indicate time and space – hot clock, convert these points cloud into the Semantic BIM in GBXML format, so that it is also a stark contrast to them. The

paper also proposes the potential preparations and methods for measuring the actual thermal resistance of the building assembly and maps them to GBXML-based representations. It also introduces the latest works of the IT Drive building automation system (BAS) for energy-saving purposes. Finally, the technical gap is determined and actions needed in future research are discussed [5].

In this paper, the GM changes the first-order differential in the model to the fractional differential and proposes a fractional-based GM. Compared to the traditional GM, the recognition of fractional order is increased, thus increasing the difficulty of parameter recognition. Therefore, the author establishes a fractional gray model based on the model of the fractional system and calculates the error of the fractional GM.

2 Establishment of prediction model

2.1 Fractional system model

There are currently three definitions of fractional order, using Caputo calculus, and the α differential of the function $x(t)$ can be expressed as follows:

$$D^\alpha x(t) = \frac{d^\alpha x(t)}{dt^\alpha} = \frac{1}{\Gamma(r-\alpha)} \int_0^t \frac{x^{(r)}(\tau)}{(t-\tau)^{(\alpha+1-r)}} d\tau \quad r-1 < \alpha < r. \quad (1)$$

In the expression, r – positive integer; α – score; and $\Gamma()$ – gamma function. The function $\Gamma()$ is intended for the following calculation:

$$\Gamma(\beta) = \int_0^\infty \tau^{\beta-1} e^{-\tau} d\tau. \quad (2)$$

Its Laplace transformation can be represented as follows:

$$L[D^\alpha x(t)] = s^\alpha Y(s) - \sum_{i=0}^{r-1} s^{\alpha-1-i} x^{(i)}(0). \quad (3)$$

In this paper, zero initial conditions are considered, when the Laplace transforms to the following:

$$L[D^\alpha x(t)] = s^\alpha Y(s). \quad (4)$$

For the calculation of s^α , usually using the integer-order integral, we use the phase front filter to simulate the output results of s^α . Assuming that the fit frequency band is (ω_b, ω_h) , the transfer function of the following filter can be constructed as follows:

$$s^\alpha \approx \frac{G^\alpha}{s} \prod_{i=1}^N \frac{1+s/\omega_i^e}{1+s/\omega_i}. \quad (5)$$

In this expression, $2N+1$ is the selected filter order.

$$G^\alpha = \omega_b^{1-\alpha}, \quad \omega_i = \delta \omega_i^e, \quad \omega_{i+1}^e = \eta \omega_i, \quad \alpha = 1 - \frac{\log \delta}{\log \delta \eta}.$$

Equation (5) may also be expressed in the form of an equation of state:

$$M_{IzI} = A_{IzI} + B_I u(t) \quad w = C_{IzI}(t) \quad (6)$$

In this expression, $z_I(t) - N+1$ is the dimensional state vector.

$$M_I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ -\delta & 1 & & \vdots \\ \vdots & \vdots & \vdots & 0 \\ 0 & \dots & -\delta & 1 \end{bmatrix}, \quad B_I = \begin{bmatrix} G^\alpha \\ 0 \\ \vdots \\ 0 \end{bmatrix};$$

$$A_I = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \omega_1 & -\omega_1 & & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & \omega_N & \omega_{-N} \end{bmatrix}, \quad z_I(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_{N+1}(t) \end{bmatrix};$$

$$C_I = [0 \dots 01].$$

Equation (6) can be converted to the following:

$$\begin{aligned} \dot{z}_I(t) &= A z_I(t) + B u(t) \\ w &= C z_I(t). \end{aligned} \quad (7)$$

Here, $A = M_I^{-1} A_I$, $B = M_I^{-1} B_I$, $C = C_I$.

2.2 Gray model with fractional order

Set the original time series as $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$. Its primary cumulative generation sequence is $x = [x(1)x(2)\dots x(n)]$, where $x(k) = \sum_{i=1}^k x^{(0)}(i)$.

