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## Badminton players' trajectory under numerical calculation method

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### Abstract

The trajectory tracking of badminton players' arm shots can be used effectively to enhance the player's shot quality. To track the image trajectory of the batting arm, it is necessary to calculate the body posture ratio and tightness of the target area of the batting arm, to filter the background interference of the image segment of the batting and to complete the efficient tracking of the trajectory of the arm batting image. The traditional method combines the adaptive threshold segmentation method to extract the hitting arm target from the background, but ignores filtering out the background interference of the hitting image fragment. This paper proposes a trajectory tracking method based on the morphological operator of the batter image of the arm stroke. The method consists of (1) differentially calculating the image sequence of the hitting arm during two consecutive shots, (2) estimating the Gaussian model parameters of the differential image of the hitting arm during the hitting process, (3) extracting the outline of the moving target of the hitting arm during the hitting process and then calculating the body posture ratio of the hitting target area and compactness, (4) filtering the background interference of the shot image fragment, (6) constructing a global matching approximation function of the moving target and (7) finally determining the motion trajectory of the badminton arm of the batter. Simulation results show that the proposed method can effectively track the target of the hitting arm during the hitting process and generate a continuous trajectory of the hitting arm.

**Keywords:** badminton, arm hitting, trajectory tracking

**AMS 2010 codes:** 11Y99

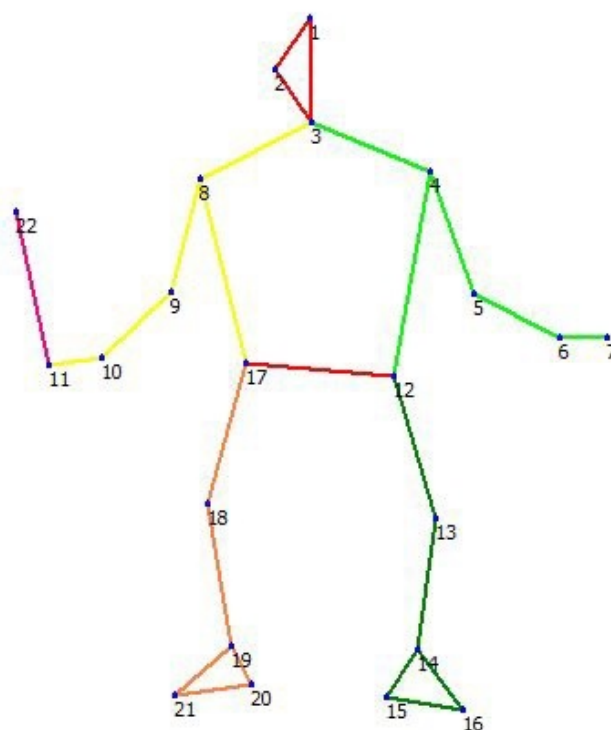
## 1 Introduction

With the continuous innovation of the technical and tactical level of competitive badminton and the continuous improvement of performance in/from computer simulation, higher requirements have been put forth on the abilities of badminton players. Experts believe that from the perspective of sports mechanics and technical

