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Human gait modelling and tracking based on motion functionalisation

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Abstract

This paper proposes a mathematical function movement model based on the gait movement of the human body and, in particular, on the trajectory of the limbs during human movement. The article systematically measures and experimentally deals with the trajectory of the limbs of 40 students in the walking movement. The linear high-order polynomial fitting method eliminates the motion error. Simultaneously, the linear relationship least square method is used to obtain the expression of the limb motion function. Finally, the mathematical model of the limb motion trajectory is obtained. It is verified through experiments that the model proposed in the thesis can calculate the law of limb movement and movement parameters of any person under normal walking movement. This research has high research value for human movement rehabilitation and the design of wearable equipment.

Keywords: Human motion, motion functionalisation, gait modelling, least square method, polynomial fitting method

AMS 2010 codes: 62J12

1 Introduction

With the rapid development of biomedical engineering technology at home and abroad, higher and higher requirements have been put forward for obtaining human motion parameters. Since the 19th century, various research methods for extracting gait parameters have emerged at home and abroad [1], involving visual tracking of human motion. To avoid the shortcomings associated with these methods, the techniques used in this paper realise the detection, positioning and tracking of motion by analysing the video sequence shot by the camera

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without human intervention or with minimal intervention. On this basis, the human body posture and motion parameters are obtained for further semantic analysis and behavioural understanding. In short, the paper aims to realise the automation of motion analysis without infringing on the analysed person, to liberate people from tedious labour.

In the field of functional modelling of gait movement, some scholars fit the gait movement of the human body with b-spline. Some scholars assume that the human body movement obeys the characteristics of a sinusoidal signal and only estimates the signal amplitude, frequency and phase. The former is currently mainly used in robotics. Because this method ignores the periodicity of human motion, it limits its modelling accuracy and the application value of the model. At the same time, the latter is due to the simplicity of the model, making it unable to reflect the differences between people accurately. Therefore, the question arises as to the optimal means to increase the parameters of the model based on consideration of the periodicity of human motion, while ensuring the accuracy of the modelling and its results; the difference between one human motion and another needs to be accurately reflected by adjusting the model parameters to perform human motion modelling, thus facilitating actual human motion to serve as the basis for tracking applications.

In the process of upright walking of the human body, the limbs play an important balancing role. When the limbs produce abnormal movement trajectories, it will directly affect the movement characteristics of the gait [2]. For this reason, the movement trajectory characteristics of the limbs need to be judged, including standard features, which are of great significance in the fields of rehabilitation medicine and sports training.

2 Three-dimensional motion trajectory measurement system of human limbs

Gait movement is a natural human movement that can be seen everywhere. Through careful observation, we can find that the gait has prominent periodic characteristics. The two strides during normal walking can be regarded as a gait cycle, and the human gait movement is a repeated process of such a gait cycle. We need to extract the four critical states in the gait cycle to get a complete gait cycle. In the gait movement model, the uncertain changes are the eight joint angle parameters of the upper arm and the longitudinal body axis, the upper arm and the forearm, the thigh and the longitudinal body axis, the upper leg and the lower leg and the changes of the two ankle joints and the lower leg [3]. The changing relationship is approximately unchanged. We use 'maker' to locate the absolute joint position, calculate the rotation angle of the human body's degree of freedom through the joint position and then model the respective degree of movement through curve fitting.

The three-dimensional motion trajectory measurement system of the limbs is composed of two parts: a trajectory detection system and a plantar force detection system, which mainly includes a treadmill, a support frame, a pull-wire displacement sensor, a PC, a Dspace system and a force measurement insole and interface card. In the experimental measurement process, the subject only needs to walk naturally on the treadmill at a given speed. The operator observes the subject and starts to record the data after the subject is observed to be walking smoothly. The displacement sensor is measured by the interface card, Dspace system and Matlab software. The output signal of the load cell is collected in real-time, and matrix operation is performed. The movement trajectory curve of the limb in the coronal, sagittal and transverse planes is displayed on the computer screen through the ControlDesk interface. All experimental data are recorded after the measurement is completed [4]. The Matlab software processes the data stored in the computer for secondary data processing to obtain the characteristic parameters such as the amplitude, phase, frequency and other related gait parameters of the limbs in the coronal, sagittal and transverse planes, as well as the gait cycle and other related gait parameters.