The gray model can be represented as

$$\frac{dx(t)}{dt} + b_1 y(t) = b_2. \quad (8)$$

The pending constants b_1 and b_2 in Eq. (8) are determined as follows:

$$[b_1 b_2]^T = [E_1^T E_1]^{-1} E_1^T E_2. \quad (9)$$

In Eq. (9), $E_1 = \begin{bmatrix} -0.5(x(1) + x(2)) & 1 \\ -0.5(x(2) + x(3)) & 1 \\ \vdots & \vdots \\ -0.5(x(n-1) + x(n)) & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$.

It can be seen from Eq. (8) that the gray model is an input-free ordinary differential equation, which is changed here in two aspects: one is to increase the input term u ; and the second is the variable ordinary differential equation for the fractional differential equation, namely:

$$\frac{d^\alpha x(t)}{dt^\alpha} + b_1 x(t) = b_2 u(t). \quad (10)$$

To maintain the characteristics of the original gray model, here, the input item u may use the unit step function, namely:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases} \quad (11)$$

Using Laplace transform on both sides of Eq. (10), the transfer function of the system is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_2}{s^\alpha + b_1}. \quad (12)$$

If the frequency domain of $z(t)$ is defined as

$$Z(s) = \frac{1}{s^\alpha + b_1} U(s), \quad (13)$$

the output variable $x(t)$ can be expressed as follows:

$$\begin{aligned} D^\alpha z(t) &= -b_1 z(t) + u(t) \\ x(t) &= b_2 z(t). \end{aligned} \quad (14)$$

The above formula can be written equivalent to the following:

$$\begin{aligned} \dot{z}_I(t) &= G^\alpha [-b_1 z_{N+1}(t) + u(t)] \\ x(t) &= b_2 z_{N+1}(t) \end{aligned} \quad (15)$$

That is to say,

$$\begin{aligned} \dot{z}_I(t) &= \tilde{A} z_I(t) + \tilde{B} u(t) \\ x &= \tilde{C} z_I(t) \\ \tilde{A} &= A - b_1 B C \\ \tilde{B} &= B \\ \tilde{C} &= b_2 C \end{aligned} \quad (16)$$

2.3 The GM(1,1) model

The Gray system theory is the study of “little data, uncertainty” theory. The GM(1,1) model, established based on this theory, is currently the most widely used gray model, which helps to solve the current situation of energy consumption data of ‘small samples and poor information’ [6, 7]. The main modeling steps are as follows:

1. Set the original sequence as $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ and build the original sequence with Eq. (17):

$$x^{(1)}(k) = \sum_{j=1}^k x^{(0)}(j), (j, k = 1, 2, \dots, n). \quad (17)$$

2. Fit the differential equation in Eq. (18):

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u. \quad (18)$$

The values of the parameters a and u are obtained from the formula $[a, u]^T = (B^T B)^{-1} B^T Y$, where the matrixes B and Y are known.

3. To solve the above differential equation, the time response function, namely, Eq. (19), is considered:

$$\hat{x}^{(1)}(t+1) = \left[x^{(0)}(1) - \frac{u}{a} \right] e^{-at} + \frac{u}{a}. \quad (19)$$

4. The above function is reduced and the prediction equation, namely, Eq. (20) is obtained:

$$\hat{x}^{(0)}(t+1) = -a \left[x^{(0)}(1) - \frac{u}{a} \right] e^{-at}. \quad (20)$$

2.4 The GM-BP neural network model

The energy consumption of buildings is seriously affected over time and has the characteristics of cyclical changes. It is difficult to achieve a good prediction effect using the GM(1,1) model or BP neural network model. This paper combines both to obtain the GM-BP neural network prediction model, which uses the GM(1,1) model (weakened by the ‘small sample’ and ‘poverty information’ data) but also fully exerts the maturity of the BP neural network algorithm [8,9].

