Applied Mathematics and Nonlinear Sciences

<https://www.sciendo.com>

Research on tourism income index based on ordinary differential mathematical equation

Xiaokun Xiong^{1†}¹ Department of Economics and Trade, Shijiazhuang Vocational Technology Institute, Shijiazhuang, Hebei 050081, China

Submission Info

Communicated by Juan Luis García Guirao

Received June 17th 2021

Accepted September 24th 2021

Available online December 27th 2021

Abstract

Economic benefits continue to increase with the rapid development of the tourism industry. The level of tourism development is an indicator to measure the maturity of a country's tourism industry. The article takes China's coastal provinces as the research object and uses finite element ordinary differential mathematical equations to explore the income indicators of the tourism industry in coastal cities. In the dynamic process, the article portrays the development trend of tourism in different regions. It analyses the income levels and indicators of tourism development in coastal cities under different time and space conditions. The research results show that we can divide the time zone to count the base period value and average growth rate of domestic tourism in each province to classify the regional response type of domestic tourism development.

Keywords: finite element, ordinary differential mathematical equation, tourism economy, income index, time and space difference, coastal area

AMS 2010 codes: 74S05

1 Introduction

Tourism is an integral part of the tertiary industry and consumer economy. It plays an essential role in international trade and the national economy. Many countries or regions have adopted tourism as an effective means to reduce regional differences. The Chinese economy is highly concentrated in the coastal areas [1], which have superior locations, rich tourism resources and unique climatic conditions. The development of inbound tourism is conducive to the industrial transformation of the region. It promotes the industrial transfer of the three significant east, middle and west areas to achieve coordinated regional development [2].

The study of regional differences in inbound tourism by foreign academic circles began in the mid-1970s. Domestic scholars started the study of regional differences in inbound tourism at a later time. They mainly analysed time and space differences, regional differences in tourism, regional differences in the provincial inbound

[†]Corresponding author.Email address: xmqxxk123@163.com

tourism economy and development models, endowments of tourism resources and spatial distribution of tourism flows. This article studies the temporal and spatial differences of inbound tourism in coastal areas.

We look forward to a valuable exploration of inbound tourism from this aspect.

2 Time discontinuous Galerkin method: Establishment of the space-time finite element method format

We assume that the spatial region $\Omega \subseteq R^d (d = 1, 2)$ is bounded and has a smooth boundary. The time interval $J = [0, T]$. Let us remember that X and Y are separable Hilbert spaces. Consider the following abstract parabolic equation:

$$\begin{cases} u_t + Au = f(x, t), t \in J, x \in \Omega, \\ Bu = 0, t \in J, x \in \partial\Omega, \\ u(0, x) = u_0(x), x \in \Omega, \end{cases} \quad (1)$$

where A is a self-adjoint, positive definite and compact inverse elliptic operator with a domain of $D(A) \subset X$. B is the boundary operator. For this, we first discretise the time interval J . We set $0 = t^0 < t^1 < \dots < t^N = T, I_n = (t^n, t^{n+1})$ in the article. We also set $T_n^n = \{\tau\}$ to be a regular division of the space region Ω . Here, $h\tau$ represents the length of unit τ and $-h_n = \max_{\tau \in T_n^n} h\tau, h = \max_n h_n$. The division of each space-time slice Q^n can be different, and T_n^n and T_n^{n-1} can take different nodes on the interface $t = t^n (1 \leq n \leq N-1)$.

We assume that $\|\cdot\|_Y$ and $(\cdot, \cdot)_Y$ are the norm and inner product of space Y , respectively. $\|\cdot\|_X$ is the norm of space X . When $X = H^s(\Omega), Y = L^2(\Omega)$, the above norm and inner product are abbreviated as $\|\cdot\|_s$, respectively [3].

We define $L^2(I_n; Z)$ as the set of all strongly measurable functions $v : I_n \rightarrow Z$. We make $\|v\|_{Z,n}^2 = \int_{I_n} \|v(t)\|_Z^2 dt$, where $Z = X$ (or Y), $I_n \subseteq J$. When $Z = H^s(\Omega)$ is abbreviated as $\|\cdot\|_{s,n}$, we assume that $C(\bar{I}_n; Z)$ is the set of all continuous functions $v : I_n \rightarrow Z$. We make $\|v\|_{Z,n}^2 = \max_{t_n \leq t \leq t^{n+1}} \|v(t)\|_Z^2$, where $Z = X$ (or Y) $I_n \subseteq J$.

