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Optimisation of Modelling of Finite Element Differential Equations with Modern Art Design Theory

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Abstract

A bridge structure is one of the most expressive forms of art design. The artistic expression of bridge structure combines different concepts of structural design and architectural art design. Finite element differential equations are widely used in bridge art design theory and based on these features, the paper adopts the bridge modal parameter recognition algorithm and uses the finite element model to modify and realise the bridge's artistic design. The simulation results show the feasibility of the author's attempt to use the finite element differential equation as the bridge structure art design carrier. After the finite element differential equation modelling, the bridge art structure correction is highly consistent with the experimental results.

Keywords: Bridge art design, finite element model modified differential equation modelling, constrained optimisation, modal parameters

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1 Introduction

The bridge is one of the most symbolic and expressive public buildings in the urban landscape. Because of its open and shared use attributes, it has attracted more attention from society. In recent years, with the continuous improvement of the quality of urban construction in China, higher requirements have been put forward on the landscape and function of bridges. In the finite element model of bridge structure damage identification, health

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diagnosis and bridge working condition assessment and prediction, a reliable and more accurate finite element analysis model is the basis. However, most finite element models are established based on structural design drawings, implying more idealised assumptions and simplifications [1]. therefore, thalamic characteristics an

This phenomenon is a specific difference between the established finite element model and the actual structure. When the difference is significant, the calculated result of the model will be different from the actual measurement result, even exceeding the accuracy allowed in the project. In this case, the finite element model needs to be revised. Regarding the finite element model revision, scholars from various countries have conducted extensive research [2]. The general approach is to minimise the residuals of various tests/calculations of the structure. They proposed a series of structural model modification algorithms for different modification objects according to different optimisation objectives and different optimisation constraints.

This paper proposes a parametric model correction algorithm based on optimisation design theory. This algorithm takes the minimum weighted sum of the frequency residual and the measured degree of freedom residuals as the optimisation of calculation objective. The correction objects are the geometrical, physical and mechanical parameters of the components in the finite element model [3]. The algorithm imposes constraints on the modification parameters and frequency changes based on engineering experience, and further reduces the problem of finite element model modification to constrained optimisation. The problem is solved using an optimised iterative algorithm based on gradient descent.

2 Theory and Method

2.1 Objective function

In this paper, frequency and mode shape are jointly used to construct the residual and objective functions.

$$F(P) = W_\omega F_\omega(P) + W_\phi F_\phi(P) \quad (1)$$

P is the vector of parameter set to be corrected $P = [p_1, p_2, \dots, p_n]$. There are a total of parameters to be corrected. $F_\omega(P)$ is the residual frequency term. $F_\phi(P)$ is the mode shape residual term. W_ω is the weight of the frequency correction term. W_ϕ is the weight of the mode shape correction term. The residual frequency term $F_\omega(P)$ is obtained by the weighted summation of the relative difference between the measured frequency and the calculated frequency of each order

$$F_\omega(P) = r(\omega)^T W_\omega r(\omega) = \sum_{i=1}^{m_1} W_{\omega i} \left(\frac{\omega_{ei} - \omega_{ai}}{\omega_{ei}} \right)^2 \quad (2)$$

where m_1 is the measured frequency order used. Note that ω_{ei} and ω_{ai} are the measured frequency and calculated frequency of the i -th order of the structure, respectively.

Similarly, the modal residual term is defined as

$$F_\phi(P) = \sum_{i=1}^{m_2} W_{\phi i} r_i(\phi)^T r_i(\phi) \quad (3)$$

where m_2 is the measured mode order used and $r_i(\phi)$ is the residual vector of the i -th mode shape, which is defined as follows:

$$r_i(\phi) = \frac{\phi_e^i}{\phi_e^{ir}} - \frac{\phi_a^i(P)}{\phi_a^{ir}(P)} \quad (4)$$

Where ϕ_e^i and $\phi_a^i(P)$ are respectively the incomplete measured mode shape and the calculated mode shape of the i order corresponding to the degree of freedom of the measuring point. ϕ_e^{ir} and $\phi_a^{ir}(P)$ are the components of the measured mode shape and the calculated mode shape on the reference degrees of freedom, respectively [4]. The reference degree of freedom is generally selected at the position of the mode amplitude, as shown in Figure 1.

Fig. 1 The normalisation of the i mode shape and the reference degree of freedom r .