In addition, the combined model does not require many factors that affect energy consumption and simply provides accurate and reliable energy consumption history data to conduct public buildings’ energy consumption prediction research. At the same time, in addition to the support of the MATLAB neural network toolbox, this paper provides a more convenient and fast process of establishing a GM-BP neural network prediction model. The main steps in the modeling of the GM-BP combination prediction model are as follows:

1. Gray: once the original energy consumption data are accumulated, the accumulation sequence weakens the randomness of the raw data and highlights the overall development trend.
2. Data preprocessing: normalize the input and output data within the $[-1,1]$ interval, and the normalization formula is presented in Eq. (21):

$$y = 2 \frac{x - x_{\min}}{x_{\max} - x_{\min}} - 1. \quad (21)$$

In the expression, x represents the raw data; $x_{\max}$ and $x_{\min}$ are the maximum and minimum levels, respectively; and y stands for the preprocessed data.

3. Initialization: initialize the network weights w and threshold a .
4. Network training: the method of training the process uses the method of vacancies. The specific process is as follows: 1: the raw data are divided into $N \ m + 1$ lengths, and they have a coincidence data segment (by the data value of the first M time). For training the $M + 1$ time value, each data segment is a training sample; in the second step, input the value of the network input bit, and the value of the network output bit at $m + 1$ time, successively advance the construction of the repeat data segment Sample matrix (sample matrix contains N rows and $m + 1$ columns); in the third step, calculate the neural network output; calculate the neural network reverse propagation; fifth step tests whether the exit conditions are satisfied; if the conditions are satisfied, output the weight; otherwise, repeat Step 3.
5. Save the training model and enter a new sample for the test to output the forecast results.
6. Antinormalization: according to Eq. (21), the anti-in-treatment processing can be conducted to yield Eq. (22):

$$x = x_{\min} + \frac{(y + 1)(x_{\max} - x_{\min})}{2} \quad (22)$$

7. Whitening: Transform the obtained data to obtain the corresponding energy consumption predictive value.
8. Model verification: compare the forecasted and actual values, draw a comparison map of the forecasted results, and test the effectiveness of the combined prediction model.

3 Application of prediction model in buildings’ energy saving prediction

3.1 Experimental data

This paper takes a community in Taiyuan, Shanxi Province, as the experimental object, and the number of residences in 2018 was about 297,300. The residential stock is divided into six residential types: detached

houses; semidetached houses that are single families; terraced houses that share partition walls with neighbors; villas; and second floor apartments, often with a separate entrance. Also, there are two different configuration apartments, hospitality apartments and innhouse apartments.

After repeated testing, it was understood that when $M = 6$, $n = 6$ (the sample matrix is of six rows and seven columns), i.e., 6 months ago, the GM-BP combination model predicted the effect of 6 months after training.

The GM (1, 1) model and the GM-BP combination model (established by the MATLAB tool) predict the monthly power consumption (energy consumption) of the building in the second half of 2018, and the forecasted results are shown in Figure 1.

GM, gray model; BP, backpropagation.

Fig. 1 Comparison of actual and forecasted values of each model.

In order to evaluate the effectiveness of various models, this paper evaluates the prediction accuracy and stability of three models using three performance indicators of maximum relative error absolute value $E_{\max}$ (see Eq. (23)), mean relative error E_{ave} (Eq. (24)), and root mean square error [RMSE] (RMSE (see Eq. (25))).

$$E_{\max} = \max \left| \frac{\tilde{Y}(i) - Y(i)}{Y(i)} \right|; \quad (23)$$

$$E_{ave} = \frac{1}{n} \sum_{i=1}^n \frac{\tilde{Y}(i) - Y(i)}{Y(i)}; \quad (24)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n [\tilde{Y}(i) - Y(i)]^2}. \quad (25)$$

In the equations, n is the total number of samples; $\tilde{Y}$ is the predicted value for the i th sequence; and $Y(i)$ is the actual value of the i th sequence.

