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Design of Morlet wavelet neural network to solve the non-linear influenza disease system

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Abstract

In this study, the solution of the non-linear influenza disease system (NIDS) is presented using the Morlet wavelet neural networks (MWNNs) together with the optimisation procedures of the hybrid process of global/local search approaches. The genetic algorithm (GA) and sequential quadratic programming (SQP), that is, GA-SQP, are executed as the global and local search techniques. The mathematical form of the NIDS depends upon four groups: susceptible $S(y)$, infected $I(y)$, recovered $R(y)$ and cross-immune individuals $C(y)$. To solve the NIDS, an error function is designed using NIDS and its leading initial conditions (ICs). This error function is optimised with a combination of MWNNs and GA-SQP to solve for all the groups of NIDS. The comparison of the obtained solutions and Runge–Kutta results is presented to authenticate the correctness of the designed MWNNs along with the GA-SQP for solving NIDS. Moreover, the statistical operators using different measures are presented to check the reliability and constancy of the MWNNs along with the GA-SQP to solve the NIDS.

Keywords: Non-linear influenza disease system, Morlet wavelet neural networks, sequential quadratic programming, Runge–Kutta, genetic algorithms, numerical measures

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1 Introduction

Influenza is a serious disease caused by viruses that affect the lungs, the upper breathing organs, throat, bronchi and nose. Its recovery rate is very high, but only with proper medical care. A high infection rate is noticed in older adults or in those with severe kidney, heart and lung problems, diabetes or cancer. The epidemic rate of influenza is estimated to be between 5% and 15% per year of the population, caused by upper respiratory tract disease. Worldwide, annual epidemics are seen with 3–5 million illness cases and the number of deaths is estimated at around 250,000–500,000 [1]. In mathematical form, epidemiological systems are demonstrated by the ordinary autonomous non-linear differential systems with assumptions of the time-independent parameters. Such biological models involve the variable state to the infected, recovered, susceptible and transmitted vectors.

A number of schemes have been tested to solve the non-linear influenza disease system (NIDS). A few of them are as follows. Astuti et al. [2] suggested a differential transformation step-by-step scheme to solve the influenza virus disease-resistant system. Alzahrani et al. [3] proposed a numerical approach for solving a fractional influenza pandemic system. Erdem et al. [4] discussed the influenza SIQR model using the imperfect quarantine system. Sun et al. [5] stated an optimisation based multi-objective system to allocate patients during an influenza epidemic. González-Parra et al. [6] designed a fractional epidemiologic system based on simulating an influenza A epidemic. Ghanbari et al. [7] worked towards examining the two systems of avian influenza epidemics related to the derivatives of fractal–fractional with memorabilia and power from Mittag–Leffler. Tchuente et al. [8] attempted to enhance the media coverage impacts on the dynamics of human influenza communications. Schulze-Horsel et al. [9] worked on the dynamics of infection as well as virus-induced apoptosis using the influenza vaccine production in cellular philosophy. Hovav et al. [10] discussed the system of network flow to manage the inventory and allocating of influenza vaccines in a healthcare supply chain management. Patel et al. [11] applied genetic algorithms (GAs) for optimal vaccination strategies for the pandemic influenza system. Kanyiri et al. [12] worked on the applications of optimal control for influenza along with the antiviral resistance and pulmonary congestion.

NIDS is divided into four groups: (i) susceptible $S(y)$, (ii) infectious $I(y)$, (iii) recovered $R(y)$ and (iv) cross-immune $C(y)$. The mathematical form of NIDS along with the initial conditions (ICs) is given as follows [13]:

$$\begin{cases} S'(y) = \mu - (\mu + \beta I(y))S(y) + \gamma C(y), & S(0) = u_1, \\ I'(y) = \beta (\sigma C(y) + S(y))I(y) - (\alpha + \mu)I(y), & I(0) = u_2, \\ R'(y) = \alpha I(y) + \beta (1 - \sigma)I(y)C(y) - (\delta + \mu)R(y), & R(0) = u_3, \\ C'(y) = \delta R(y) - (\mu + \beta I(y) + \gamma)C(y), & C(0) = u_4, \end{cases} \quad (1)$$

where, u_1, u_2, u_3 and u_4 are the ICs, β represents the rate of transmission from the individuals of $S(y)$ to $I(y)$, σ is the exposed individuals based on cross-immune that are shifted in a unit time to the communicable subpopulation [14]. Moreover, the infectious, infected and cross-immune are defined as δ^{-1} , γ^{-1} and α^{-1} , respectively.

The purpose of the current investigation is to treat NIDS numerically by using the Morlet wavelet neural networks (MWNNs) together with the optimisation procedures in the hybrid process of global/local search approaches. The GA and sequential quadratic programming (SQP), that is, GA-SQP, are executed as global and local search techniques. There are various applications in which stochastic computing approaches have been applied, such as COVID-19-based SITS dynamics [15, 16], singular fractional models [17, 18], prey-predator model [19], delay singular functional model [20, 21], dengue fever model [22], higher order non-linear singular systems [23–25], non-linear mosquito release system in heterogeneous atmosphere [26] and multi-singular differential systems [27, 28]. Keeping in view these recognised submissions, the authors are motivated to solve NIDS using the MWNNs and GA-SQP. Some motivational factors of MWNNs using the GA-SQP are briefly as follows:

- The proposed MWNNs using GA-SQP provides impressive numerical solutions of NIDS.

