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Optimal preview repetitive control for impulse-free continuous-time descriptor systems

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Abstract

This paper concerns the design problem of optimal preview repetitive control (OPRC) for impulse-free continuous-time descriptor systems. First, by using a linear transformation, the descriptor system is transformed into a normal system with relatively low dimensionality and an algebraic system. Based on this, an augmented system which contains the error vector and the derivative of the normal system is constructed. Then, applying the lift technique and introducing a new performance index, the OPRC designing problem is converted into a regulation problem. Based on the optimal control theory, the regulator problem of the augmented system is solved. Furthermore, the explicit OPRC law for the original system is deduced. Different from the existing results, the preview feed-forward and error integral compensations are added into the OPRC system, which can significantly improve the tracking performance. Finally, a numerical example simulation result verifies the validity of the proposed method.

Keywords: repetitive control, preview control; optimal control; continuous-time descriptor systems

1 Introduction

Descriptor systems (also known as singular systems), which are first proposed by Rosenbrock in the 1970s [1] are a class of more general dynamic systems than general ones. They have been widely used in many fields in recent years [2–4], and increasing attention has been paid to the study of descriptor systems [5–7]. Moreover, many characteristics of analysis and synthesis in general systems theory also have been extended to descriptor systems [8, 9].

Preview control (PC) is a kind of feed-forward control method. By fully utilizing the known future knowledge of references or disturbances, it can improve the tracking performance of the controlled system [10, 11]. For discrete-time systems, by using a difference operator, the preview compensation can be constructed as part of the augmented discrete-time systems. Therefore, the PC problem of discrete-time systems can be converted into

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a stabilisation problem. So far, the most extensive researches of PC for discrete-time systems are based on linear quadratic regulator (LQR) or linear matrix inequality (LMI) [12–14]. Moreover, the problems of H PC [15], adaptive PC [16] and sliding mode-based PC [17] are also investigated.

For continuous-time systems, the preview compensation cannot be used directly as a part of the augmented dynamic systems. An alternative method is to use the derivatives of the state equation and the tracking error. However, the obtained augmented dynamic systems are non-autonomous ones. As a result, the difficulty and complexity are greatly increased. By differentiating both the output error signal and the two sides of the state equation, the optimal PC problem of continuous-time systems is solved in [18–20]. By taking the derivative of the control input, a new optimal PC was deduced [21]. The optimal PC problem of impulse-free descriptor systems was studied [23]. More recently, for continuous-time descriptor multi-agent systems, a cooperative optimal preview tracking problem was considered in [24] by using the LQR technique.

On the other hand, repetitive control (RC) is an effective technique for improving the performance of systems that track periodic reference or reject periodic disturbance [25]. It was originated by Inoue et al. [25] and subsequently developed by many researchers, such as Hara [26] and Doh [27]. Nowadays, a great deal of research has been focused on the theory and applications of RC, and various structures and algorithms have been devised (see, for example, [28–32]). The combination of preview and RC, which is called preview RC, can significantly improve the control performance of closed-loop systems. Since the relationships between the system and the future signal as well as the output of the RC were established by the difference operator, preview RC for discrete-time systems has become very popular in many research fields.

A discrete-time preview RC method [34] was first presented based on LQ optimal control. Using ARMAX models, a new design method for discrete-time preview RC was proposed [35]. The design method of discrete-time sliding mode preview repetitive servo systems was introduced [36, 37]. Very recently, an LMI-based robust guaranteed-cost preview RC for uncertain discrete-time systems was proposed [38].

It is worth pointing out that although there have been fruitful results about discrete-time preview RC, there have been only a few studies on designing preview RC for continuous-time systems. This paper focuses on the preview RC problem of linear continuous-time descriptor systems. The main contributions are as follows: (1) The descriptor system is transformed into a restricted equivalent one with an algebraic system by taking non-singular linear transformation.

Based on the lift technique, an augmented system is constructed, which contains the output of the basic RC; (2) Different from the existing PC methods, the augmented state vector and the derivative of the tracking error, as well as the derivative of the control input, are introduced into the quadratic performance index; (3) Based on the optimal control, the regulator problem of the augmented system is solved, from which the explicit optimal preview repetitive controller (OPRC) for the original system is deduced.

The rest of this paper is organised as follows. Section 1 presents the problem formulation and assumptions. The OPRC is derived in Section 2. A numerical example is provided in Section 3. Finally, some conclusions are drawn in Section 4.

1.1 Problem Formulation and Assumptions

Fig. 1 Basic repetitive-control system.