2.2 Optimisation constraints

To avoid losing the physical meaning of modifying the parameters during the optimisation process, constraints to the optimisation iteration are added as follows:

$$\begin{aligned} |\omega_{ei} - \omega_{ai}| &\leq UL, \quad i = 1, 2, \dots, m \\ p_k^L &\leq p_k \leq p_k^U, \quad k = 1, 2, \dots, n \end{aligned} \quad (5)$$

Equation (5) is the frequency constraint condition. UL is the maximum value of the frequency change of the state variable in each iteration [5]. Each parameter sets the upper limit and the lower limit p_k^L . The value can be determined according to engineering experience and actual conditions.

2.3 Optimised solution

The general form of the constrained optimisation problem is

$$\min : F = F(x) \quad (6)$$

Restrictions:

$$\begin{aligned} x_i^L &\leq x_i \leq x_i^U, \quad (i = 1, 2, \dots, n) \\ g_i &\leq g_i^U, \quad (i = 1, 2, \dots, m_1) \\ h_i^L &\leq h_i(x), \quad (i = 1, 2, \dots, m_2) \\ w_i^L &\leq w_i(x) \leq w_i^U, \quad (i = 1, 2, \dots, m_3) \end{aligned} \quad (7)$$

In the formula: x_i is the optimisation design variable, and $g_i, h_i, w_i(x)$ is the state variable. In the optimisation calculation, we deal with the constrained optimisation problem by converting it into an unconstrained optimisation problem [6]. This paper uses the penalty function method to convert the above-constrained optimisation problem to unconstrained optimisation problem. The objective function after the penalty function is introduced in the experiment is recorded as

$$Q(x, q) = F(x) + \sum_{i=1}^n P_x(x_i) + \left[q \sum_{i=1}^{m_1} P_g(g_i) + \sum_{i=1}^{m_2} P_h(h_i) + \sum_{i=1}^{m_3} P_w(w_i) \right] \quad (8)$$

P_x is the penalty function term of the optimisation design variable constraint condition. P_g, P_h, P_w are the penalty function terms of the constraint condition of the state variable. The more common and effective method to solve the unconstrained optimisation problem expressed by formula (8) is the iterative algorithm based on gradient descent. The iterative calculation starts from a particular initial point $X^{(0)}$ of the design variable [7]. At each iteration step k , the gradient of the objective function is calculated to determine the iterative calculation direction

of the design variables. Iterate repeatedly until the convergence condition is met. Whether the iteration converges or not can be judged from equations (9) and (10).

$$|F^{(k+1)} - F^{(k)}| \leq tol \quad (9)$$

$$|F^{(k)} - F^b| \leq tol \quad (10)$$

where tol is allowable error of objective function and F^b is the optimal value of the objective function. Equation (9) indicates that the difference between the value of the objective function at the iteration $k + 1$ and the previous iteration step k is less than the allowable error of the objective function [8]. Equation (10) indicates that the difference between the value of the objective function at step k and the optimal value (known) of the objective function is less than the allowable error of the objective function.

3 Calculation examples

The three-span continuous beam is shown in Figure 2. Its total length is 11 m. The span layout is 3 + 5 + 3 m. The rectangular cross-section is 0.2 m × 0.2 m. The corresponding moment of inertia is $I = 1.33 \times 10^{-4}$ m. The area is $A = 0.04$ m². The material is $E = 3.0 \times 10^4$ MPa. $\rho = 2500$ kg/m³.

Fig. 2 Three-span continuous beam/cm calculated by model modification simulation.

The experiment uses plane beam elements to establish a finite element analysis model. It is divided into 22 units and 23 nodes at equal intervals [9]. We calculated the first eight modal frequencies and modal shapes using initial structural parameters that are mentioned above. At the same time, these values are used as the calculated values of the initial structural modal parameters. To simulate the unknown parameters, we reduce the moments of inertia of units 2, 9, 12 and 20 by 30%, 40%, 50% and 40%, respectively. And the experiment increases the density of all unit materials by 20% when other parameters remain unchanged. On this basis, the first eight modal frequencies and modal shapes after the parameter change are calculated and used as the simulation measured values [10]. The starting point of the parameter correction iteration is the initial structure parameter, and the ideal target of the correction is the changed parameter value. Among them, the parameter correction iteration start point is the initial structure parameter.