For $s, m = 0, 1, \dots$ and $v \in H^m(\Omega)$, we define the discrete norm $\|h_n^s v\|_{m,h} = (\sum_{r \in T_h^n} h_\tau^{2s} \|v\|_{m,\tau}^2)^{\frac{1}{2}}$. Let $u(0, x) = u_0(x)$ have the following equation:

$$-\int_J (u, \phi_t)_Y dt + \int_J A(u, \phi) dt = (u, \phi(0))_Y + \int_J (f, \phi)_Y dt \quad (2)$$

$$|A(v, \omega)| \leq c \|v\|_X \|\omega\|_X, \forall v, \omega \in X$$

$$A(v, \omega) \geq c \|v\|_X^2, \forall v \in X;$$

Space $D(J; C^\infty) := \{\phi : \phi \in C^1(J; C^\infty(\Omega)) \text{ and } \phi(T) = 0\}$.

We define the finite element space

$$S_h^n = \{\psi \in X : \psi|_\tau \in P_r(\tau), \forall \tau \in T_h^n\} \quad (3)$$

$$V_{hk} = \left\{ \phi : \phi|_{Q_n} = \sum_{j=0}^{q-1} t^j \psi_j(x), \forall \psi_j \in S_h^n \right\} \quad (4)$$

$$V_{hk}^n = \{\phi|_{Q_n} : \phi \in V_{hk}\} \quad (5)$$

by replacing u in the first term at the left end of the weak form (2) with a function $U \in V_{hk}$. The article is divided into points on each time slice I_n to get

$$-\int_J (U, \phi_t)_Y dt = -\sum_{n=0}^{N-1} \left((U, \phi)_Y|_{t_+^{n+1}} - \int_{I_n} (U_t, \phi)_Y dt \right) = \int_J (U_t, \phi)_Y dt + \sum_{n=1}^{N-1} ([U^n], \phi^n)_Y + (U_+^0, \phi^0)_Y \quad (6)$$

where $\phi^n = \phi(\cdot, t^n) \in D(J; C^\infty)$. $U_\pm^n = \lim_{s \rightarrow 0^\pm} U(x, t^n + s)$ ($0 \leq n \leq N-1$). $[U^n] = U_\pm^n - U_-^n$.

$$\begin{cases} \int_J [(U_t, \phi)_Y + A(U, \phi)] dt + \sum_{n=1}^{N-1} ([U^n], \phi_+^n)_Y + (U_\pm^n, \phi_+^n)_Y = (u_0, \phi_+^n)_Y + \int_J (f, \phi)_Y dt, \forall \phi \in V_{hk}^n \\ U^0 = u_0, n = 0, 1, \dots, N-1 \end{cases} \quad (7)$$

Also, note that values of $\phi \in V_{hk}^n$ are independent of each other in each subinterval I_n ($0 \leq n \leq N-1$); so, in subinterval $I_n = (t^n, t^{n+1})$, correspondingly

$$\int_{I_n} [(U_t, \phi)_Y + A(U, \phi)] dt + ([U^n], \phi_+^n)_Y = \int_{I_n} (f, \phi)_Y dt \quad (8)$$

Further, the first item on the left end of the equal sign in Eq. (8) can be obtained after partial integration of time t to obtain the equivalent format (II), so that $U \in V_{hk}$ is obtained for $n = 0, 1, \dots, N-1$

$$(U^{n+1}, \phi^{n+1})_Y + \int_{I_n} [-(U, \phi_t)_Y + A(U, \phi)] dt = (U^n, \phi_+^n)_Y + \int_{I_n} (f, \phi)_Y dt \quad (9)$$

3 Comprehensive measurement and temporal-spatial characteristic model of urban tourism development efficiency in coastal urban agglomerations

This article assumes that there are K and L input indicators and output indicators for M cities' input and output efficiencies, respectively. x_{mk} and y_{ml} ($x_{mk}, y_{ml} > 0$) respectively represent the standardised value of input and output of the k th and l th elements of tourism in the m th city. ξ is the non-Archimedean infinitesimal quantity. λ_m ($\lambda_m \geq 0$) is the calculation weight variable. s^- ($s^- \geq 0$) is the slack variable, which represents the amount of input reduced when relative efficiency is achieved. s^+ ($s^+ \geq 0$) is the residual variable, which represents the increased output when effective. Therefore, the DEA model for measuring input-output efficiency of urban tourism is represented as follows [4]:

$$\begin{cases} \min \left(\theta - \varepsilon \left(\sum_{k=1}^K s^- + \sum_{l=1}^L s^+ \right) \right) \\ s.t. \sum_{m=1}^M x_{mk} \lambda_m + s^- = \theta x_k^m, k = 1, 2, \dots, K \\ \sum_{m=1}^M y_{ml} \lambda_m - s^+ = y_l^m, l = 1, 2, \dots, L \\ \lambda_m \geq 0, m = 1, 2, \dots, M \end{cases} \quad (10)$$