When the subject walks naturally on the treadmill, subjective factors and the influence of the environment will cause the subject to randomly experience irregular gait on the track, such as when the walking line swings from side to side or back and forth, which results in three-dimensional movement of the limbs. The curve has obvious, irregular up and down fluctuations. This process can be observed in real-time on the computer screen. To eliminate the measurement error caused by irregular gait, we obtain the standard limb movement trajectory and various characteristic parameters, and analyse the same using secondary offline data processing. In the

process, a high-order polynomial fitting method is first used to fit the actual measurement data to obtain the error curve and the error data at each sampling point [5]. This curve reflects the macroscopic position of the limbs during walking. This article aims to study the microscopic movement of the limbs based on this macroscopic movement. Then, the actual measurement data at each time point and the error value are reversely superimposed. The obtained data is the relative movement trajectory of the final limb, and thereafter we draw each set of curves.

3 The trajectory of the limbs in the walking experiment

The paper selects 40 students of different heights as the measurement objects, and each subject walks naturally on a treadmill at a given speed. Figure 1 shows the trajectory of the limbs when one of the subjects walks at different speeds. Figure 2 shows the limb movement trajectories of nine randomly selected subjects of different heights walking at a fixed speed [6]. The abscissa is the percentage of the gait cycle, and the ordinate is the displacement y_p of the limbs from the centre position in the left and right directions.

Fig. 1 The trajectory of the limbs of the same subject walking at different speeds.

Fig. 2 Limb movement trajectories of subjects of different heights walking at a fixed speed.

This result shows that the walking speed and the height of the subject have a particular influence on the trajectory of the limbs and show a regular trend of changes in parameters such as the motion amplitude a_y , period T_y and initial phase ϕ_y .

4 Establishment of body motion trajectory model

4.1 Model analysis of motion amplitude

Table 1 shows the experimental measurement results of the limb motion amplitude of the above nine subjects walking at different speeds [7]. We use height as the abscissa and motion amplitude as the ordinate and plot the measurement results at each speed on the same graph. As shown in Figure 3, the ‘*’ point corresponds to the data in Table 1, h is the height of the subject and a_y is the amplitude of limb movement. The minimum two multiplication method is used to fit the data to accurately express the relationship between height and amplitude at different speeds. The fitting process is realised by Matlab software. The straight line pointed by the arrow in Figure 3 is the primary fitting curve at the corresponding speed. The expression of the fitting function is

$$a_y = ah + b \quad (1)$$

In the formula, a and b are the fitting coefficients.

Fig. 3 Fitting curves of the relationship between height and amplitude at different speeds.

Table 1 The amplitude of limb movement when subjects of different heights walk at different speeds.

Serial number	h/m	v/m · s ⁻¹							
		0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
1	1.86	44.03	42.25	38.04	34.35	32.52	29.45	24.43	22.03
2	1.84	40.64	35.24	33.48	31.33	28.82	25.13	24.26	21.26
3	1.68	34.34	31.25	28.85	24.58	23.12	22.55	19.4	19.43
4	1.64	31.88	30.5	25.24	24.85	22.45	20.95	16.24	16.11
5	1.624	30.28	25.44	24.23	22.05	18.32	15.99	14.66	14.03
6	1.6	26.84	24.88	22.55	18.45	18.32	15.23	16.41	14.19
7	1.684	24.18	22.85	20.55	15.88	14.44	14.31	14.1	11.4
8	1.64	22.16	16.86	16.84	14.48	14.82	13.34	11.94	10.99
9	1.614	16.68	14.1	14.22	13.44	12.26	11.42	10.44	9.94

It can be seen from Figure 3 that there is a fitting error between the fitted curve and the measured value. The expression for calculating the fitting error Δe_y of the measuring point is

$$\Delta e_y = a_y^* - a_y(h^*) \quad (2)$$

The formula a_y^* is the measured value and $a_y(h^*)$ is the fitting function value obtained when the height is taken

from the measured value. The expression for calculating the variance e_y of each set of fitting functions is

$$e_y = \sqrt{\frac{\sum \Delta e_y^2}{m}} \quad (3)$$

In the above equation, m is the number of subjects, i.e. the number of measurement points at each speed. We take the average of the fitting variance of each speed, and the expression of the average variance Δe of all fitting functions can be obtained as

$$\Delta e = \frac{\sum e_y}{n} \quad (4)$$

where n is the number of fitting functions, i.e. the number of speed values. The expression for calculating the average relative error ε_y is

$$\varepsilon_y = \frac{\Delta e}{\Delta a_y^*} \quad (5)$$

In the above equation, Δa_y^* is the average value of all amplitude measurements, and the expression is