- Steady, reliable and stable numerical outcomes of NIDS authenticate the worth of the proposed form of MWNNs using GA-SQP.
- The values of the absolute error (AE) show the best performances, which demonstrate the consistency of the proposed MWNNs using GA-SQP.
- The numerical performance of the scheme is certified using different statistical annotations to solve the NIDS for multiple independent runs.
- The proposed MWNNs using GA-SQP is smoothly executed to solve NIDS with inclusive, easy-to-understand and smooth operations.

The remainder of the paper is categorised as follows: Section 2 depicts the designed MWNNs using the GA-SQP methodology along with statistical procedures. Section 3 provides the results and simulation. Section 4 deals with the final comments and future research directions.

2 Methodology: MWNNs–GA-SQP

The structure of the MWNNs–GA-SQP is presented in this section based on two phases for solving NIDS:

- A merit function is proposed based on MWNNs using the GA-SQP to solve NIDS.
- Some major settings are provided to improve the merit function using the methodology of GA-SQP.

2.1 Structure of MWNNs

The mathematical design to solve NIDS is divided into four groups: susceptible (S), infectious (I), recovered (R) and cross-immune (C). The performance through the proposed results of these groups are $\hat{S}$, $\hat{I}$, $\hat{R}$ and $\hat{C}$, written as:

$$\begin{aligned} [\hat{S}(y), \hat{I}(y), \hat{R}(y), \hat{C}(y)] &= \begin{bmatrix} \sum_{i=1}^m r_{S,i} L(w_{S,i}y + s_{S,i}), & \sum_{i=1}^m r_{I,i} L(w_{I,i}y + s_{I,i}), \\ \sum_{i=1}^m r_{R,i} L(w_{R,i}y + s_{R,i}), & \sum_{i=1}^m r_{C,i} L(w_{C,i}y + s_{C,i}) \end{bmatrix}, \\ [\hat{S}'(y), \hat{I}'(y), \hat{R}'(y), \hat{C}'(y)] &= \begin{bmatrix} \sum_{i=1}^m r_{S,i} L'(w_{S,i}y + s_{S,i}), & \sum_{i=1}^m r_{I,i} L'(w_{I,i}y + s_{I,i}), \\ \sum_{i=1}^m r_{R,i} L'(w_{R,i}y + s_{R,i}), & \sum_{i=1}^m r_{C,i} L'(w_{C,i}y + s_{C,i}) \end{bmatrix}, \end{aligned} \quad (2)$$

W is the unknown weight vector, given as:

$W = [W_S, W_I, W_R, W_C]$, for $W_S = [r_S, \omega_S, s_S]$, $W_I = [r_I, \omega_I, s_I]$, $W_R = [r_R, \omega_R, s_R]$ and $W_C = [a_C, \omega_C, b_C]$, where

$$\begin{aligned} r_S &= [r_{S,1}, r_{S,2}, \dots, r_{S,m}], & r_I &= [r_{I,1}, r_{I,2}, \dots, r_{I,m}], & r_R &= [r_{R,1}, r_{R,2}, \dots, r_{R,m}], \\ r_C &= [r_{C,1}, r_{C,2}, \dots, r_{C,m}], & \omega_S &= [\omega_{S,1}, \omega_{S,2}, \dots, \omega_{S,m}], & \omega_I &= [\omega_{I,1}, \omega_{I,2}, \dots, \omega_{I,m}], \\ \omega_R &= [\omega_{R,1}, \omega_{R,2}, \dots, \omega_{R,m}], & \omega_C &= [\omega_{C,1}, \omega_{C,2}, \dots, \omega_{C,m}], & s_S &= [s_{S,1}, s_{S,2}, \dots, s_{S,m}], \\ s_I &= [s_{I,1}, s_{I,2}, \dots, s_{I,m}], & s_R &= [s_{R,1}, s_{R,2}, \dots, s_{R,m}], & s_C &= [s_{C,1}, s_{C,2}, \dots, s_{C,m}]. \end{aligned}$$

The updated form of NIDS using the Morlet function $L(y) = (1 + \exp(-y))^{-1}$ [29–31] is given as:

$$[\hat{S}(y), \hat{I}(y), \hat{R}(y), \hat{C}(y)] = \begin{bmatrix} \sum_{i=1}^m r_{S,i} \cos(1.75(w_{S,i}y + s_{S,i})) e^{-0.5(w_{S,i}y + s_{S,i})^2}, \\ \sum_{i=1}^m r_{I,i} \cos(1.75(w_{I,i}y + s_{I,i})) e^{-0.5(w_{I,i}y + s_{I,i})^2}, \\ \sum_{i=1}^m r_{R,i} \cos(1.75(w_{R,i}y + s_{R,i})) e^{-0.5(w_{R,i}y + s_{R,i})^2}, \\ \sum_{i=1}^m r_{C,i} \cos(1.75(w_{C,i}y + s_{C,i})) e^{-0.5(w_{C,i}y + s_{C,i})^2} \end{bmatrix},$$