Figure 1 shows the basic configuration of a continuous-time RC system, where $G(s)$ is the controlled plant, $r(t)$ is a periodic reference input with period L , and $C_R(s)$ is an RC. The output of the RC, $v(t)$, is the tracking error.

$$v(t) = \begin{cases} v(t-L) + e(t), & t \geq L, \\ e(t), & t < L, \end{cases} \quad (1)$$

where

$$e(t) = r(t) - y(t). \quad (2)$$

This paper focuses on OPRC designing of a plant with a relative degree of zero, which is described by the following regular and impulse-free continuous-time descriptor system.

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases} \quad (3)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^q$ is the control input, and $y(t) \in \mathbb{R}^p$ is the output of the plant. $A, B, C, D \neq 0$ are constant matrices with appropriate dimensions. E is a singular matrix with $\text{rank}(E) = m < n$. In addition, the corresponding initial conditions for system (3) are $x(t) = 0, u(t) = 0$ for $t \leq 0$. Let $r(t) \in \mathbb{R}^p$ be a periodic reference signal to be tracked by $y(t)$ and the period is L . Further, $r(t)$ is piecewise differentiable. Moreover, we assume that the desired output is previewable in the sense that the future values $r(\tau), t \leq \tau \leq t + l_r$ are available at each instant of time t , where $l_r (l_r < L)$ is the preview length.

The objective of this paper is to develop an RC with preview compensation such that in the steady-state, the output vector $y(t)$ of the system (3) tracks the desired output $r(t)$ without static error, namely,

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} (r(t) - y(t)) = 0. \quad (4)$$

The following assumptions and lemmas are needed.

Assumption A1. Assume that system (3) is regular and there exists a s_0 such that $\det(s_0 E - A) \neq 0$.

Assumption A2. The matrix pair (E, A) is impulse-free. That is, for any $s \in \mathbb{C}$, $\deg(\det(sE - A)) = \text{rank} E$.

Assumption A3. The matrix pair (E, A, B) is stabilisable and (E, A, C) is detectable.

Assumption A4. The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is of full row rank.

Remark 1. By the Popov-Belevitch-Hautus (PBH) rank test [2, 3], system (3) is stabilisable if and only if for any complex s satisfying $\text{Re}(s) \geq 0$, the matrix $[sE - A \ B]$ is of full row rank. Similarly, system (3) is detectable if and only if for any complex s satisfying $\text{Re}(s) \geq 0$, the matrix $\begin{bmatrix} sE - A \\ C \end{bmatrix}$ is of full column rank.

Lemma 1. [3] (E, A) is regular and impulse-free if and only if $\text{rank} \begin{bmatrix} E & 0 \\ A & E \end{bmatrix} = n + \text{rank} E$.

Lemma 2. [3] Suppose (E, A) is regular and impulse-free, then there exist two nonsingular matrices U and S satisfying

$$UES = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad UAS = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad UB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad CS = [C_1 \ C_2].$$

2 OPRC Design

2.1 Restricted Equivalent Form of Control System

In this subsection, we transform system (3) into a restricted equivalent form by non-singular linear transformation.

Taking non-singular linear transformation $x(t) = S\bar{x}(t)$ and substituting it into the system (3), it gives

$$ES\dot{\bar{x}}(t) = AS\bar{x}(t) + Bu(t). \quad (5)$$

Left multiplying U on both sides of (5), we obtain

$$UES\dot{\bar{x}}(t) = UAS\bar{x}(t) + UB u(t). \quad (6)$$

Now, we decompose UES , UAS and UB into block matrices and denote

$$UES = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad UAS = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad UB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

Therefore, (6) can be expressed as

$$\begin{cases} \dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t), \\ 0 = A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t). \end{cases} \quad (7)$$

Denote

$$CS = [C_1 \ C_2]. \quad (8)$$

The observation equation in (3) is transformed into

$$y(t) = CS\bar{x}(t) + Du(t) = C_1x_1(t) + C_2x_2(t) + Du(t). \quad (9)$$

Combining (7) and (9), it gives the following restricted system

$$\begin{cases} \dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t), \\ 0 = A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t), \\ y(t) = C_1x_1(t) + C_2x_2(t) + Du(t). \end{cases} \quad (10)$$

Remark 2. Note that system characteristics are not changed by the non-singular linear transformation.

Therefore, the regularity, impulse-free, stability and detectability of systems (3) and (10) are equivalent. On the other hand, it follows from assumption A2 that system (10) is impulse-free, which implies that A_{22} is non-singular [3].

Remark 3. It is worth noting that $D \neq 0$ in Eq. (10) is necessary. In fact, it has been pointed out by Inoue et al. in [25] that only the controlled plant with a relative degree of zero ($D \neq 0$), the closed-loop system can be asymptotically stable under the RC (1).