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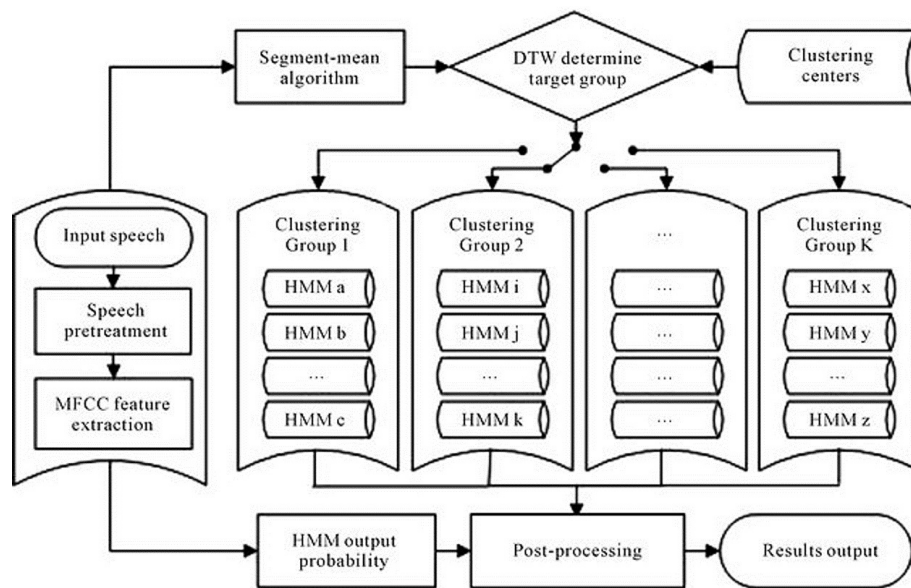
characteristics of badminton sports, the batting arm has the functions of maintaining body balance, improving body movement speed and assisting in exertion of force compared with the entire sports process of the athlete during the batting process. By studying the role and trajectory of the batting arm in the process of badminton, it can provide scientific and effective training methods for badminton enthusiasts on the one hand, and help to improve and develop the technical and tactical theory of badminton players on the other. When conducting in-depth analysis of the trajectory tracking of the arm shot image, it is necessary to track the motion target of the shot arm. However, most current methods are difficult to extract the active contour of the batting arm during the tracking of the batting arm, and there is a problem of large tracking error. In this case, how to effectively extract the contour of the target of the batting arm's movement and how it affects the entire stroke of the badminton player have been studied. The real-time and accurate tracking of arm batting image trajectory tracking during the ball process has become a major problem to be solved in the current sports field and has attracted widespread attention. Figure 1 shows a 21-point human joint model of Hideki Matsui, Japan [1].



**Fig. 1** Black point human joint model of Matsui Japan

At present, the badminton player's arm trajectory tracking method during the ball hitting process includes: The literature proposes a method based on HMM clustering for the badminton player's arm trajectory tracking method. This method first trains an HMM model for each motion trajectory of the battering arm of the badminton player, then calculates the distance between the pair of arm targets and blurs the corresponding trajectory characteristics of the motion behaviour of the batting arm after dimensionality reduction. Figure 2 shows the basic method steps of HMM clustering. Finally, Hidden Markov model is used to accurately predict the trajectory of the player's hitting arm during the hitting process. This method can obtain the complete trajectory of the batting arm, but cannot guarantee the accuracy of the target association of the batting arm. The literature proposes a method for trajectory analysis of arm shot images of badminton players based on adaptive threshold segmentation [2,3]. This method first uses the principle of the hexagonal vertebral body model and an adaptive threshold

segmentation method to extract the target of the hitting arm of the badminton player from the background, and calculates and predicts the motion trajectory of the hitting arm during the hitting process. This method is difficult to deal with the problem when the hitting arm is completely blocked leading to the poor tracking effect of the hitting arm. The literature proposes a method for tracking analysis of the image of badminton shots of a badminton player during a shot with a low-view camera. This method first uses the information/parameters of the badminton player's hitting arm to obtain the three-dimensional information of the feature points, and then distinguishes the stable feature points from the unstable feature points with the height of the hitting arm's hitting process. Tracking is performed to complete the trajectory tracking of the batting arm. This method has fast tracking efficiency, but it is difficult to determine the state of the moving target of the hitting arm [4].



**Fig. 2** Basic method steps of HMM clustering

In order to solve the problems existing in the traditional method, a trajectory tracking method of arm shot image of badminton player based on morphological operator is proposed. The experimental results show that the proposed method can effectively track the target of the hitting arm during the hitting process, and the tracking effect is better.