$$\text{Frequency error: } \omega\% = \left| \frac{\omega_e - \omega_a}{\omega_e} \right| \%. \text{ Vibration shape error: } \|\phi\|_{\frac{1}{2}}\% = \left(\frac{\|\phi_e - \phi_a\|_{\frac{1}{2}}}{\|\phi_e\|_{\frac{1}{2}}} \right) \%.$$

The calculated and measured values of the modal parameters before model modification and their errors are shown in Table 1. It can be seen from Table 1 that the calculated value of the modal parameters has a significant error with the actual measured value [11]. For example, the maximum error of frequency is 18.26%, and the maximum error of mode shape is 38.55%. Therefore, the experiment is divided into two cases.

Case 1. The target mode adopts the first eight orders. The trimming parameters only include the moments of inertia of elements 2, 9, 12 and 20 and the material parameter density ρ . We set a total of 5 parameters.

Case 2. The target mode adopts the first eight orders. The inertia of all elements is selected as the design parameter, plus the material density ρ , a total of 23 parameters.

Table 1 Comparison of calculated and measured modal parameters before correction

Order	Frequency calculation value	Frequency measured value	Frequency error	Mode shape difference
1	18.503	15.674	18.05	2.7
2	39.549	33.74	17.22	12.69
3	46.472	39.854	16.61	10.2
4	68.545	58.921	16.33	11.2
5	78.746	71.885	9.54	-
6	122.81	103.85	18.26	17.78
7	153.04	133.7	14.47	38.55
8	167.12	146.14	14.36	36.81

Table 2 Comparison of calculated and measured values of modal parameters after correction

Order		Frequency calculation value	Frequency measured value	Frequency error	Mode shape difference
1	Situation 1	15.672	15.674	0.010	0.089
	Situation 2	15.433		1.540	0.330
2	Situation 1	33.725	33.740	0.040	0.650
	Situation 2	33.089		1.929	0.260
3	Situation 1	39.842	39.854	0.030	0.540
	Situation 2	39.103		1.880	0.310
4	Situation 1	58.932	58.921	0.020	0.170
	Situation 2	57.933		1.677	0.500
5	Situation 1	71.922	71.885	0.050	-
	Situation 2	71.510		0.521	
6	Situation 1	103.850	103.850	0	0.11
	Situation 2	102.280		1.512	0.93
7	Situation 1	133.750	133.700	0.04	0.15
	Situation 2	131.350		1.758	0.72
8	Situation 1	146.170	146.140	0.02	0.45
	Situation 2	143.700		1.67	0.91

Table 3 Parameter modification results

Category		Before correction $\times 10^{-4}$	After correction $\times 10^{-5}$	True value $\times 10^{-5}$	Error/%
1	Situation 1	1.33	9.309	9.31	0.011
	Situation 2		9.204		1.138
2	Situation 1	1.33	7.974	7.98	0.075
	Situation 2		7.788		2.4
3	Situation 1	1.33	6.624	6.65	0.391
	Situation 2		6.61		0.6
4	Situation 1	1.33	7.899	7.98	1.015
	Situation 2		7.937		0.53
5	Situation 1	2500	2997.1	3000	0.097
	Situation 2		3031.6		1.05

Modified calculation constraints: (1) Moment of inertia constraint: $5 \times 10^{-5} \leq I_i \leq 1.8 \times 10^{-4}$. (2) Density constraint: $2200 \leq \rho \leq 3200$. (3) Frequency constraint: $\{|\omega_{ei} - \omega_{ai}| \leq 20, i = 1, 2, 3, 4 | \omega_{ei} - \omega_{ai}| \leq$

50, $i = 5, 6, 7, 8$. The correction results are shown in Tables 2 and 3.

Tables 2 and 3 show that the calculated values of the modal parameters (frequency, mode shape) after the two cases of the proposed algorithm are very close to the measured values [12]. The maximum error of case 1 frequency is 0.05% and the maximum error of mode shape is 0.65%. The correction values of the five parameters are also very close to the actual values of the parameters. The maximum error is 1.015%. The maximum error of the case 2 frequency is 1.929%. Further, the maximum error of mode shape is 0.93%. The correction values of the five parameters are also relatively close to the actual values of the parameters. The maximum error is 2.4%. In case 1, the correction parameters are directly carried out on the five parameters with errors, and it only takes a few iterations to converge to the actual value. In case 2, there are 23 correction parameters and the algorithm can still effectively complete the model correction function [13]. However, the error of the parameter correction results has increased, and the iterative calculation workload has also increased. As shown in Figure 3, case 1 (Figure 3b) converges at the 9th iteration step, while Case 2 (Figure 3b) takes more than 15 iterations to converge.