2.2 Construction of the Augmented System and Problem Transformation

Since A_{22} is non-singular, from the second equation of (10), we get

$$x_2(t) = -A_{22}^{-1}A_{21}x_1(t) - A_{22}^{-1}B_2u(t). \quad (11)$$

Substituting (11) into the first and the third equation of (10), respectively, we have

$$\dot{x}_1(t) = \bar{A}x_1(t) + \bar{B}u(t), \quad (12)$$

where

$$\bar{A} = A_{11} - A_{12}A_{22}^{-1}A_{21}, \quad \bar{B} = B_1 - A_{12}A_{22}^{-1}B_2, \quad (13)$$

and

$$y(t) = \bar{C}x_1(t) + \bar{D}u(t), \quad (14)$$

where

$$\bar{C} = C_1 - C_2A_{22}^{-1}A_{21}, \quad \bar{D} = D - C_2A_{22}^{-1}B_2. \quad (15)$$

Thus, system (7) is decomposed into a normal system with relatively low dimensionality and an algebraic equation as following.

$$\begin{cases} \dot{x}_1(t) = \bar{A}x_1(t) + \bar{B}u(t), \\ y(t) = \bar{C}x_1(t) + \bar{D}u(t). \end{cases} \quad (16)$$

Taking the derivative on both sides of (2), we get

$$\dot{e}(t) = \dot{r}(t) - \bar{C}\dot{x}_1(t) - \bar{D}\dot{u}(t). \quad (17)$$

Let

$$z(t) = \begin{bmatrix} \dot{x}_1(t) \\ e(t) \end{bmatrix} \in \mathbb{R}^{(m+p) \times 1} \quad (18)$$

be the augmented vector. Combining (16), (17) and (18), we have

$$\dot{z}(t) = \hat{A}z(t) + \hat{B}\dot{u}(t) + \hat{D}\dot{r}(t), \quad (19)$$

where

$$\hat{A} = \begin{bmatrix} \bar{A} & 0 \\ -\bar{C} & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} \bar{B} \\ -\bar{D} \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad (20)$$

and $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ are defined in (13) and (15).

For $\chi(t) \in \{z(t), u(t), r(t), x(t), v(t)\}$, assume that $\chi(t) = 0$ for $t < 0$ and denote

$$\Delta\chi(t) := \chi(t) - \chi(t-L). \quad (21)$$

Therefore, we can derive

$$\Delta\dot{z}(t) = \hat{A}\Delta z(t) + \hat{B}\Delta\dot{u}(t) + \hat{D}\Delta\dot{r}(t), \quad (22)$$

and

$$\dot{e}(t) = \dot{e}(t-L) - \bar{C}\Delta\dot{x}_1(t) - \bar{D}\Delta\dot{u}(t) + \Delta\dot{r}(t), = \dot{e}(t-L) - \hat{C}\Delta z(t) - \bar{D}\Delta\dot{u}(t), \quad (23)$$

where $\hat{C} = [\bar{C} \ 0]$.

Eqs (22) and (23) are augmented dynamic systems. Correspondingly, we wish to find the optimal controller that minimises the following performance index [39]

$$J = \lim_{t_a \rightarrow \infty} \frac{1}{2} \int_{t_0}^{t_a} \left(\Delta z^T(t) Q_z \Delta z(t) + \dot{e}^T(t) Q_e \dot{e}(t) + \Delta \dot{u}^T(t) R \Delta \dot{u}(t) \right) dt \quad (24)$$

subject to Eqs (22) and (23), where $R \in \mathbb{R}^{q \times q}$, $Q_e \in \mathbb{R}^{p \times p}$ are positive-definite matrices, and $Q_z = \text{diag}[Q_x \ Q_{xe}]$, $Q_x \in \mathbb{R}^{m \times m}$, $Q_{xe} \in \mathbb{R}^{p \times p}$ are nonnegative-definite and positive-definite matrices, respectively.

Remark 4. If we can derive a control input $\Delta \dot{u}(t)$ such that the closed-loop system of (22) is asymptotically stable, then we can get $\lim_{t \rightarrow \infty} \Delta z(t) = 0$ and $\lim_{t \rightarrow \infty} \Delta e(t) = 0$. Since $r(t)$ is a periodic reference signal, naturally $\lim_{t \rightarrow \infty} e(t) = 0$ holds.

Remark 5. Borrowed from [39], we use performance index (24) to evaluate system (22), which is different from that of PC, the term $\dot{e}^T(t) Q_e \dot{e}(t)$ is evaluated in Eq. (24). Since $e(t) = v(t) - v(t - L)$, where $v(t)$ is the output of the basic RC, by solving the optimal control law $\Delta \dot{u}(t)$ of Eq. (22) under performance index (24), the OPRC $u(t)$ of the system (3) can be obtained.

Now, the design problem is transformed into the design of a control input $\Delta \dot{u}(t)$ such that the closed-loop system of Eq. (22) is asymptotically stable and the performance index (24) is minimised. This is a standard PC problem.