Eq. (10) is the DEA model under constant returns to scale (CRS). Assuming $\theta_m = 1$, the tourism industry of the m th city is at the best production frontier. The overall efficiency of the city's tourism input and output is the best. Conversely, $\theta_m < 0.5$ indicates that the overall efficiency is invalid [5]. If $0.5 \leq \theta_m < 0.7$ and $0.7 \leq \theta_m < 1$, it indicates that the overall efficiency is medium and good, respectively. This model makes $\theta_m = \theta_{PTE} \times \theta_{SE}$ (θ_{PTE} and θ_{SE} are both $> \theta_m$). θ_{PTE} and θ_{SE} are the pure technical efficiency indices [6]. According to the research of Ma Zhanxin et al., we set $\theta_{PTE}(\theta_{SE}) = 1$ as the most efficient; $0.8 \leq \theta_{PTE}(\theta_{SE}) < 1$ implies good efficiency; $0.6 \leq \theta_{PTE}(\theta_{SE}) < 0.8$ indicates medium efficiency and $\theta_{PTE}(\theta_{SE}) < 0.6$ is invalid for efficiency. The principle of the model is to fit the original data through sampling and obtain its process situation.

$$\{\hat{\theta}_{kb}^*, b = 1, 2, \dots, B\} \quad (11)$$

In this paper, the value of B is taken as 1,000. This involves the confidence interval range of the city tourism efficiency measurement after correction [7]. To directly compare the changes in the correction measures of

urban tourism comprehensive efficiency and its decomposition efficiency in different periods, we introduce the Malmquist total factor index model. Assuming that any time t is T , the m th city uses k types of the tourism input factor x_{mk}^t . We obtain the first tourism output factor y_{ml}^t . This article borrows the distance function concept of the reciprocal of technical efficiency proposed by American scholar Farrell to obtain the input distance function under technology $L^t(y^t|C, S)$:

$$D_i^t(y_{ml}^t, x_{mk}^t) = 1/F_i^t(y_{ml}^t, x_{mk}^t|C, S) \quad (12)$$

This function is understood as the ratio of the input-output array (y_{ml}^t, x_{mk}^t) of any city's tourism industry close to the lowest input point in the general state [8]. Changes in technical efficiency from time t to $t+1$ are represented as follows:

$$M_i^{t+1} = D_i^{t+1}(x_{mk}^t, y_{ml}^t)/D_i^{t+1}(x_{mk}^{t+1}, y_{ml}^{t+1}) \quad (13)$$

$$M_i^t = D_i^t(x_{mk}^t, y_{ml}^t)/D_i^t(x_{mk}^{t+1}, y_{ml}^{t+1}) \quad (14)$$

According to the geometric mean of the measurement results of Eqs (13) and (14), we write the Malmquist index model in different periods as follows:

$$\theta_{TFP}^t = M_0(x_{mk}^{t+1}, y_{ml}^{t+1}, x_{mk}^t, y_{ml}^t) \quad (15)$$

It is generally believed that each efficiency index > 1 indicates that each efficiency is improved. If it is ≤ 1 , there is no change or decrease in efficiency.

4 The characteristics of temporal and spatial differences in the inbound tourism economy in China's coastal areas

4.1 The characteristics of overall change in economic differences of inbound tourism

4.1.1 The absolute difference continues to expand, while the relative difference shows a downward trend

The evolution trend of the overall difference in tourism economy in China's coastal areas is shown in Figure 1. It can be roughly divided into two stages: The first stage is the slow expansion of the absolute difference from 2006 to 2012. The difference increased from 765.18 in 2006 to 1,452.52 in 2012, and the value of the absolute difference increased by 89.83%. The second stage is from 2013 to 2021, and the absolute difference expands at a relatively rapid rate, increasing from 1,238.43 in 2013 to 3838.61 in 2021. The increase was as high as 209.96%.

Although the overall trend is slow, it is still pronounced [9]. All coefficients of variation > 1 indicate that the economic development of inbound tourism in coastal areas is still uneven. Furthermore, the differences between provinces and cities are still quite obvious, and there are still significant differences in the overall coordinated development of regions.