Table 1 presents the calculation results of each model evaluation index. According to the table analysis, the GM-BP combination model is optimal in terms of error control and is the ideal choice [10, 11]. Although the GM(1,1) model is simple and easy, the prediction accuracy is greatly reduced and the strain capacity is poor; the BP model involves short training time, the prediction accuracy is lacking, and the model stability is insufficient. The GM-BP combination model has excellent results for various types of building predictions.

Table 1 Calculation results of the evaluation indexes of each model.

Error	GM(1,1) model	GM-BP neural network model
Maximum relative error, absolute value, E_{max} , %	46.12	0.47
Average relative error, ε_{ave} , %	7.69	0.08
RMSE ($\times 10^6$)	66.10	1.46

GM, gray model; BP, backpropagation; RMSE, root mean square error.

3.2 Analysis of the prediction error

The 200 prediction experiments on natural gas and power consumption for various residential types and time periods were recorded and compared with the actual measurements. Figure 2 depicts a scatterplot of the predicted and measured values of the average residential natural gas consumption on a specific area, and Figure 3 depicts the scatterplot of the predicted and measured power consumption. Among them, the transverse coordinates represent the actual measured value, the longitudinal coordinates represent the value predicted by this model, and the dotted line represents the dividing line of $\pm 20\%$ error between the predicted value and the mea-

Fig. 2 Distribution scatterplot of prediction and measurement of residential average natural gas consumption. **Fig. 3** Distribution scatterplot of predicted and measured values of average residential power consumption.

sured value. It is seen that 18% of the forecasted values showed deviation from the measured values exceeding $\pm 20\%$ for natural gas consumption and going down to 8% [12]. The larger differences in the gas consumption prediction are attributed to the accuracy of the original data set and the gas measurement procedures, as well as changes in building features. For natural gas, the average absolute percentage error in the model prediction is 13% and 9% for electric energy, which shows that the prediction accuracy of the prediction model has reached about 90% and thus this parameter has a high prediction accuracy. These results provide a preliminary assessment of the distribution of error spaces in statistical analysis, by performing further analysis, obtaining predictions for larger regions, and detecting possible spatial patterns of energy consumption throughout the city.

3.3 Analysis of buildings' energy-saving potential

Table 2 shows the energy-saving potential of space heating and domestic hot water in different building years and various types of residences after implementing renovation measures. It can be seen that the larger the building's age, the greater is the building's energy savings. For example, homes built before 2016 had the highest energy-saving potential, with an average percentage of 56%, while homes built after 2018 had only 3%. For building types, apartments – inns (26%), villas (29%), and townhouses (20%) – have greater energy savings potential as these are the major parts of the current residential stock. Independent housing and nonindependent housing contribute only 3%–12% to the total energy conservation potential, respectively, indicating that implementing improvement measures in single-family houses does not bring greater energy conservation efficiency to the whole city.

Table 2 Energy-saving potential after residential renovation of different years and types of buildings.

	Building life				Building type			
	<2016	2016–2017	2017–2018	>2018	Detached house	Semidetached dwelling	Row house	Villa
Energy-saving rate, %	56	24	17	3	3	12	20	29

The traditional gray model is an input-free ordinary differential equation, which has two aspects: increasing the input term and the variable ordinary differential equation is fractional differential equation. Combined with the engineering examples of the fractional gray model and the hyperbolic model, the error between the fractional gray theory prediction results and the project measurement results shows that the prediction models are more accurate and better fit the measured results.

4 Conclusions

The traditional prediction models based on the gray theory involve integer orders, with discontinuity, and are greatly different from the measured data. In view of this, this paper improves the prediction effect of the GM by changing the integer-order differential in the model as the fractional differential. The biggest difference between this model and the traditional model is that the recognition of the fractional order is added, first combined with the gray theory, then input is introduced, and the ordinary differential equation is transformed to get the differential equation; finally, the model is compared with the measured data and the traditional gray theory, which shows that the prediction model is better.

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