2.3 Existence Conditions for the Optimal Controller of the Augmented Dynamic System

The following lemmas related to stabilisability (controllability) and detectability (observability) give the existence conditions of the optimal control law for the augmented dynamic system.

Lemma 3. For the augmented dynamic system (22), where $\hat{A}, \hat{B}$ are defined in Eq. (20), the matrix pair $(\hat{A}, \hat{B})$ is stabilisable (controllable) if and only if the matrix pair $(\bar{A}, \bar{B})$ is stabilisable (controllable) and the matrix $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}$ is of full row rank.

Proof. It suffices to show that for any complex s satisfying $\text{Re}(s) \geq 0$,

$$\begin{aligned} & \text{rank} [sI_{\bar{n}} - \hat{A} \hat{B}] \\ &= \text{rank} \begin{bmatrix} sI_m - \bar{A} & 0 & \bar{B} \\ \bar{C} & sI_p - \bar{D} \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} 0 & sI_m - \bar{A} & \bar{B} \\ sI_p & \bar{C} & -\bar{D} \end{bmatrix} \\ &= \bar{n} = m + p. \end{aligned} \quad (25)$$

For any $s \neq 0$ and $\text{Re}(s) \geq 0$, sI_p is invertible, we have

$$\text{rank} [sI_{\bar{n}} - \hat{A} \hat{B}] = p + \text{rank} [sI_m - \bar{A} \bar{B}]. \quad (26)$$

For $s = 0$, we obtain

$$\begin{aligned} & \text{rank} [sI_{\bar{n}} - \hat{A} \hat{B}] \\ &= \text{rank} \begin{bmatrix} 0 & -\bar{A} & \bar{B} \\ 0 & \bar{C} & -\bar{D} \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}. \end{aligned} \quad (27)$$

It follows from Eqs (26) and (27) that the conclusion of Lemma 3 is true.

Lemma 4. [23] The matrix pair $(\bar{A}, \bar{B})$ is stabilisable if and only if system (3) is stabilisable.

Lemma 5. The matrix $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}$ is of full row rank if and only if the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is of full rank

Proof. Since A_{22} is non-singular, by some calculation, we have

$$\left\{ \begin{bmatrix} I_m & -A_{12}A_{22}^{-1} & 0 \\ 0 & I_{n-m} & 0 \\ 0 & -C_2A_{22}^{-1} & I_p \end{bmatrix} \begin{bmatrix} U & 0 \\ 0 & I_p \end{bmatrix} \right\} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \left\{ \begin{bmatrix} S & 0 \\ 0 & I_{n-m} \end{bmatrix} \begin{bmatrix} I_m & 0 & 0 \\ -A_{22}^{-1}A_{21} & I_{n-m} & -A_{22}^{-1}B_2 \\ 0 & 0 & I_p \end{bmatrix} \right\} = \begin{bmatrix} \bar{A} & 0 & \bar{B} \\ 0 & A_{22} & 0 \\ \bar{C} & 0 & \bar{D} \end{bmatrix},$$

where U, S are defined as in Lemma 2, $\bar{A}, \bar{B}$ are defined as in (13) and $\bar{C}, \bar{D}$ are defined as in (15).

Clearly, the matrices

$$\begin{bmatrix} I_m & -A_{12}A_{22}^{-1} & 0 \\ 0 & I_{n-m} & 0 \\ 0 & -C_2A_{22}^{-1} & I_p \end{bmatrix}, \quad \begin{bmatrix} U & 0 \\ 0 & I_p \end{bmatrix}$$

and

$$\begin{bmatrix} S & 0 \\ 0 & I_{n-m} \end{bmatrix}, \quad \begin{bmatrix} I_m & 0 & 0 \\ -A_{22}^{-1}A_{21} & I_{n-m} & -A_{22}^{-1}B_2 \\ 0 & 0 & I_p \end{bmatrix}$$

are invertible, so we can get

$$\text{rank} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \text{rank} \begin{bmatrix} \bar{A} & 0 & \bar{B} \\ 0 & A_{22} & 0 \\ \bar{C} & 0 & \bar{D} \end{bmatrix} = n - m + \text{rank} \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}. \quad (28)$$

Therefore, the matrix $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}$ is of full row rank if and only if $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is of full row rank.

Based on Lemma 3 to Lemma 6, we have the following Lemma 6.