Fig. 1 The evolution trend of the overall difference of tourism economy in China's coastal areas

4.2 The polarisation of regional inbound tourism economy is obvious

Through the analysis of the Herfindahl index (see Figure 1), it can be seen that the overall concentration of the inbound tourism economy of the coastal provinces gradually decreases, and the phenomenon of differentiation becomes more and more apparent. It can be known by calculating the ratio of the foreign exchange income of tourism in various provinces and cities relative to the average level in coastal areas. From a horizontal perspective, the foreign exchange income index of tourism is >1 . Before 2005, it was concentrated in Jiangsu Province, Shanghai and Guangdong Province [10]. After 2005, it was concentrated in the three provinces and cities of Shanghai, Fujian and Guangdong. The remaining provinces and cities generally had index <0.5 . From a vertical perspective, the tourism foreign exchange indexes of Hebei, Shanghai, Fujian, Guangdong, Guangxi and Hainan show a fluctuating and rising trend. Other places showed volatility and decline [11]. However, the rate of increase or decrease in the index of each province and city is different. This shows that the inbound tourism economy in China's coastal areas is polarised. For example, tourism's foreign exchange income index in Guangdong in 2021 is 4.957, while that of Hebei is only 0.139. The difference between the two is as much as 35 times. Thus, the polarisation of the inbound tourism economy is evident.

4.3 The evolution characteristics of inter-provincial, inter-regional and intra-regional differences in the inbound tourism economy

4.3.1 On the whole, regional differences show an upward trend of volatility

Figure 2 shows the evolution of the differences between provinces, cities and regions of the inbound tourism economy in China's coastal areas from 2006 to 2021. The regional differences in the inbound tourism economy show a trend of rising volatility. For example, the regional difference index increased from 0.052 in 2006 to 0.063 in 2021. At the same time, there are two high periods and two low periods.

Fig. 2 The evolution of regional differences in tourism economic provinces and cities in China's coastal areas today

4.3.2 Viewed by stages, the causes of regional differences fluctuate and vary

The regional differences from 2006 to 2008 showed a low-level downward trend. Regional differences mainly caused the reason why TWR was higher than TBR. It can be seen from Figure 3 that the internal differences in the coastal areas of the Yangtze River Delta are higher than those in the Bohai Rim and the Pan-Pearl River Delta. Although the difference in the coastal area of the Yangtze River Delta shows a downward trend, it is still significantly higher than the other two regions. During this period, the total number of inbound tourists in Shanghai accounted for 45.98% of the total coastal area of the Yangtze River Delta.

From 1999 to 2012, regional differences (TP) showed a high upward trend. TWR and TBR were roughly the same, and regional differences were composed of intra-regional and inter-regional differences (see Figure 2).

Fig. 3 The evolution of regional differences in the inbound tourism economy in China's coastal areas from 2006 to 2021

4.4 Comparison of the contribution rate of differences between and within the three regions to the differences in inbound tourism in coastal regions

It can be seen from Table 1 that the contribution rate of the three significant regional and intra-regional differences to the regional differences in China's coastal inbound tourism from 2006 to 2021 shows the following characteristics. The inter-regional difference first increased and then decreased, while the intra-regional difference first decreased and then increased. The inter-regional differences showed a fluctuating upward trend, and the intra-regional differences showed a fluctuating downward trend. The inter-regional differences and inter-regional differences constitute the main aspects that affect the regional differences of inbound tourism in coastal areas.

Table 1 Theil Index of China's Coastal Inbound Tourism Scale

Years	2006	2015	2021
TP	0.052	0.061	0.063
TBR	0.016	0.037	0.025
TWR	0.036	0.024	0.038
TBR contribution rate	30.77	60.66	39.68
TWR contribution rate	69.23	39.34	60.32

5 Analysis of the reasons for the temporal and spatial differences of the inbound tourism economy in China's coastal areas

5.1 The impact of economic foundation and industrial structure

The economic foundation is the foundation of the inbound tourism economy. The economic foundation of various provinces and cities affects and restricts the development of inbound tourism to a certain extent [12]. Guangdong Province ranks first in the country in terms of gross domestic product, and economic income from inbound tourism is always far ahead, followed by the three provinces and cities of Shanghai, Jiangsu and Zhejiang. In addition, tourism is an essential part of the tertiary industry. From the overall industrial structure, the proportion of tertiary industries in the Yangtze River Delta is as high as 45.88%, followed by the Pan-Pearl River Delta with 42.49% and, finally, the Bohai Rim with only 37.94%.

5.2 Tourism resource endowment and natural conditions

Taking 5A-level scenic spots as an example, there are 61 national 5A-level tourist attractions in coastal areas. There are 27 in the Yangtze River Delta, accounting for 44% of the total. However, from a regional perspective, high-level tourism resources are not an absolute condition that determines tourism income, and natural factors also play a huge role. Take the Jiangsu coast as an example. Although Jiangsu Province has the most significant number of 5A-level scenic spots, it lags behind Shanghai in terms of the overall inbound tourism economy [13]. Natural conditions are also one of the reasons. The coastal areas of Jiangsu are large areas of wetlands and tidal flats, and there are few beaches with better quality for recreation. This affects its inbound tourism development to a certain extent.