## 2 Tracking principle of badminton player's arm shot image

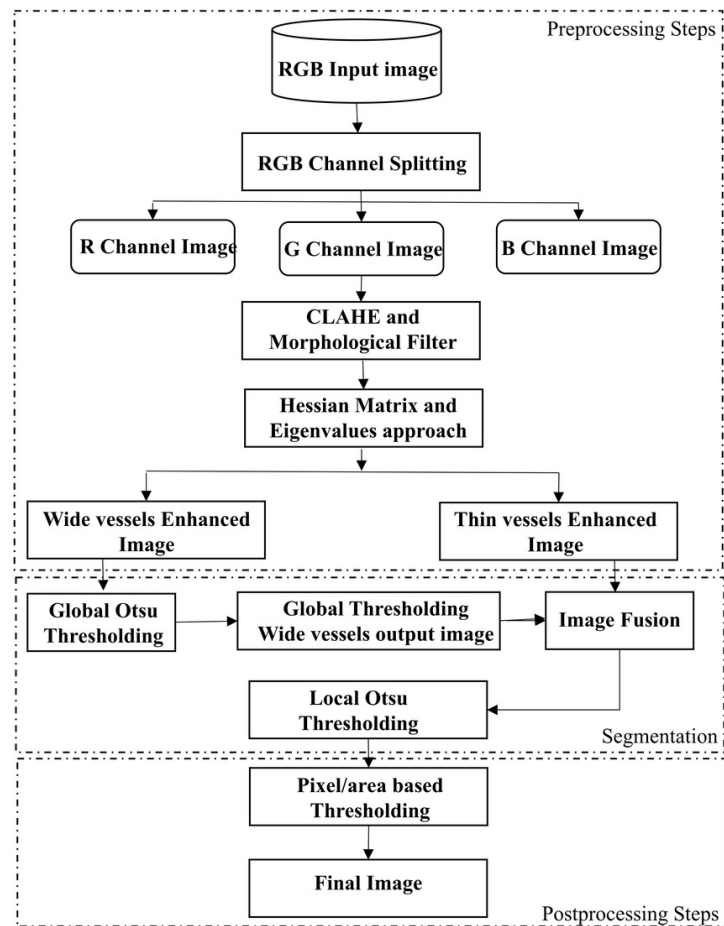
During the trajectory tracking of the arm hitting image during the badminton stroke, the image processing and matching technology are first used to extract the position coordinates and velocity components of the badminton player's hitting arm on the two-dimensional plane, are calculated. Using the theory of minimum variance estimation and the analysis of the motion trajectory relationship of the hitting arm, the recursive relationship of the motion trajectory is found and a Kalman filter state estimation model is obtained to complete the estimation of the motion trajectory of the hitting arm of the player during the stroke. The specific process is described in the following text. Figure 3 shows that adaptive threshold segmentation algorithm [5].

Suppose that  $(x_{k-1}, y_{k-1})$  and  $(x_k, y_k)$  represent the position coordinates of the badminton player's hitting arm at the  $k-1$  and  $k$  moments in the two-dimensional plane X-axis and Y-axis directions, respectively, and  $(v_x, v_y)$  represents the hitting arm in the two-dimensional plane X axis. Figure 4 shows that Algorithm simulation diagram and Y axis of the figure represents velocity components;  $t$  represents the sampling period, then the estimated relationship between the motion trajectory of the badminton arm during the stroke of the badminton

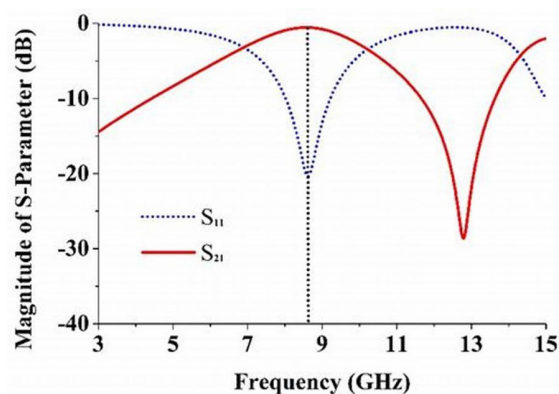
player can be expressed as

$$\begin{cases} x_k = x_{k-1} + tv_x \\ y_k = y_{k-1} + tv_y \end{cases} \quad (1)$$

Based on the recursive type of the trajectory of the hitting arm of the badminton player during the hitting process, a Kalman filtering state estimation model of the hitting arm movement system of the player during the hitting



**Fig. 3** Adaptive threshold segmentation algorithm



**Fig. 4** Algorithm simulation diagram

process can be stated as follows

$$x_k' = Ax_{k-1} + B \quad (2)$$

In the formula,  $x_k'$  represents the state vector of the motion system of the battering arm predicted by the badminton player at the time of  $k$ ,  $B$  represents the input matrix, and  $A$  represents the transition matrix of the motion state during the stroke of the battering arm. Badminton player is given by Eq. (3) Partial error covariance equation of the trajectory of the batting arm during the batting process [6]

$$P_k' = AP_{k-1}AQ_{k-1} \quad (3)$$

In the formula,  $P_k'$  and  $P_{k-1}$  represents the error covariance matrices of  $x_k'$  and  $x_{k-1}$ , respectively and  $Q_{k-1}$  represents the state correction equation of the prediction and correction part of the motion trajectory of the batting arm at time  $k - 1$ .