Lemma 6. *For the augmented dynamic system (22), the martrix pair $(\hat{A}, \hat{B})$ is stabilisable (controllable) if and only if the system (3) is stabilisable (controllable) and the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is of full row rank.*

Lemma 7. *If Q_{xe} is positive-definite and the martrix pair $(\bar{C}, \hat{A})$ is detectable (observable), then the martrix pair $(Q_z^{\frac{1}{2}}, \hat{A})$ is detectable (observable).*

Proof. For the detectability of the martrix pair $(Q_z^{\frac{1}{2}}, \hat{A})$, it suffices to show that for any complex s satisfying $\text{Re}(s) \geq 0$, the matrix $\begin{bmatrix} Q_z^{\frac{1}{2}} \\ \hat{A} - sI_{m+p} \end{bmatrix}$ is of full column rank. Note that

$$\begin{bmatrix} Q_z^{\frac{1}{2}} \\ \hat{A} - sI_{m+p} \end{bmatrix} = \begin{bmatrix} Q_x^{\frac{1}{2}} & 0 \\ 0 & Q_{xe}^{\frac{1}{2}} \\ \bar{A} - sI_m & 0 \\ -\bar{C} & -sI_p \end{bmatrix}. \quad (29)$$

Since Q_{xe} is positive definite, we get

$$\text{rank} \begin{bmatrix} Q_z^{\frac{1}{2}} \\ \hat{A} - sI_{m+p} \end{bmatrix} = \text{rank} \begin{bmatrix} Q_x^{\frac{1}{2}} & 0 \\ 0 & Q_{xe}^{\frac{1}{2}} \\ \bar{A} - sI_m & 0 \\ -\bar{C} & -sI_p \end{bmatrix} = \text{rank} \begin{bmatrix} Q_x^{\frac{1}{2}} \\ 0 \\ \bar{A} - sI_m \\ \bar{C} \end{bmatrix} + p. \quad (30)$$

If $(\bar{C}, \bar{A})$ is detectable, then $\text{rank} [\bar{A}^T - sI_m \bar{C}^T] = m$ for any $\text{Re}(s) \geq 0$, which completes the proof.

Lemma 8. [23] The matrix pair $(\bar{C}, \bar{A})$ is detectable if and only if system (3) is detectable. Combining Lemmas 7 and 8, we have the following Lemma 9.

Lemma 9. For the augmented dynamic system (22), if system (3) is detectable, then the pair $(Q_z^{\frac{1}{2}}, \hat{A})$ is detectable.

Based on the theory of optimal control, we obtain the main result of this paper as follows.

Theorem 1. For continuous-time descriptor system (3), if the assumptions from A1 to A4 hold, then the OPRC law $u(t)$ is given by

$$u(t) = F_x x(t) + F_e \int_0^t e(\tau) d\tau + F_v v(t) + f(t), \quad (31)$$

where

$$f(t) := -R^{-1} \hat{B}^T \int_0^{l_r} e^{(\bar{A}_c^T \tau)} K \hat{D} r(t + \tau) d\tau. \quad (32)$$

The feedback gains F_e , F_v and F_x are given by

$$\begin{aligned} F_e &= -R^{-1} \hat{B}^T K_e, \\ F_v &= R^{-1} \bar{D}^T Q_e, \\ F_x &= -R^{-1} \hat{B}^T K_x [I_m \ 0], \end{aligned}$$

where $[K_x \ K_e] = K$ is the unique nonnegative definite solution of the algebraic Riccati equation (ARE)

$$\hat{A}^T K + K \hat{A} - K \hat{B} R^{-1} \hat{B}^T K + Q_z = 0, \quad (33)$$

where $\bar{D}$ is defined in (15), and $\hat{A}$, $\hat{B}$, $\hat{D}$ are defined in (12). Moreover, $\bar{A}_c$ in (32) is a stable matrix defined by

$$\bar{A}_c = \hat{A} - \hat{B} R^{-1} \hat{B}^T K. \quad (34)$$

Proof. For system (22) and performance index (24), the Hamilton function is chosen as

$$H = \frac{1}{2} \dot{e}^T(t) Q_e \dot{e}(t) + \frac{1}{2} \Delta z^T(t) Q_z \Delta z(t) + \frac{1}{2} \Delta \dot{u}^T(t) R \Delta \dot{u}(t) + \lambda^T(t) (\hat{A} \Delta z(t) + \hat{B} \Delta \dot{u}(t) + \hat{D} \Delta \dot{r}(t)), \quad (35)$$

where $\lambda(t)$ is the adjoint vector. Then,

$$\frac{\partial H}{\partial \Delta \dot{u}(t)} = R \Delta \dot{u}(t) - \bar{D}^T Q_e \dot{e}(t) + \hat{B}^T \lambda, \quad (36)$$

$$\frac{\partial H}{\partial \Delta z(t)} = Q_z \Delta z(t) - \hat{C}^T Q_e \dot{e}(t) + \hat{A}^T \lambda, \quad (37)$$

and

$$\frac{\partial^2 H}{\partial \Delta \dot{u}^2(t)} = R + \bar{D}^T Q_e \bar{D} > 0. \quad (38)$$