$$[\hat{S}'(y), \hat{I}'(y), \hat{R}'(y), \hat{C}'(y)] = \frac{d}{dy} \begin{bmatrix} \sum_{i=1}^m r_{S,i} \cos(1.75(w_{S,i}y + s_{S,i})) e^{-0.5(w_{S,i}y + s_{S,i})^2}, \\ \sum_{i=1}^m r_{I,i} \cos(1.75(w_{I,i}y + s_{I,i})) e^{-0.5(w_{I,i}y + s_{I,i})^2}, \\ \sum_{i=1}^m r_{R,i} \cos(1.75(w_{R,i}y + s_{R,i})) e^{-0.5(w_{R,i}y + s_{R,i})^2}, \\ \sum_{i=1}^m r_{C,i} \cos(1.75(w_{C,i}y + s_{C,i})) e^{-0.5(w_{C,i}y + s_{C,i})^2} \end{bmatrix}. \quad (3)$$

A merit function is given as:

$$\Xi = \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 + \Xi_5, \quad (4)$$

$$\Xi_1 = \frac{1}{N} \sum_{i=1}^N [\hat{r}'_i - \mu + (\beta \hat{I}_i + \mu) \hat{S}_i - \gamma \hat{C}_i]^2, \quad (5)$$

$$\Xi_2 = \frac{1}{N} \sum_{i=1}^N [\hat{I}'_i - \beta \hat{S}'_i \hat{I}'_i + \mu \hat{I}'_i + \sigma \hat{I}'_i - \alpha \hat{R}_i]^2, \quad (6)$$

$$\Xi_3 = \frac{1}{N} \sum_{i=1}^N [(\hat{R}')_i + \alpha \hat{I}'_i - \beta(1 - \sigma) \hat{C}'_i \hat{I}'_i + (\mu + \sigma) \hat{R}'_i]^2, \quad (7)$$

$$\Xi_4 = \frac{1}{N} \sum_{i=1}^N [\hat{C}'_i - \delta \hat{R}'_i + (\gamma + \beta \hat{I}'_i + \mu) \hat{C}'_i]^2, \quad (8)$$

$$\Xi_5 = \frac{1}{4} [(\hat{S}'_0 - u_1)^2 + (\hat{I}'_0 - u_2)^2 + (\hat{R}'_0 - u_3)^2 + (\hat{C}'_0 - u_4)^2], \quad (9)$$

where $\hat{S}_i = S(y_i)$, $\hat{I}_i = I(y_i)$, $\hat{R}_i = R(y_i)$ and $\hat{C}_i = C(\mu_i)$. In Systems 5–8, Ξ_1, Ξ_2, Ξ_3 and Ξ_4 indicate the merit functions based on System (1), whereas Ξ_5 represents the merit function based on ICs.

2.2 Optimisation measures: MWNNs–GA–SQP

This section provides a detailed procedure of the designed MWNNs together with GA–SQP for solving NIDS. The designed MWNNs structure using GA–SQP for solving NIDS is shown in Figure 1.

GA was first applied by professor John Holland in 1975 [21] to present a simple representation of natural selection. GA grows with the population of applicant results. A genetic-based search initiates with a random (initial) population, then operators, as crossover, selection and mutation. It is applied one after another to get a new chromosome generation in which the projected excellence over all the chromosomes is improved over that of the preceding generation. This procedure is repeated till the termination standard is encountered, and the best values of the chromosomes of the final generation are described as the terminal solution. The evolutionary algorithms based on GA are broadly applied by researchers due to their capability of controlling the effectiveness, robustness, divergence-free, not to become fixed in local minima, consistent and efficient as compared with other mathematical heuristic solvers. Recently, GA is being applied in the network anomaly detection system [32], wellhead back pressure control system [33], optimising bank lending decisions [34], green vehicle routing systems [35], adaptive anomaly-based intrusion detection system [36], population initialisation with dispatching rules [37], heat conduction system [38], path planning in a dynamic field [39], Thomas–Fermi system [40] and non-linear HIV infection system [41].

SQP is one of the efficient, local search, speedy and rapid optimisation scheme generally applied to solve constrained/unconstrained systems. SQP is executed in numerous optimisation models of numerous complexes

Fig. 1 Proposed framework of MWNNs using the GA-SQP to solve the biological-based NIDS. GA, genetic algorithm; MWNNs, Morlet wavelet neural networks; SQP, sequential quadratic programming.

as well as non-stiff systems. Presently, it is used to investigate the guidewire deformation in blood vessels [42], in the power system stabiliser design [43], optimal control of rapid cooperative rendezvous [44], 3D deformable prostate model pose estimation in minimally invasive surgery [45], deterministic constrained production optimisation of hydrocarbon reservoirs [46], prediction differential system [47] and in the optimisation of an auxetic jounce bumper [48]. To switch the sluggishness of GA, hybridisation of the GA-SQP process is implemented along with the necessary steps, as provided in Table 1.

Table 1 Optimisation through MWNNs–GA–SQP to solve NIDS.