$$\Delta a_y^* = \frac{\sum a_y^*}{nm} \quad (6)$$

Table 2 shows the fitting coefficients a , b , variance e_y , average variance Δe and relative fitting error ε_y between the measured value and the fitted value at different speeds in Figure 3. With speed as the abscissa, and the fitting coefficients a and b as the respective ordinates, the position of the “*” point is shown in Figure 4. The straight line is the fitting curve of the two, and the fitting function is as follows:

$$fa(v) = -73.07v + 134.27 \quad (7)$$

$$fb(v) = 106.51v - 195.47 \quad (8)$$

Similarly, the relative fitting error of coefficient a is 1.50%, and the relative fitting error of coefficient b is 1.86%. Substituting Eqs (7) and (8) into Eq. (1), it can be inferred that when healthy young people walk naturally on the treadmill, the mathematical model of the amplitude of the limbs in the left and right directions is as follows:

$$a_y(h, v) = -73.07vh + 134.27h + 106.51v - 195.47 \quad (9)$$

Fig. 4 Fitting coefficient and fitting curve of relationship with walking speed.

Table 2 Fitting coefficients and errors at different speeds.

$v/m \cdot s^{-1}$	a	B	ey	Δe	$\epsilon y/\%$
0.4	104.16	−160.77	1.0683	1.0316	2.31%
0.5	88.81	−144.08	0.8648		
0.6	88.36	−118.31	1.1636		
0.7	83.85	−111.87	1.45		
0.8	77.67	−114.01	0.8586		
0.9	69.9	−101.71	1.135		
1	59.09	−95.39	1.1549		
1.1	53.91	−77.55	1.63		

Table 3 Exercise cycles of the same subject at different walking speeds.

$v/m \cdot s^{-1}$	Ty/s	$v/m \cdot s^{-1}$	Ty/s
0.4	1.94	0.8	1.35
0.5	1.6	0.9	1.21
0.6	1.45	1	1.18
0.7	1.35	1.1	1.13

Table 4 Exercise cycle of subjects with different heights when walking at a speed of 0.8 m/s.

h/m	Ty/s	h/m	Ty/s
1.97	1.17	1.684	1.15
1.94	1.14	1.67	1.13
1.735	1.13	1.644	1.16
1.72	1.14	1.614	1.11
1.705	1.15

Fig. 5 Fitting curve of the relationship between period and speed and height.

4.2 Model analysis of the movement period Ty

From the experimental data, it can be concluded that during the walking process of the subject, the limb swings once in one gait cycle, which indicates that the limb movement cycle is consistent with the gait cycle [8]. Table 3 shows the same subject in the gait cycle, with the limb movement cycles at different walking speeds. Table 4 shows the limb movement cycles when subjects with different heights walk at a speed of 0.8 m/s.

We draw the experimental data with walking speed and height as the abscissa and the limb movement period

Table 5 The initial phase of the same subject at different walking speeds.

$v/m \cdot s^{-1}$	ϕ_y	$v/m \cdot s^{-1}$	ϕ_y
0.4	-0.566	0.8	-0.442
0.5	-0.505	0.9	-0.442
0.6	-0.46	1	-0.552
0.7	-0.442	1.1	-0.552

Table 6 The initial phases of subjects with different heights when walking at a speed of 0.8 m/s.

h/m	T_y/s	h/m	T_y/s
1.87	-0.471	1.685	-0.6
1.84	-0.451	1.67	-0.566
1.735	-0.417	1.655	-0.614
1.71	-0.518	1.615	-0.487
1.705	-0.534		

as the ordinate. As shown in Figure 5, the ‘*’ points are the measurement data points. The least-squares method is also used for data fitting, and the period and walking are obtained. The fitting curve of the relationship between speed and height is shown in the Figure 5. From this, it can be seen that during the natural walking of healthy young people, the limb movement cycle is mainly affected by walking speed, and height has no effect on it. It is derived by the polynomial fitting method. The mathematical model of the relationship between the cycle and walking speed is as follows [Eq. (10)]. Similarly, the relative fitting error of the fitting function is 4.3%.

$$T_y = 1.52^2v - 3.24v + 2.87 \quad (10)$$

The walking speed is taken as 0.8 m/s and substituted into the above formula, and the gait period is 1.25 s. The average value of the measurement data in Table 3 is 1.24 s, which shows that the measured value and the fitted value are the same.