According to optimal control theory, the Hamilton function H satisfies the following canonical equations:

$$\begin{cases} \frac{\partial H}{\partial \Delta \dot{u}(t)} = 0, \\ \dot{\lambda} = -\frac{\partial H}{\partial \Delta z(t)}. \end{cases} \quad (39)$$

Therefore,

$$\begin{cases} \Delta \dot{u}(t) = -R^{-1} \hat{B}^T \lambda + R^{-1} \bar{D}^T Q_e \dot{e}(t), \\ \dot{\lambda} = -Q_z \Delta z(t) + \hat{C}^T Q_e \dot{e}(t) - \hat{A}^T \lambda. \end{cases} \quad (40)$$

Substituting the first equation of Eq. (40) into Eq. (22), it yields

$$\Delta \dot{z}(t) = \hat{A} \Delta z(t) - \hat{B} R^{-1} \hat{B}^T \lambda + \hat{B} R^{-1} \bar{D}^T Q_e \dot{e}(t) + \hat{D} \Delta \dot{r}(t). \quad (41)$$

Assume that,

$$\lambda = K(t) \Delta z(t) + \Delta g(t), \quad (42)$$

where $K(t) \in \mathbb{R}^{\bar{n} \times \bar{n}}$, $\Delta g(t) \in \mathbb{R}^{\bar{n} \times 1}$ and

$$K(t_a) = 0, \quad \Delta g(t_a) = 0. \quad (43)$$

Using Eq. (40) in Eq. (42), we can obtain

$$\dot{K}(t) = -Q_z - \hat{A}^T K(t) + K(t) \hat{B} R^{-1} \hat{B}^T K(t) - K(t) \hat{A}, \quad (44)$$

and

$$\Delta \dot{g}(t) = -[\hat{A}^T - K(t) \hat{B} R^{-1} \hat{B}^T] \Delta g(t) - K(t) \hat{D} \Delta \dot{r}(t) + [\hat{C}^T Q_e - K(t) \hat{B} R^{-1} \bar{D}^T Q_e] \dot{e}(t). \quad (45)$$

From Lemma 3 to Lemma 9 and Assumption A3, A4 that $(\hat{A}, \hat{B})$ is stabilizable and $(Q_z^{\frac{1}{2}}, \hat{A})$ is detectable. Let $t_a \rightarrow \infty$. Then it is well known that the solution $K(t)$ of (44) converges to a constant matrix K , which is the unique positive semi-definite solution of the ARE of (33).

Now, let $\Phi(t, \tau)$ be a fundamental matrix solution of

$$\Delta \dot{g}(\tau) = -\bar{A}_c^T \Delta g(\tau)$$

with $\Phi(t, t) = I$, where $\bar{A}_c = \hat{A} - \hat{B} R^{-1} \hat{B}^T K$. Then, for $t \in [t, t_a]$, the solution of (45) is given by

$$\Delta g(t_a) = \Phi(t_a, t) \Delta g(t) + \int_t^{t_a} \Phi(t_a, \tau) m(\tau) d\tau, \quad (46)$$

where

$$m(t) = -K \hat{D} \Delta \dot{r}(t) + [\hat{C}^T Q_e - K \hat{B} R^{-1} \bar{D}^T Q_e] \dot{e}(t). \quad (47)$$

It follows from $\Delta g(t_a) = 0$ and (46), we have

$$\Delta g(t) = - \int_t^{t+l_r} \Phi^{-1}(t_a, t) \Phi(t_a, \tau) m(\tau) d\tau. \quad (48)$$

As $\bar{A}_c$ is stable, with $t + l_r \rightarrow \infty$, and $\Phi(t_a, \tau)$ reduces to $e^{-\bar{A}_c^T(\tau-t)}$. Therefore,

$$\Delta g(t) = - \int_t^{t+l_r} e^{\bar{A}_c^T(\tau-t)} m(\tau) d\tau = - \int_0^{l_r} e^{\bar{A}_c^T \sigma} m(t + \sigma) d\sigma. \quad (49)$$

On the other hand, substituting Eq. (42) with $K(t) = K = [K_x \ K_e]$ into Eq. (40), we have

$$\begin{aligned} \Delta \dot{u}(t) &= -R^{-1} \hat{B}^T \lambda + R^{-1} \bar{D}^T Q_e \dot{e}(t) \\ &= -R^{-1} \hat{B}^T (K \Delta z(t) + \Delta g(t)) + R^{-1} \bar{D}^T Q_e \dot{e}(t) \\ &= -R^{-1} \hat{B}^T K_x \Delta x_1(t) - R^{-1} \hat{B}^T K_e \Delta e(t) - R^{-1} \hat{B}^T \Delta g(t) + R^{-1} \bar{D}^T Q_e \dot{e}(t). \end{aligned} \quad (50)$$