$$x_k = x_k' + K_k(z_k - H_k x_k') \quad (4)$$

In the formula,  $z_k$  represents the vector of the arm movement system of the athlete during the stroke. The image matching technology is used to obtain the target position and speed information of the arm of the badminton athlete. Then, the coordinate position of the hitting arm is obtained by  $z_k = (x\omega_k, y\omega_k, xv_k, yv_k)^T$  characterisation.  $H_k$  represents the observation matrix of the system. The Kalman filter gain coefficient  $K_k$  of the correction part is given by Equation (5).

$$K_k = P_k' H_k (H_k P_k' H_k + R_k)^{-1} \quad (5)$$

In the formula,  $R_k$  represents the covariance matrix of the observed noise of the badminton player's hitting arm system at the time of  $k$ ,  $\omega_k$  and  $v_k$  and represent the zero-mean Gaussian white noise vector independent of each other. The covariance of the correction part correction equation is obtained by using the formula (6), which is stated as follows:

$$P_k = (I - K_k H_k) P_k' \quad (6)$$

In the formula,  $P_k$  represents the error covariance matrix of the estimated motion trajectory of the badminton player at the time of  $k$ , and  $I$  represent the identity matrix [7].

To sum up, it can be explained that the principle of trajectory tracking of the arm hitting image of badminton players can be used to efficiently track the trajectory of the arm striking image.

### 3 Tracking method of badminton player's arm shot image

#### 3.1 Arm movement target detection during athlete's batting process

During the trajectory tracking of the arm shot image of the badminton player during the shot, a differential calculation on the image sequence of the shot arm during two consecutive shots is performed first, and a Gaussian model for the gray distribution of the difference image is established, and then the expectation maximisation algorithm is used to estimate Gaussian model parameters of the differential image of the batting arm during the batting process and the a boundary detection operator is introduced to construct the boundary image of the batting arm movement during the batting process. The outline of the badminton player's batting arm movement target is extracted [8].

First, a Gaussian mixture model was used to model the difference image of the battering arm of the badminton player's batting process, and a boundary detection operator was introduced to construct a new batting arm boundary detection image.

$$D(x) = I(x, t + 1) - I(x, t) \quad (7)$$

In the formula,  $I(x, t + 1)$ ,  $I(x, t)$  represents the difference image of the gray image of the batter arm of the badminton player at the time of  $t$  and  $t + 1$ , respectively, and  $x$  represents the image pixel.

Assume that  $p_{D|s}(d|s), p_{D|m}(d|m)$  represents the conditional distribution of the difference image gray level when the image of the hitting arm of the original badminton player is the background and the target conditions. Set the gray distribution of the differential image of the hitting arm during the impact process to the mixed distribution  $P_D(d)$  of

$$\begin{aligned} p_{D|s}(d|s), p_{D|m}(d|m) \\ P_D(d) = P_s p_{D|s}(d|s) + P_m p_{D|m}(d|m) \end{aligned} \quad (8)$$

In the formula,  $P_s, P_m$  represents the prior probability that the hitting arm belongs to the background and the target, and  $m$  represents the corresponding dimension of the histogram. The expectation maximisation algorithm is used to estimate the parameter  $P_s, P_m$  in Equation (8), assuming that the player hits the ball. The pixel  $x_1$  of the hitting arm differential image belongs to the background or moving target, and at least one pixel  $x_2$  of the neighbouring pixels of the difference image belongs to the hitting arm moving target or background, then it is defined as the boundary of the hitting arm target during the hitting process, assuming that differential image pixels  $x_1$  and  $x_2$  belong to the background or moving target [9], and are defined as non-boundary. The following two energy terms  $E_t(x_1, x_2), E_s(x_1, x_2), E_t$  are introduced to characterise the extent to which  $x_1$  belongs to the boundary of the battering arm of the badminton player, and  $E_s$  indicates that  $x_1$  is non-boundary. The degree of the boundary, according to the above-mentioned energy term, the function of detecting the boundary of the battering arm during the battering process is defined by

$$F(x) = \max \{E_t(x_1, x_2)/E_s(x_1, x_2)\}; x_2 \in n(x) \quad (9)$$

In the formula,  $n(x)$  represents the 8-connected neighbourhood of the differential image pixel  $x$  of the battering arm during the battering process. The active contour model is used to construct the boundary image of the athlete's battering arm during the battering process, and then the contour of the badminton player's battering arm movement target is extracted. The active contour model is defined as a curve  $x(s)' = [x(s), y(s)]$ , and the badminton battering player's battering arm image during the battering process move on plane  $I(x, y)$  to minimise its total energy RE

$$E = \int_0^1 [E_{\text{int}}(x(s)') \alpha + E_{\text{ext}}(x(s)') \beta] P_s \quad (10)$$