<p>Process of GA starts</p> <p>Inputs: The number of chromosome are selected as: $W = [u, w, b]$</p> <p>Population: Set of chromosomes are given as:</p> <p>$W = [W_S, W_I, W_R, W_C]$, for $W_S = [r_S, \omega_S, s_S]$, $W_I = [r_I, \omega_I, s_I]$, $W_R = [r_R, \omega_R, s_R]$ and $W_C = [r_C, \omega_C, s_C]$.</p> <p>Output: Global weights (GA) vectors are $W_{B.GA}$</p> <p>Initialisation: To select the variables, adjust the $W_{B.GA}$.</p> <p>Evaluation of FIT: Modify FIT (Ξ) in the population (P) for Eqs 4–9.</p> <p>Stopping process: Stop if [$\Xi = 10^{-21}$], [Generations = 100], [TolFun = 10^{-21}], [StallLimit = 130], [TolCon = 10^{-19}] & [PopSize = 180] achieved.</p> <p>Go to [storage]</p> <p>Ranking: Rank specific W in the selected population for Ξ.</p> <p>Storage: Save $W_{B.GA}$, Ξ, iterations, function counts and time.</p> <p>End of GA</p>
<p>SQP process starts</p> <p>Inputs: $W_{B.GA}$.</p> <p>Output: The best weight values of the GA–SQP are designated as $W_{GA.SQP}$.</p> <p>Initialise: Assignments, $W_{B.GA}$, iterations and other performances.</p> <p>Stopping standards: Terminate if one can achieve [$\Xi = 10^{-18}$], [TolFun = 10^{-21}], [Iterations = 500], [TolCon = TolX = 10^{-18}] & [MaxFunEvals = 180000].</p> <p>FIT Assessment: Compute Ξ and W for Eqs 4–9.</p> <p>Adjustments: Regulate ‘fmincon’ for SQP and Ξ for Eqs 4–9.</p> <p>Accumulate: Transform $W_{GA.SQP}$, time, iterations, Ξ and function counts.</p> <p>SQP End</p>

GA, genetic algorithm; Max, maximum; MWNNs, Morlet wavelet neural networks; NIDS, non-linear influenza disease system; SQP, sequential quadratic programming.

2.3 Performance operators

The mathematical notations using the statistical-based operators, including ‘mean absolute deviation (MAD)’, ‘variance account for (VAF)’, ‘Theil’s inequality coefficient (TIC)’ and ‘semi interquartile (S.I.R)’, along with

their global performances accessible to solve the biological based NIDS, are written as:

$$\left\{ \begin{aligned} [\text{VAF}_S, \text{VAF}_I, \text{VAF}_R, \text{VAF}_C] &= \begin{bmatrix} \left(1 - \frac{\text{var}(S_k - \hat{S}_k)}{\text{var}(S_k)}\right) \times 100, \\ \left(1 - \frac{\text{var}(I_k - \hat{I}_k)}{\text{var}(S_k)}\right) \times 100, \\ \left(1 - \frac{\text{var}(I_k - \hat{I}_k)}{\text{var}(I_k)}\right) \times 100, \\ \left(1 - \frac{\text{var}(R_k - \hat{R}_k)}{\text{var}(R_k)}\right) \times 100 \end{bmatrix}, \\ [\text{EVAF}_S, \text{EVAF}_I, \text{EVAF}_R, \text{EVAF}_C] &= \begin{bmatrix} 100 - \text{VAF}_S, 100 - \text{VAF}_I, \\ 100 - \text{VAF}_R, 100 - \text{VAF}_C \end{bmatrix}. \end{aligned} \right. \quad (10)$$

$$\left\{ \begin{aligned} \text{S.I.R} &= -\frac{1}{2} (Q_1 - Q_3), \\ Q_1 &= 1^{\text{st}} \text{ quartile} \ \& \ Q_3 = 3^{\text{rd}} \text{ quartile}, \end{aligned} \right. \quad (11)$$

$$[\text{TIC}_S, \text{TIC}_I, \text{TIC}_R, \text{TIC}_C] = \begin{bmatrix} \frac{\sqrt{\frac{1}{n} \sum_{k=1}^n (S_k - \hat{S}_k)^2}}{\left(\sqrt{\frac{1}{n} \sum_{k=1}^n S_k^2} + \sqrt{\frac{1}{n} \sum_{k=1}^n \hat{S}_k^2}\right)}, \frac{\sqrt{\frac{1}{n} \sum_{k=1}^n (I_k - \hat{I}_k)^2}}{\left(\sqrt{\frac{1}{n} \sum_{k=1}^n I_k^2} + \sqrt{\frac{1}{n} \sum_{k=1}^n \hat{I}_k^2}\right)}, \\ \frac{\sqrt{\frac{1}{n} \sum_{k=1}^n (R_k - \hat{R}_k)^2}}{\left(\sqrt{\frac{1}{n} \sum_{k=1}^n R_k^2} + \sqrt{\frac{1}{n} \sum_{k=1}^n \hat{R}_k^2}\right)}, \frac{\sqrt{\frac{1}{n} \sum_{k=1}^n (C_k - \hat{C}_k)^2}}{\left(\sqrt{\frac{1}{n} \sum_{k=1}^n C_k^2} + \sqrt{\frac{1}{n} \sum_{k=1}^n \hat{C}_k^2}\right)} \end{bmatrix}, \quad (12)$$

$$[\text{MAD}_S, \text{MAD}_I, \text{MAD}_R, \text{MAD}_C] = \begin{bmatrix} \frac{\sum_{k=1}^n |S_k - \hat{S}_k|}{\sum_{k=1}^n |R_k - \hat{R}_k|}, & \frac{\sum_{k=1}^n |I_k - \hat{I}_k|}{\sum_{k=1}^n |C_k - \hat{C}_k|} \end{bmatrix} \quad (13)$$

where, $\hat{S}$, $\hat{I}$, $\hat{R}$ and $\hat{C}$ represent the approximate forms of the solutions.