Integrating (50) over $[-L, t]$ and noting that $\Delta u(-L) = 0$, $\Delta x(-L) = 0$, $e(-L) = 0$, we get

$$\begin{aligned} \Delta u(t) = & -R^{-1}\hat{B}^T K_x \Delta x_1(t) + R^{-1}\bar{D}^T Q_e e(t) - R^{-1}\hat{B}^T K_e \int_{-L}^t \Delta e(\tau) d\tau \\ & + R^{-1}\hat{B}^T \int_{-L}^t \int_0^{l_r} e^{\bar{A}_c^T \sigma} m(\tau + \sigma) d\sigma d\tau. \end{aligned} \quad (51)$$

Furthermore,

$$\int_{-L}^t \int_0^{l_r} e^{\bar{A}_c^T \sigma} m(\tau + \sigma) d\sigma d\tau = \int_0^{l_r} e^{\bar{A}_c^T \sigma} \int_{-L}^t m(\tau + \sigma) d\tau d\sigma. \quad (52)$$

Note that $\dot{e}(t)$ is unpreview and $r(-L + \sigma) = 0$ for $0 \leq \sigma \leq l_r$. Therefore, Eq. (52) can be reduced to

$$\int_{-L}^t \int_0^{l_r} e^{\bar{A}_c^T \sigma} m(\tau + \sigma) d\sigma d\tau = - \int_0^{l_r} e^{\bar{A}_c^T \sigma} \int_{-L}^t K \hat{D} \Delta \dot{r}(\tau + \sigma) d\tau d\sigma = - \int_0^{l_r} e^{\bar{A}_c^T \sigma} K \hat{D} [\Delta r(t + \sigma)] d\sigma. \quad (53)$$

Substituting Eq. (53) into Eq. (51), we have

$$\Delta u(t) = -R^{-1}\hat{B}^T K_x \Delta x_1(t) - R^{-1}\hat{B}^T K_e \int_{-L}^t \Delta e(\tau) d\tau + R^{-1}\bar{D}^T Q_e e(t) - R^{-1}\hat{B}^T \int_0^{l_r} e^{\bar{A}_c^T \sigma} K \hat{D} \Delta r(t + \sigma) d\sigma. \quad (54)$$

Note that $e(\tau) = 0$ for $\tau \leq 0$, and

$$\int_{-L}^t \Delta e(\tau) d\tau = \int_{-L}^t e(\tau) d\tau - \int_{-L}^t e(\tau - L) d\tau = \int_0^t e(\tau) d\tau - \int_0^{t-L} e(\tau) d\tau. \quad (55)$$

Moreover, for all $t > 0$, we have $e(t) = v(t) - v(t - L) = \Delta v(t)$. So

$$\begin{aligned} \Delta u(t) = & -R^{-1}\hat{B}^T K_x \Delta x_1(t) - R^{-1}\hat{B}^T K_e \left[\int_0^t e(\tau) d\tau - \int_0^{t-L} e(\tau) d\tau \right] \\ & + R^{-1}\bar{D}^T Q_e \Delta v(t) - R^{-1}\hat{B}^T \int_0^{l_r} e^{\bar{A}_c^T \delta} K \hat{D} \Delta r(t + \delta) d\delta. \end{aligned} \quad (56)$$

As a result,

$$\begin{aligned} u(t) = & -R^{-1}\hat{B}^T K_x x_1(t) - R^{-1}\hat{B}^T K_e \int_0^t e(\tau) d\tau + R^{-1}\bar{D}^T Q_e v(t) \\ & - R^{-1}\hat{B}^T \int_0^{l_r} e^{\bar{A}_c^T \delta} K \hat{D} r(t + \delta) d\delta = F_x x(t) + F_e \int_0^t e(\tau) d\tau + F_v v(t) + f(t), \end{aligned} \quad (57)$$

where $F_x = -R^{-1}\hat{B}^T K_x [I_m \ 0]$, $F_e = -R^{-1}\hat{B}^T K_e$, $F_v = R^{-1}\bar{D}^T Q_e$,

$$f(t) = -R^{-1}\hat{B}^T \int_0^{l_r} e^{\bar{A}_c^T \tau} K \hat{D} r(t + \tau) d\tau. \quad (58)$$

The proof is completed.

Remark 6. From Theorem 1, it is easy to see that the OPRC law (31) is composed of four parts. The first one, $F_x x_1(t)$ is a state feedback, which can improve the control performance during one repetition period. The second, $F_e \int_0^t e(\tau) d\tau$ is called integrator, which is a feedback of integral of the tracking error. This integrator ensures that the output of the closed-loop system tracks the reference signal without static error. The third, $F_v v(t)$ is RC. This is exactly the particularity of the RC system and can improve the learning performance between repetition periods. The last, $f(t)$ is a preview compensation of the reference signal, which can be used to improve the tracking performance.