4.3 Model analysis of initial phase ϕ_y

It can be seen from Figures 1 and 2 that at the beginning of the gait cycle, the limb is located on the left side of the intermediate balance position (taking the left heel landing time as the gait cycle dividing point), and then the first wave peak is reached through the balance position; so is the initial phase of the sine curve [10]. Table 5 shows the initial phase ϕ_y of the limb movement trajectory of the same subject at different walking speeds. Table 6 shows that subjects with different heights all walk at a speed of 0.8 m/s. The initial phase ϕ_y of each time is calculated according to the method described above, and the initial phase expression of the limb swing at the beginning of the gait cycle is given by:

$$\phi_y = 0.48v - 0.76 \quad (11)$$

Similarly, the relative error of this fitting function is 1.8%.

4.4 Model analysis of limb movement trajectory

The paper uses a sine function to establish a mathematical model of the limb movement trajectory, and the expression is

$$y_p = a_y \sin\left(\frac{2\pi}{T_y}t + \phi_y\right) \quad (12)$$

The paper substitutes the models in Eqs (9), (10) and (11) of the amplitude, period and initial phase obtained in the above sections into Eq. (12), respectively, to obtain the trajectory of the limbs in the left and right directions during the natural walking of healthy young people. The mathematical model is as follows:

$$y_p = (-73.07vh + 134.27h + 106.51v - 195.47) \cdot \sin\left(\frac{2\pi}{1.52v^2 - 3.24v + 2.87}t + 0.48v - 0.76\right) \quad (13)$$

5 Accurate human motion tracking experiment results

An important factor that affects the accuracy of human motion is whether the model can reflect the absolute motion of the human body [11]. In this part of the research, we use the previously established model as a tool for particle state prediction. The algorithm is as follows:

5.1 Initialisation

When tracking the first frame, we don't have any tracking history information to guide the estimation of the state. If the blind search is used, it will inevitably involve a heavy computational cost and affect the tracking efficiency. Therefore, we need a specific method to roughly estimate the state of the human body in the first frame [12]. The prior knowledge we can use is the periodic function model of joint angles in gait motion obtained through model learning. We select a particle set of 500 particles to be distributed according to Gaussian at the initialisation time.

5.2 Forecast

The dynamic model uses continuity assumptions and prior knowledge (motion model and motion constraints) to predict the human pose represented by the particle state in the current frame according to the motion function of each joint [13]. A particle set of 500 particles is selected for error correction, and the corrected particle set has a $\mu(0, 0.5)$ Gaussian distribution.

5.3 Particle trade-off

We match the motion regions extracted from the video with the motion regions represented by the particles to detect the degree of matching for all particles. Regional matching of the head, upper body and arm parts is carried out. The movement of the head and the upper body torso is relatively simple, and the movement of the arm is more complicated, but the phenomenon of occlusion between the torso and the torso is serious when walking, and the functionalisation of human movement can be in the arm. We can predict the movement of the arm more accurately during occlusion [14]. Therefore, in the feature matching stage, we only use region matching for the head, upper body and arms. Subsequently, regional matching and edge matching of the legs is carried out. Among the changes of various parts of the human body during walking, the movement of the legs is the part that best reflects the characteristics of human walking. Therefore, in the field of human motion tracking, especially gait tracking, the accuracy of leg tracking is higher. In the research carried out in this paper, the feature matching of the legs uses both edge features and regional features, as well as the final region matching error E_r , and edge matching error and feature error, which determine the proportion of regional matching error and edge matching error in the feature error.

5.4 Particle update

The particle update process in this paper is different from the re-sampling update of the ordinary particle filter [15]. In the tracking study of this part, the particle state represents the error; so, the difference between the particle states at adjacent moments is not significant; thus, in the process of particle updating, the re-sampling operation is no longer needed. The generation of the new particle set is also sampled according to the Gaussian

distribution. The mean of this Gaussian distribution is the output of the best particle state at the last moment, and the variance remains unchanged. The tracking effect is shown in Figure 6.

Fig. 6 Human motion gait modelling and tracking effect.

6 Conclusion

Based on the experimental data, this article establishes a mathematical model of the trajectory of the limbs of healthy young people when they walk naturally. This model has a particular general significance and application value. In the case of there being no actual measurement, as long as the height of a certain young man is given together with the walking speed, this model can be used to obtain the physical characteristics of the limbs that the person should have under the condition of physical health. In rehabilitation training for patients with walking dysfunction, these parameters can provide a basis for rehabilitation evaluation. The parameters obtained from the model are compared with the actual morbid parameters to evaluate the rehabilitation effect. The research on the mathematical model of the human limb movement trajectory has particular scientific significance and application value in many fields such as sports, anthropology, aerospace and ergonomics.

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