3 Simulations and numerical results

The comparative presentations of the obtained numerical results and the Runge–Kutta solutions are specialised to check the accuracy of the MWNNs–GA–SQP. Furthermore, the statistical representations are specified to check the precision, accuracy and reliability of the proposed scheme. The efficient form of NIDS using the suitable parameters is accessible as:

$$\left\{ \begin{aligned} S'(y) &= 0.02 - (50I(y) + 0.02)S(y) + 0.5C(y), & S(0) &= 0.8, \\ I'(y) &= 50(S(y) + 0.05C(y))I(y) - 73.02I(y), & I(0) &= 0.1, \\ R'(y) &= 73I(y) + 47.5C(y)I(y) - 1.02R(y), & R(0) &= 0.04, \\ C'(y) &= R(y) - (0.52 + 50I(y))C(y), & C(0) &= 0.06, \end{aligned} \right. \quad (14)$$

A merit function based on NIDS (14) is written as:

$$\begin{aligned} \Xi &= \frac{1}{N} \sum_{k=1}^N \left(\left[\hat{S}'_k - 0.02 + 50\hat{S}_k\hat{I}_k + 0.02\hat{S}_k - 0.5\hat{C}_k \right]^2 + \left[\hat{I}'_k - 50\hat{S}_k\hat{I}_k - 2.5\hat{C}_k\hat{I}_k + 73.02\hat{I}_k \right]^2 \right) \\ &\quad + \frac{1}{4} \left[(\hat{S}_0 - 0.8)^2 + (\hat{I}_0 - 0.1)^2 + (\hat{R}_0 - 0.04)^2 + (\hat{C}_0 - 0.06)^2 \right]. \end{aligned} \quad (15)$$

The demonstration of the performance is presented to solve NIDS using the designed MWNNs–GA–SQP for multiple trials along with 30 variations. The obtained solutions of NIDS are stated in the form of best weight

vector values, which are presented in Eqs 16–19. The graphical representations of these best weight vector values are illustrated in Figure 2.

$$\begin{aligned}\hat{S}(\Xi) = & -17.16 \cos(1.75(0.0090\Xi + 0.1085)) e^{-0.5(0.0090\Xi + 0.1085)^2} \\ & + 0.383 \cos(1.75(-17.533\Xi - 3.0806)) e^{-0.5(-17.53\Xi - 3.0806)^2} \\ & + 1.2417 \cos(1.75(8.6442\Xi + 4.1544)) e^{-0.5(8.6442\Xi + 4.1544)^2} \\ & - 3.7922 \cos(1.75(-11.40\Xi - 14.176)) e^{-0.5(-11.40\Xi - 14.176)^2} \\ & + 3.4256 \cos(1.75(0.2127\Xi - 2.2218)) e^{-0.5(0.2127\Xi - 2.2218)^2},\end{aligned}\quad (16)$$

$$\begin{aligned}\hat{I}(\Xi) = & -17.1643 \cos(1.75(-20\Xi - 2.5219)) e^{-0.5(-20.00\Xi - 2.5219)^2} \\ & + 0.3836 \cos(1.75(0.0461\Xi - 0.6859)) e^{-0.5(0.0461\Xi - 0.6859)^2} \\ & + 1.2417 \cos(1.75(1.4619\Xi + 0.8549)) e^{-0.5(1.4619\Xi + 0.8549)^2} \\ & - 3.4256 \cos(1.75(-9.906\Xi - 2.1412)) e^{-0.5(0.1054\Xi + 0.4437)^2} \\ & + 3.4256 \cos(1.75(2.3213\Xi - 6.8886)) e^{-0.5(-0.1424\Xi + 0.518)^2},\end{aligned}\quad (17)$$

$$\begin{aligned}\hat{R}(\Xi) = & -1.0285 \cos(1.75(16.848\Xi + 1.1773)) e^{-0.5(16.848\Xi + 1.1773)^2} \\ & + 1.7527 \cos(1.75(0.3206\Xi + 17.2099)) e^{-0.5(0.3206\Xi + 17.2099)^2} \\ & + 1.4105 \cos(1.75(17.2958\Xi + 1.5052)) e^{-0.5(17.2958\Xi + 1.5052)^2} \\ & - 1.6372 \cos(1.75(0.07120\Xi + 2.3310)) e^{-0.5(0.07120\Xi + 2.3310)^2} \\ & + 0.6731 \cos(1.75(-0.663\Xi - 12.5775)) e^{-0.5(-0.663\Xi - 12.5775)^2},\end{aligned}\quad (18)$$

$$\begin{aligned}\hat{C}(\Xi) = & -1.6283 \cos(1.75(0.3292\Xi - 2.1560)) e^{-0.5(0.3292\Xi - 2.1560)^2} \\ & - 6.6645 \cos(1.75(-0.0350\Xi - 0.4678)) e^{-0.5(-0.035\Xi - 0.4678)^2} \\ & - 2.9774 \cos(1.75(0.0916\Xi + 2.47260)) e^{-0.5(0.0916\Xi + 2.47260)^2} \\ & - 0.4609 \cos(1.75(-4.359\Xi + 13.907)) e^{-0.5(-4.3590\Xi + 13.9070)^2} \\ & - 0.1666 \cos(1.75(1.53431\Xi + 2.5169)) e^{-0.5(1.53431\Xi + 2.5169)^2},\end{aligned}\quad (19)$$