Fig. 2 The configuration of closed-loop system.

Remark 7. Noting that there have been several studies on RC systems [28, 32, 33, 41]. Comparing with the existing results, the integrator compensation $F_e \int_0^t e(\tau) d\tau$, as well as the preview $f(t)$ compensation, is innovatively added into the configuration of the closed-loop system showing as in Figure 2, which can significantly improve the tracking performance (see, the numerical example in Section 3).

Remark 8. The proposed preview RC Eq. (31) in Theorem 1 depends on the model, the performance against modelling and parameter uncertainties is quite important. It is hard to directly solve it for uncertain systems. An alternative approach was provided in Ref. [40], which is an LMI formulation of the robust LQR problem in the presence of uncertainties.

3 An Illustrative Example

Example. The mathematical model of PMSM in the $d-q$ reference coordination is described as [41, 42]:

$$\begin{cases} J \frac{d\omega}{dt} = \frac{3p\phi_f}{2} i_q - B\omega - T_L, \\ L \frac{di_q}{dt} = -Ri_q - p\omega Li_d - p\phi_f \omega + u_q, \\ L \frac{di_d}{dt} = -Ri_d + p\omega Li_q + u_d, \end{cases} \quad (59)$$

where ω is the rotor angular speed, i_d and i_q are the $d-q$ axis currents, u_d and u_q are the $d-q$ axis voltages. R and L denotes the stator resistance and inductance per phase respectively, p is the number of pole pairs, ϕ_f is the permanent magnet flux, J is the rotor moment of inertia, B is the viscous friction factor and T_L also represents the load torque.

To decouple the speed and currents, the vector control strategy of $i_d = 0$ is used. Assume that $T_L = 0$, $L = 0$, system (59) can be reduced to

$$\begin{cases} 0 = -Ri_q - p\phi_f \omega + u_q, \\ J \frac{d\omega}{dt} = \frac{3p\phi_f}{2} i_q - B\omega. \end{cases}$$

Therefore, the corresponding continuous-time descriptor system becomes

$$\begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \frac{di_q}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -R & -p\phi_f \\ \frac{3p\phi_f}{2} & -B \end{bmatrix} \begin{bmatrix} i_q \\ \omega \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_q, \\ y = [1 \ 0] \begin{bmatrix} i_q \\ \omega \end{bmatrix} + u_q. \end{cases} \quad (60)$$

The parameters of PMSM system are set as

$$p = 4, R = 1, \phi_f = 0.25, J = 0.005. \quad (61)$$

Given the structure of E , let

$$U = S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (62)$$

and the weight matrix in the performance index function be

$$R = 1, \quad Q_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q_e = 5.$$

The feedback gains of the OPRC law are obtained as follows:

$$F_x = -0.2574, \quad F_v = 10, \quad F_e = -2.2361.$$

The desired signal $r(t)$ is set as

$$r(t) = \begin{cases} 600, & t \in \{[0.0, 1.0], (2.0, 3.0], (4.0, 5.0]\}, \\ 900, & t \in \{(1.0, 2.0], (3.0, 4.0], (5.0, 6.0]\}. \end{cases}$$

The numerical simulation results are shown in Figure 3 to Figure 6. Figures 3 and 4 show the output response and tracking error with different preview lengths. It can be seen that the tracking quality of the output signal can be improved by adjusting the preview lengths. But when the preview lengths reach a certain degree, there is almost no effect on the output response.

For comparison, the LMI-based RC [28,29] simulation is also provided. The simulation results between RC and OPRC with preview lengths $l_r = 1.0$ are shown in Figures 5 and 6. It can be observed from the two figures that the OPRC has a shorter settling time and a smaller tracking error than that of the LMI-based RC method.

Fig. 3 Output responses with different preview lengths.

Fig. 4 Tracking errors with different preview lengths.

Fig. 5 Output responses comparison of RC and OPRC. OPRC, optimal preview repetitive control; RC, repetitive control.

Fig. 6 Tracking errors comparison of RC and OPRC. OPRC, optimal preview repetitive control; RC, repetitive control.

4 Conclusions

This paper considered the design method of OPRC for impulse-free continuous-time descriptor systems. Based on the characteristics of impulse-free descriptor systems, the descriptor system was transformed into a restricted equivalent form. By using the lift technique and the construction of the augmented system, the OPRC designing problem was converted into a regulator problem. The optimal controller for the augmented system was designed by making use of standard optimal control theory, and the OPRC of the original system was then obtained, in which the integrator compensation was innovatively added. The effectiveness of the proposed method has been shown by a numerical example.

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