In the formula,  $E_{\text{int}}(x(s)')$  represents its internal energy,  $\alpha$  and  $\beta$  respectively represent the elasticity and smoothness coefficient of the control curve itself, and  $E_{\text{ext}}(x(s)')$  represents the external energy, that is,

$$E_{\text{ext}}(x(s)') = -|\nabla [G_\sigma(x, y) * I(x, y)]|^2 \quad (11)$$

In the formula,  $G_\sigma(x, y)$  represents the Gaussian function of the standard deviation of the battering arm image as  $\sigma$ ,  $\nabla$  represents the gradient operator, and  $*$  represents the convolution operation.

### 3.2 Trajectory tracking of arm hitting images based on morphological operators

During the trajectory tracking of the arm shot image of the badminton player, after analysing the connectivity of the foreground image in Section 3.1, a morphological operator was used to calculate the body posture ratio and tightness of the target area of the shot arm of the player during the shot. The badminton player's shot image fragment background interferer, according to the position and size of the hitting arm target during the hitting process, constructs the global matching approximation function of the moving target, thereby realising the badminton player's hitting arm movement trajectory optimisation during the hitting process. The specific process is as follows: Description [10, 11]:

The background of the moving target obtained by the segmentation processing of the image of the badminton arm of the badminton player during the hitting process usually has a background interferer. The body interferer



and closeness of the morphological operator are used to filter the background interferer of the shot arm image.

$$\begin{cases} R = H/W \\ C = A/P^2 \end{cases} \quad (12)$$

In the formula,  $R$  stands for the posture ratio,  $H$  and  $W$  stand for the height and width of the target contour of the hitting arm, respectively.  $C$  stands for the tightness value and  $P$  stands for the area perimeter. When setting up the colour histogram of the badminton player's motion target of the battering arm, choose a smaller weight for the pixels that are far away from the centre of the outline of the battering arm and a larger value for the pixels that are closer to the centre of the outline of the battering arm. Weights to obtain an improved histogram model of target colour for batting arm movement

$$\begin{cases} \omega^v = \left( 1 / \sum_{i=1}^s \eta(r^i) \right) \sum_{i=1}^s \eta(r^i) \delta(h(\chi^i) - v) \\ r^i = \text{dis}(\chi^i, o) / d_{\max} \end{cases} \quad (13)$$

In the formula,  $\omega^v$  represents the corresponding  $v$  component of the colour histogram  $\omega$ , and represents the proportion of pixels with a pixel value of  $v$  in the outline of the hitting arm target during the whole shot process. Total points,  $\eta(r^i)$  represents the weight function,  $\delta$  represents  $\omega^v$  which only counts the pixels whose pixel value is  $v$ ,  $\chi^i$  represents the  $i$  pixel point in the contour target of the hitting arm,  $h(\chi^i)$  represents the pixel value of  $\chi^i$  and  $o$  represents the contour target of the striking arm during the stroke. At the centre point,  $\text{dis}(\chi^i, o)$  is the distance between  $\chi^i$  and  $o$ ,  $d_{\max}$  is the maximum value of  $\text{dis}(\chi^i, o)$  and  $r^i$  is the ratio of  $\text{dis}(\chi^i, o)$  and  $d_{\max}$ . Equation (14) below is used to calculate the histogram approximation of the target colour of the batting arm outline

$$\psi(\omega, \xi) = \sum_{v=1}^V \sqrt{\omega^v \xi^v} \quad (14)$$

In the formula,  $\omega$ ,  $\xi$  represents the two colour histograms of the badminton player's hitting arm movement target to be matched during the batting process, and  $\xi^v$  represents the  $v$  component of  $\xi$ , thereby forming a global badminton player's hitting arm movement target during the hitting process using Matching approximation function:

$$\text{sim}(a, b) = \alpha \varphi(a, b) + \beta \phi(a, b) + \gamma \psi(a, b) \quad (15)$$

$$\varphi(a, b) = (d_a + d_b) / (d_a + d_b + \text{dis}(a_o + b_o)) \quad (16)$$

$$\phi(a, b) = (2S_a S_b) / (S_a^2 + S_b^2) \quad (17)$$

In the formula,  $\varphi(a, b)$  represents the approximate degree of the centre position of the moving target of the hitting arm to be matched, and  $\phi(a, b)$  represents the