The obtained outputs are calculated using Eqs 16–19 within the range of 0–1, to indicate the numerical outcomes for each group of NIDS. Figure 2(a–d) illustrates the weight vector plots based on the best solutions to solve NIDS. A comparison of the best and mean solutions using MWNNs–GA–SQP with the reference Runge–Kutta results are presented in Figure 2(e–h) for solving NIDS. It is indicated in these plots that the obtained results through MWNNs–GA–SQP overlapped with the reference solutions for each group of NIDS. This overlapping of the results indicates the excellence and precision of the designed MWNNs–GA–SQP. The AE plots for each group of NIDS are provided in Figure 3. One can find that the best values of the AE for the groups susceptible $S(y)$, infected $I(y)$, recovered $R(y)$ and cross-immune $C(y)$ based on NIDS is found to be around 10^{-02} – 10^{-03} , 10^{-02} – 10^{-04} , 10^{-04} – 10^{-05} and 10^{-02} – 10^{-03} , respectively, whereas the mean values of the AE for the groups $S(y)$, $I(y)$, $R(y)$ and $C(y)$ are found to be around 10^{-01} – 10^{-02} , 10^{-01} – 10^{-03} , 10^{-01} – 10^{-04} and 10^{-01} – 10^{-03} , respectively. Figure 4 signifies the convergence performances in terms of EVAF, MAD and TIC statistical operators for solving each group of NIDS. It is observed that the convergence performances of the best EVAF, MAD and TIC statistical operator of the $S(y)$ group lie around 10^{-02} – 10^{-03} , 10^{-01} – 10^{-02} and 10^{-05} – 10^{-06} , respectively. The best values of the $I(y)$ group lie around 10^{-02} – 10^{-03} , 10^{-03} – 10^{-04} and 10^{-05} – 10^{-06} for the operators EVAF, MAD and TIC, respectively. The best values of the $R(y)$ group lie around 10^{-01} – 10^{-02}

(a) Best weights of $S(y)$ (e) Results of $S(y)$ (b) Best weights of $I(y)$ (f) Results of $I(y)$ (c) Best weights of $R(y)$ (g) Results of $R(y)$ (d) Best weights of $C(y)$ (h) Results of $C(y)$

Fig. 2 Best weight vectors set and best/mean results comparison with reference solutions to solve NIDS. NIDS, non-linear influenza disease system.

(a) AE values of group $S(y)$ (b) AE values of group $I(y)$ (c) AE values of group $R(y)$ (d) AE values of group $C(u)$

Fig. 3 Best and mean values of the AE for each group of NIDS. AE, absolute error; NIDS, non-linear influenza disease system.

Fig. 4 Statistical performances based on EVAF, MAD and TIC operators for solving NIDS. MAD, mean absolute deviation; NIDS, non-linear influenza disease system; TIC, Theil's inequality coefficient.

Fig. 5 Convergence performances based on TIC, EVAF and RMSE operators for solving NIDS. NIDS, non-linear influenza disease system.

Table 2 Statistical performances for NIDS-based $S(y)$.

y	$S(y)$					
	Max	Min	Med	Mean	S.I.R	STD
0	9.97407E−02	6.23657E−06	9.67647E−02	3.15861E−03	6.56937E−04	1.11176E−03
0.1	3.82797E−03	2.46073E−03	2.63760E−03	3.28652E−02	3.74939E−04	1.50172E−01
0.2	1.59218E−03	2.35321E−03	2.86086E−04	3.36978E−02	4.41197E−03	1.41380E−01
0.3	7.52762E−04	2.33924E−03	1.43482E−04	3.35508E−02	7.04131E−03	1.27429E−01
0.4	1.17681E−03	1.74469E−03	1.49107E−04	3.20838E−02	8.72803E−03	1.14515E−01
0.5	8.45225E−04	1.43288E−03	8.54966E−05	3.07057E−02	9.01342E−03	1.02999E−01
0.6	9.64376E−04	6.38340E−04	6.69333E−05	2.97428E−02	9.36168E−03	9.27501E−02
0.7	1.03468E−03	1.08089E−03	8.11832E−05	2.88456E−02	9.76687E−03	8.35818E−02
0.8	1.04732E−03	9.68113E−04	6.50947E−05	2.78143E−02	1.02683E−02	7.53055E−02
0.9	9.93449E−04	6.87790E−04	6.98467E−05	2.65912E−02	1.08763E−02	6.77636E−02
1	8.66289E−04	3.87491E−04	6.32949E−05	2.51101E−02	1.15130E−02	6.08310E−02

Max, maximum; Med, median; Min, minimum; NIDS, non-linear influenza disease system; S.I.R, semi interquartile.