5.3 The influence of location conditions and the level of opening to the outside world

From the perspective of spatial interaction theory, location is the basis of regional development. The Bohai Rim is close to Japan and South Korea, and the coastal areas of the Pan-Pearl River Delta take advantage of being close to Hong Kong, Macao and Taiwan. In addition, an open policy is also a prerequisite for the development of inbound tourism. Affected by government policies, the opening hours and levels of the provinces and cities in China's coastal areas are not consistent. Opening to the outside world affects the city's popularity to a certain extent, and the city's popularity harms the regional inbound tourism economy.

6 Conclusion

The absolute difference in the inbound tourism economy of the coastal provinces and cities has an expanding trend of fluctuations and increases. On the other hand, the relative difference shows a downward trend. As a result, the degree of concentration is gradually decreasing, and the phenomenon of polarisation is gradually becoming apparent. From the perspective of different stages, the differences between and within regions affected by multiple factors fluctuate. In-depth analysis of the reasons for the above differences has found that the economic foundation, industrial structure, resource endowments, location conditions, the level of opening to the outside world and essential events all have a more significant impact on regional differences.

References

- [1] Chen, C., Zhang, X., Zhang, G., & Zhang, Y. A two-grid finite element method for nonlinear parabolic integro-differential equations. *International Journal of Computer Mathematics.*, 2019. 96(10): 2010-2023
- [2] Sangwan, V., & Kaur, B. An exponentially fitted numerical technique for singularly perturbed Burgers-Fisher equation on a layer adapted mesh. *International Journal of Computer Mathematics.*, 2019. 96(7): 1502-1513
- [3] Xu, J., Yin, K., & Liu, L. State-Continuity Approximation of Markov Decision Processes via Finite Element Methods for Autonomous System Planning. *IEEE Robotics and Automation Letters.*, 2020. 5(4): 5589-5596
- [4] Deka, B., & Roy, P. Weak Galerkin finite element methods for parabolic interface problems with nonhomogeneous jump conditions. *Numerical Functional Analysis and Optimization.*, 2019. 40(3): 259-279
- [5] Reddy, J. N. A dual mesh finite domain method for the numerical solution of differential equations. *International Journal for Computational Methods in Engineering Science and Mechanics.*, 2019. 20(3): 212-228
- [6] Kohl, N., Thönnies, D., Drzisga, D., Bartuschat, D., & Rüde, U. The HyTeG finite-element software framework for scalable multigrid solvers. *International Journal of Parallel, Emergent and Distributed Systems.*, 2019. 34(5): 477-496
- [7] Pradhan, S. P., & Siddique, T. Stability assessment of landslide-prone road cut rock slopes in Himalayan terrain: a finite element method based approach. *Journal of Rock Mechanics and Geotechnical Engineering.*, 2020. 12(1): 59-73
- [8] Mahoui, S., Moulay, M. S., & Omrane, A. Finite element approach to linear parabolic pointwise control problems of incomplete data. *International Journal of Systems Science.*, 2020. 51(14): 2597-2609
- [9] Pirbastami, S., & Pepper, D. W. A two-dimensional finite element recursion relation for the transport equation using nine-diagonal solvers. *Numerical Heat Transfer, Part B: Fundamentals.*, 2020. 78(3): 125-140
- [10] de Assis, R., Pazim, R., Malavazi, M., Petry, P., de Assis, L. & Venturino, E. A Mathematical Model to describe the herd behaviour considering group defense. *Applied Mathematics and Nonlinear Sciences.*, 2020. 5(1): 11-24

- [11] Aghili, A. Complete Solution for The Time Fractional Diffusion Problem With Mixed Boundary Conditions by Operational Method. *Applied Mathematics and Nonlinear Sciences.*, 2020. 6(1): 9-20
- [12] Wu, C. P., Lin, C. H., & Wang, Y. M. Nonlinear finite element analysis of a multiwalled carbon nanotube resting on a Pasternak foundation. *Mechanics of Advanced Materials and Structures.*, 2019.26(18): 1505-1517
- [13] Ansari, M. I., & Kumar, A. Bending analysis of functionally graded CNT reinforced doubly curved singly ruled truncated rhombic cone. *Mechanics Based Design of Structures and Machines.*, 2019. 47(1): 67-86.