The batting arm movement target contains the approximation of the total number of pixels,  $\alpha$ ,  $\beta$ ,  $\gamma$  represents the weighting coefficient of  $\varphi(a, b)$ ,  $\phi(a, b)$ ,  $\psi(a, b)$  respectively, and  $a$  and  $b$  represent the two batting arm movement target feature points to be matched.  $d_a$ ,  $d_b$  represent the radius of the batting arm movement targets  $a$  and  $b$ , respectively,  $\text{dis}(a_o + b_o)$  represents the distance between the centres of  $a_o$ ,  $b_o$  and  $S_a$ ,  $S_b$  represents the total number of pixels in the batting arm movement targets  $a$  and  $b$ , respectively.

The badminton players' stage/action on the moving target during the hitting process obtained by the above detection is matched with the currently detected battering arm moving target, and the global matching method is used to realise the tracking and trajectory of the badminton player's hitting target analysis.

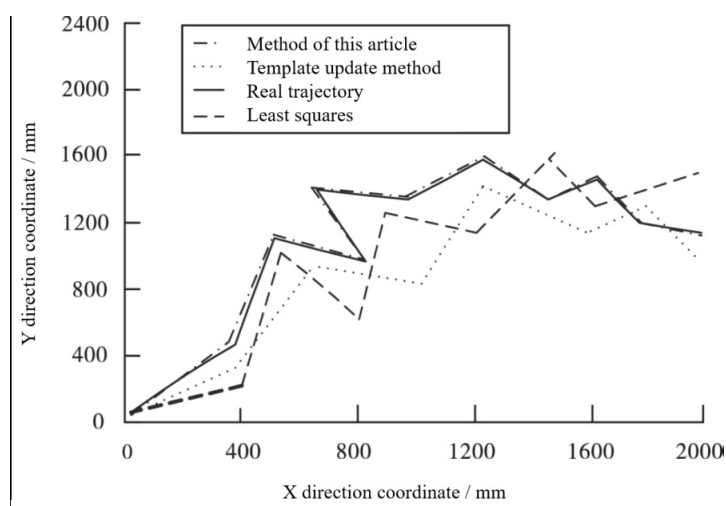
#### 4 Simulation proof

In order to prove the effectiveness of the trajectory tracking method based on the morphological operator of the badminton player's ball hitting process, a simulation is needed. The experimental subject selects 15 outstanding male athletes from Capital Institute of Physical Education. For 8 years, these athletes had no sports injuries in the experimental stage, and they were proficient in the basics of badminton batting. The camera is used to shoot the entire shot process. Within 5 m of the shooting range, the marked points are affixed to the shot arm to obtain a relatively complete shot action for each athlete, a total of 50 images. The simulation mainly adopts different methods in the MATLAB software environment under Windows 7 system to realise the detection and tracking of the badminton player's shot arm image target during the shot process.



**Fig. 5** Image of the player's batting process

Firstly, the error curve of the coordinate position of the hitting arm and the true position of the badminton player during the stroke is given in this paper, template updating method and least square linear method used is shown in Figure 6.



**Fig. 6** Tracking error curve of arm coordinate position in different methods

Analysis of Figure 6 shows that the corresponding coordinate positions of the template update method and the least squares linear method deviate greatly from the true positions, and the corresponding tracking error curve of the hitting arm shows that. But the coordinate position of the method used in this article has a small



error, indicating that this method is closer to the trajectory of the batting arm in the real batting process, and the tracking effect is better.

The methods, the template updating method and the least squares linear method are also used to track the moving target of the badminton player's hitting arm in our paper. The tracking success rates (%) of our method with the three different methods are compared. The comparison results are shown in Table 1, where the tracking success rate = accurate tracking of the total number of pixels (N)/the total number of pixels of the hitting arm target in the target area.