Table 3 Statistical performances for NIDS-based $I(y)$.

y	$I(y)$					
	Min	Max	Med	Mean	S.I.R	SD
0	1.07524E−02	1.13718E−05	6.43473E−04	1.84408E−03	1.81226E−02	1.12267E−03
0.1	1.02042E−01	1.55703E−04	9.97639E−02	2.50909E−03	2.90364E−04	7.08815E−03
0.2	1.03305E−01	2.36329E−03	1.01710E−01	7.67239E−03	9.27342E−05	2.05399E−02
0.3	1.01679E−01	2.59357E−04	1.00308E−01	1.30883E−02	1.08303E−04	3.23014E−02
0.4	1.00068E−01	1.83367E−04	9.69585E−02	1.72975E−02	6.27781E−05	4.25479E−02
0.5	9.73448E−02	1.67571E−03	9.31885E−02	2.05823E−02	5.26592E−05	5.12053E−02
0.6	9.38826E−02	1.71384E−03	9.00952E−02	2.32492E−02	5.88914E−05	5.83514E−02
0.7	8.99105E−02	1.69484E−03	8.65303E−02	2.55060E−02	5.75666E−05	6.41060E−02
0.8	8.55744E−02	1.52268E−03	8.22485E−02	2.74916E−02	5.39135E−05	6.85933E−02
0.9	8.09763E−02	1.14571E−03	7.80244E−02	2.93020E−02	4.83546E−05	7.19385E−02
1	7.61938E−02	5.69917E−04	7.35171E−02	3.10092E−02	3.62539E−05	7.42639E−02

Max, maximum; Med, median; Min, minimum; NIDS, non-linear influenza disease system; S.I.R, semi interquartile.

for the operators EVAF and MAD, while for the TIC operator, these values lie around 10^{-04} – 10^{-05} . The best values of the $C(y)$ group lie around 10^{-02} – 10^{-03} , 10^{-01} – 10^{-03} and 10^{-05} – 10^{-06} for the operators EVAF, MAD and TIC, respectively. These encouraging suggestions confirm the correctness of the MWNNs–GA–SQP for solving each group of NIDS.

The graphical plots based on the statistical measures to authorise the performance of convergence are given in Figure 5 to solve NIDS. The performance of the TIC operator using multiple executions to solve NIDS. It is observed the maximum (Max) number of trials based on the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups of the TIC operators are found around 10^{-04} – 10^{-06} , 10^{-05} – 10^{-06} , 10^{-03} – 10^{-06} and 10^{-05} – 10^{-06} , respectively. The performances of the EVAF values for the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups lie around 10^{-05} – 10^{-07} . The performances of the EVAF values for the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups lie around 10^{-01} – 10^{-04} . These best values of the executions based on the MWNNs–GA–SQP are found suitable based on the TIC, EVAF and MAD operators for solving NIDS.

The statistical representations are provided in Tables 2–5, based on the operators Minimum (Min), standard deviation (STD), Mean, Max, Median (Med) and S.I.R to authenticate the precision and accurateness for

Table 4 Statistical performances for NIDS-based $R(y)$.

y	$R(y)$					
	Max	Min	Med	Mean	S.I.R	STD
0	6.92243E-03	5.64028E-05	2.11655E-04	3.15861E-03	9.26468E-04	7.81009E-02
0.1	1.51990E-02	8.69332E-03	7.09041E-03	3.28652E-02	1.78168E-02	2.56713E-03
0.2	2.92093E-02	5.01355E-03	2.44201E-02	3.36978E-02	1.51010E-02	3.86235E-04
0.3	4.14911E-02	5.07207E-03	3.94198E-02	3.35508E-02	1.34338E-02	1.77153E-04
0.4	5.40406E-02	4.99083E-03	5.19292E-02	3.20838E-02	1.28378E-02	2.18649E-04
0.5	6.43792E-02	4.44331E-03	6.24911E-02	3.07057E-02	1.23623E-02	1.43347E-04
0.6	7.31617E-02	4.24610E-03	7.10555E-02	2.97428E-02	1.20395E-02	1.42089E-04
0.7	8.03111E-02	4.63968E-03	7.80977E-02	2.88456E-02	1.17625E-02	1.55918E-04
0.8	8.60489E-02	5.44280E-03	8.35771E-02	2.78143E-02	1.16541E-02	1.16323E-04
0.9	9.05767E-02	6.39784E-03	8.76171E-02	2.65912E-02	1.14688E-02	1.19646E-04
1	9.40773E-02	7.34102E-03	9.11587E-02	2.51101E-02	1.11275E-02	1.06769E-04

Max, maximum; Med, median; Min, minimum; NIDS, non-linear influenza disease system; S.I.R, semi interquartile.