Fig. 3 Parameter (I_2, I_9, I_{12}, I_{20}) iterative process.

The calculation examples prove that the parametric model correction algorithm proposed in this paper is feasible and effective. Satisfactory correction results can be obtained even when there are many correction parameters. However, when the parameters increase, the efficiency of iterative calculation decreases, and the amount of calculation is reduced. Larger.

4 Conclusion

The construction of the optimised objective function considers the frequency and mode shape errors and directly uses the measured mode shape components. Thus, there is no need for a complete mode vector, and the error introduced by mode expansion is avoided. Based on the experience of modern bridge art design engineering to impose constraints on the modification parameters and frequency changes, we attribute the model modification problem to the constraint optimisation problem. The problem is solved by an iterative optimisation algorithm based on gradient descent. The result of calculations of the example proves that the model correction method

proposed in this paper is feasible and effective.

References

- [1] Bu, W., & Xiao, A. An h-p version of the continuous Petrov-Galerkin finite element method for Riemann-Liouville fractional differential equation with novel test basis functions. *Numerical Algorithms.*, 2019. 81(2): 529-545
- [2] Liu, Y., Du, Y., Li, H., Liu, F., & Wang, Y. Some second-order schemes combined with finite element method for nonlinear fractional cable equation. *Numerical Algorithms.*, 2019. 80(2): 533-555
- [3] Duru, H., Tasbozan, O., & de Griño, A. K. New analytical solutions of conformable time fractional bad and good modified Boussinesq equations. *Applied Mathematics and Nonlinear Sciences.*, 2020. 5(1): 447-454
- [4] Duru, H., Kurt, A., & Tasbozan, O. New travelling wave solutions for KdV6 equation using sub equation method. *Applied Mathematics and Nonlinear Sciences.*, 2020. 5(1): 455-460
- [5] Abbaszadeh, M., & Dehghan, M. Analysis of mixed finite element method (MFEM) for solving the generalized fractional reaction-diffusion equation on nonrectangular domains. *Computers & Mathematics with Applications.*, 2019. 78(5): 1531-1547
- [6] Gunzburger, M., Li, B., & Wang, J. Convergence of finite element solutions of stochastic partial integro-differential equations driven by white noise. *Numerische Mathematik.*, 2019. 141(4): 1043-1077
- [7] Cimrman, R., Lukeš, V., & Rohan, E. Multiscale finite element calculations in Python using SfePy. *Advances in Computational Mathematics.*, 2019. 45(4): 1897-1921
- [8] Liu, F., Feng, L., Anh, V., & Li, J. Unstructured-mesh Galerkin finite element method for the two-dimensional multi-term time-space fractional Bloch-Torrey equations on irregular convex domains. *Computers & Mathematics with Applications.*, 2019. 78(5): 1637-1650
- [9] Ammi, M. R. S., Jamiia, I., & Torres, D. F. A finite element approximation for a class of Caputo time-fractional diffusion equations. *Computers & Mathematics with Applications.*, 2019. 78(5): 1334-1344
- [10] Sur, A., Pal, P., Mondal, S., & Kanoria, M. Finite element analysis in a fiber-reinforced cylinder due to memory-dependent heat transfer. *Acta Mechanica.*, 2019. 230(5): 1607-1624
- [11] Hao, Z., Park, M., Lin, G., & Cai, Z. Finite element method for two-sided fractional differential equations with variable coefficients: Galerkin approach. *Journal of Scientific Computing.*, 2019. 79(2): 700-717
- [12] Bu, W., Shu, S., Yue, X., Xiao, A., & Zeng, W. Space-time finite element method for the multi-term time-space fractional diffusion equation on a two-dimensional domain. *Computers & Mathematics with Applications.*, 2019. 78(5): 1367-1379
- [13] Chen, C., Li, K., Chen, Y., & Huang, Y. Two-grid finite element methods combined with Crank-Nicolson scheme for nonlinear Sobolev equations. *Advances in Computational Mathematics.*, 2019. 45(2): 611-630

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