**Table 1** Comparison of target tracking success rates of different hitting arm

Method	Number of image pixels	Target total number of prime points/a	Number of tracking pixels	Tracking success rate/%
Method of this article	500	357	350	0.98
Template update method	500	357	284	0.79
Least squares linear method	500	357	268	0.76

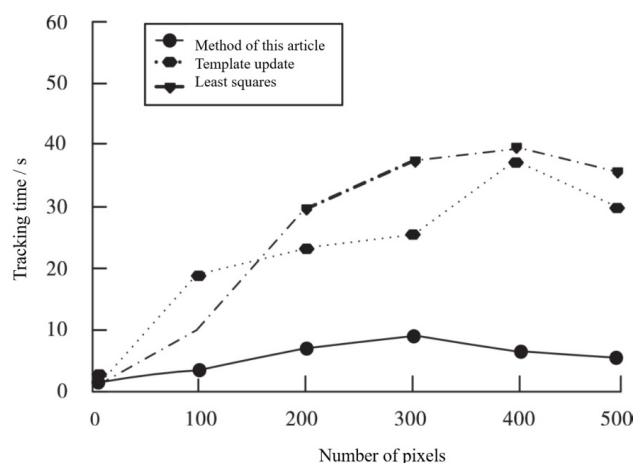
Analysis of Table 1 shows that the success rate of target tracking of the badminton player's hitting arm obtained by this method is much higher than the template update method and the least squares linear method, which is mainly because the method of this article first extracts the player's hitting arm during the hitting process. The outline of the moving target is based on the position and size of the hitting arm target of the player during the ball hitting process. The global matching approximation function of the hitting target is constructed to determine the badminton player's hitting track of the hitting arm. The success rate of target tracking during the batting arm is high.

Using the method of this paper, the template updating method and the least squares linear method, the trajectory tracking experiment of the badminton player's moving arm during the batting process is performed. The tracking time (s) of the trajectory tracking of the batting arm target corresponding to the three different methods is compared, and the comparison result is shown in Figure 7. Analysing Figure 7, it can be seen that the method used track the target's motion trajectory of the badminton player's hitting arm is of shorter time than the template update method and the least squares linear method, which is mainly because it estimates the hitting arm's differential image during the hitting process. Gaussian model parameters and the boundary detection operator are introduced to construct the boundary image of the batting arm movement. The outline of the batting arm movement target is extracted. Based on this, the morphological operator is used to calculate the body posture ratio and tightness, thus this process solves the contradiction between the amount of calculation and the amount of information to a certain extent, which makes the method used/proposed in this paper take a short time to track the target arm's motion trajectory of the badminton player.

The two-fold linear method is mainly because the method in this paper estimates the parameters of the differential image Gaussian model of the batting arm and introduces a boundary detection operator to construct a perimeter image of the batting arm movement to extract the target of the batting arm's movement during the batting process. Based on this, the morphological operator is used to calculate the body posture ratio and compactness of the hitting arm target area. This process solves the contradiction between the amount of calculation and the amount of information to a certain extent. The tracking time of the ball arm's target track is shorter.

## 5 Conclusion

Aiming at solving the problem of difficulty in extracting the active contour of the batting arm during the tracking of the batting arm when using the current method and as there is a large tracking error, a method of trajectory tracking of the ball hitting image of the badminton player based on the morphological operator is



**Fig. 7** Tracking time of arm trajectory in different ways

proposed. Simulation results show that the proposed method can effectively track the target of the hitting arm during the hitting process and generate a continuous trajectory of the hitting arm. This paper proposes a sub-array space segmentation method based on a 9-element uniform area array and applied it in a radio direction finding system. Simulation analysis and experimental verification show that:

1. The sub-array space segmentation method not only has 360-degree omnidirectional direction-finding capability, but also has the advantages of decoherent direction finding.
2. The number of decoherent signals corresponding to different sub-array space partitions is different. The more sub-array space partitions, the stronger the spatial smoothing ability to decoherent signals.
3. The actual system test verification shows that the sub-array segmentation method is effective for coherent direction finding, and the 4-sub-array segmentation is better than the 2-sub-array segmentation, while the traditional method is almost ineffective.

In practical environments, the sub-array segmentation method can basically perform coherent signal direction finding, but there are still some differences in the effect compared to the theoretical effect. The improvement of its performance depends on the improvement of hardware conditions and algorithm.

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