Table 5 Statistical performances for NIDS-based $C(y)$.

y	$C(y)$					
	Max	Min	Med	Mean	S.I.R	STD
0	1.48512E-02	2.50654E-03	1.85049E-04	1.84408E-03	3.85021E-04	2.18319E-03
0.1	1.90381E-01	2.03580E-04	1.86341E-01	2.50909E-03	3.27146E-02	8.05712E-02
0.2	1.77459E-01	1.05623E-04	1.74888E-01	7.67239E-03	2.46695E-02	8.27583E-02
0.3	1.60388E-01	7.96272E-06	1.58166E-01	1.30883E-02	2.07562E-02	8.11572E-02
0.4	1.46077E-01	5.88240E-06	1.41058E-01	1.72975E-02	1.89140E-02	7.90228E-02
0.5	1.32592E-01	5.20223E-07	1.26230E-01	2.05823E-02	1.73278E-02	7.64063E-02
0.6	1.19829E-01	3.96760E-07	1.14065E-01	2.32492E-02	1.59848E-02	7.32956E-02
0.7	1.08056E-01	1.44197E-06	1.03949E-01	2.55060E-02	1.49896E-02	6.99375E-02
0.8	9.73313E-02	1.37818E-06	9.40407E-02	2.74916E-02	1.37403E-02	6.64160E-02
0.9	8.76205E-02	1.07879E-06	8.53022E-02	2.93020E-02	1.25955E-02	6.27128E-02
1	7.88533E-02	3.73973E-07	7.71953E-02	3.10092E-02	1.09303E-02	5.88248E-02

Max, maximum; Med, median; Min, minimum; NIDS, non-linear influenza disease system; S.I.R, semi interquartile.

solving each group of NIDS. The Max values for the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups based on NIDS lie around 10^{-02} – 10^{-04} , 10^{-01} – 10^{-02} , 10^{-02} – 10^{-03} and 10^{-01} – 10^{-02} . The Min values for the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups based on NIDS lie around 10^{-03} – 10^{-06} , 10^{-03} – 10^{-05} , 10^{-03} – 10^{-05} and 10^{-04} – 10^{-07} . The Med values for the $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups based on NIDS lie around 10^{-02} – 10^{-05} , 10^{-02} – 10^{-04} , 10^{-03} – 10^{-04} and 10^{-01} – 10^{-04} . The Mean, S.I.R and STD values for $S(y)$, $I(y)$, $R(y)$ and $C(y)$ groups based on NIDS lie around 10^{-02} – 10^{-03} , 10^{-02} – 10^{-04} and 10^{-03} – 10^{-04} . These small calculated values designate the worth and performance of MWNNs–GA–SQP to solve each group of NIDS. One can find through these attained measures that the proposed MWNNs–GA–SQP is stable, precise and accurate.

The global operators of [G-TIC], [G-MAD] and [G-EVAF] for multiple independent trials to solve NIDS using the proposed MWNNs–GA–SQP are tabulated in Table 6. These global performances based on Mean lie around 10^{-02} – 10^{-03} , 10^{-06} – 10^{-07} and 10^{-01} – 10^{-02} , whereas the global S.I.R performances are found around 10^{-02} to 10^{-03} , 10^{-07} – 10^{-08} and 10^{-02} – 10^{-03} for each group of NIDS. These ideal close performances attained through global measures show the precision, accuracy and correctness of the designed MWNNs–GA–SQP for solving all the groups of NIDS.

Table 6 Global operators based on TIC, MAD and EVAF values to solve NIDS.

Index	(G-MAD)		(G-TIC)		(G-EVAF)	
	Mean	S.I.R	Mean	S.I.R	Mean	S.I.R
$S(y)$	6.66623E–02	1.60694E–03	2.89392E–06	3.44759E–08	7.20734E–02	2.11643E–02
$I(y)$	7.47583E–03	1.14516E–02	9.71475E–07	7.52206E–07	7.09781E–01	2.08050E–03
$R(y)$	9.25308E–02	7.73955E–03	4.16727E–06	2.24806E–07	6.77949E–02	2.90708E–03
$C(y)$	4.47326E–02	1.63317E–02	2.11608E–06	5.07713E–07	8.00158E–01	2.75884E–02

MAD, mean absolute deviation; NIDS, non-linear influenza disease system; S.I.R, semi interquartile.

4 Conclusions

This current work is related to solve NIDS by exploiting MWNNs using the optimisation procedures of the hybrid process of global/local search approaches. The GA as a global approach and SQP as a local search approach have been implemented as an optimisation procedure to solve the non-linear biological model. The influenza model is based on four groups: susceptible $S(y)$, infected $I(y)$, recovered $R(y)$ and cross-immune $C(y)$; the numerical results of each group of the influenza system are presented using the proposed MWNNs–GA–SQP. The Morlet wavelet function is exploited the first time to solve the non-linear influenza system using 15 numbers of variables together with the optimisation procedures of GA–SQP. For the exactness of the MWNNs–GA–SQP, a comparison of the proposed results with the reference solutions is performed. The overlapping of the results around 4–5 decimal places is noticed as a result of this comparison and this accuracy develops in the consistency of the designed MWNNs–GA–SQP. The particular performances of the MWNNs–GA–SQP through the statistical RMSE, TIC, EVAF operators are observed for 40 trials to solve NIDS. These statistical operator performances are accomplished based on the higher precision level to solve NIDS. Moreover, statistics by Max, Min, STD, S.I.R, Med and Mean were further validated to solve NIDS using the proposed MWNNs–GA–SQP. The global performances of the operators are found to be in good measure through the Mean and S.I.R operators, which prove the validity and correctness of the MWNNs–GA–SQP for solving NIDS.

In future, the proposed MWNNs–GA–SQP can be used to solve the systems of higher order models, fluids problems and non-linear biological systems [49–58].

Conflicts of interest

The authors declare no potential conflicts of interest.

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