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Composite mechanical performance of prefabricated concrete based on hysteresis curve equation

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Abstract

The angle steel-concrete column used in the new frame structure means that the rigid angle steel is placed in the column instead of the longitudinal reinforcement. The steel plate hoop welded to the longitudinal angle steel is used instead of the stirrup to form a spatial steel frame and concrete component. To ensure the normal use of existing houses in the building and adding floors, avoid transferring the load during the construction phase to the roof of the original house. The corresponding skeleton curve is extracted from the hysteresis curve of the specimen. Based on experimental research, the force performance of the normal section of the angle steel-concrete column was discussed, and the calculation formula of the normal section bearing capacity of the angle steel-concrete column was discussed.

Keywords: hysteretic curve equation, prefabricated concrete, mechanical performance

AMS 2010 codes: 34C55

1 Introduction

At present, the retrofitting of existing houses has become one of the hot issues in urban construction in my country. To ensure the normal use of existing houses in the process of building and adding floors, some scholars have put forward the idea of ‘self-supporting floors in the construction phase.’ In the construction phase, the prestressed steel truss bears the frame beam’s weight and construction load. This solves the problem that the original roof cannot bear the new construction load, ensures the normal use of the original building during the construction process, saves investment, and shortens the construction period due to the integration of structural force and construction measures. However, the angle steel of the built-in truss chord of the frame beam occupies a larger space when passing through the built-in column. This affects the arrangement of the frame column longitudinal reinforcement, the frame beam prestressed reinforcement, and the horn tube, and at the same time, it is not convenient for the prestressed reinforcement to be anchored.

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The angle steel-concrete column used in the new frame structure means that the rigid angle steel is placed in the column instead of the longitudinal steel bars. The steel plate hoop welded to the longitudinal angle steel replaces the stirrup to form a space steel skeleton and concrete component. Because there is no additional longitudinal reinforcement on the outside of the angle steel, the thickness of the concrete protective layer of the angle steel skeleton can be relatively reduced, roughly the same as that of ordinary reinforced concrete columns [1]. This increases the moment arm of the angle steel, enables the column to play a greater role, and facilitates embedded parts. At the same time, the angle steel frame can be prefabricated in the factory to simplify the construction procedure and speed up the construction progress. Good economic benefits will be achieved in actual projects.

There are not many reports on the study of the force performance of the normal section of the angle steel-concrete column. The corresponding skeleton curve is extracted from the hysteresis curve of the specimen [2]. The specimens all suffered a large eccentric bending failure. Based on experimental research, the force performance of the normal section of the angle steel-concrete column was discussed. The experiment in this article provides a reference for the elasticity analysis of the structure.

2 Test overview

2.1 Specimen design and production

The specimen is shown in Figure 1. The shear span ratios of the 6 test columns are all 3. The cross-sectional dimensions are all 200 mm × 200 mm. The clear height of the pillars is 1200 mm. Each test column has 4 Q345 angle sheets of steel $\angle 30 \times 3$, and the steel content is 1.75%. The arrangement is shown in the A-A section of Figure 1. The stirrup is made of steel plate hoops, and the steel plate hoop is 3 mm thick Q235 steel [3].

Fig. 1 Schematic diagram of the specimen

The average compressive strength f_{cu} of the concrete cube with a side length of $150\text{ mm} \times 150\text{ mm} \times 150\text{ mm}$ is 56.43 MPa. The cross-sectional dimensions of the reinforced concrete beams at both ends of the specimen are $800\text{ mm} \times 400\text{ mm} \times 300\text{ mm}$. The longitudinal reinforcement adopts HRB335-grade steel, and stirrups adapt HPB235-grade steel. Figure 2 shows the sample after molding [4]. The main parameters of the experimental study are the axial compression ratio and the specific values of the steel plate hoop ratio, as shown in Table 1.

Fig. 2 The molded test piece

Table 1 The main parameters of the test piece

Specimen number	Axial pressure ratio n_0	With hoop rate $\rho_{sv}/\%$
PPECC1	0.33	1.18
PPECC2	0.33	1.36
PPECC3	0.33	1.55
PPECC4	0.37	1.36
PPECC5	0.37	1.55
PPECC6	0.37	1.82

2.2 Test device

As shown in Figure 3, the loading device is mainly composed of an L-shaped beam, a four-bar linkage, a reaction frame, and a hydraulic servo actuator [5]. The four-bar linkage mechanism allows the L-shaped beam to move freely in the vertical and horizontal directions without rotating, thereby realizing the boundary condition that the top of the column is the embedded end. Hydraulic servo actuators apply horizontal repeated loads fixed on the reaction wall. A 1000 kN pressure sensor is arranged on the jack to measure the axial force. Rollers are arranged between the distribution beam and the L-shaped beam so that they can slide freely. Since the four-bar linkage mechanism cannot bear the horizontal and vertical loads, the horizontal and vertical loads are the specimen's horizontal shears and axial force.

Fig. 3 Loading device

2.3 Loading system

First, apply the axial load with a hydraulic jack, keep it at a constant value, and then apply the horizontal load. The application of horizontal load adopts the method of load-displacement dual control. Before the specimen yields, the load is controlled in stages until the specimen yields, which corresponds to one cycle of each load step [6]. Displacement control is adopted after the specimen yields. The multiple of the yield displacement is taken as the level difference for controlled loading. The horizontal load is cycled three times for each controlled displacement until failure. Keep the loading and unloading speed consistent during the test to ensure the stability of the test data.

3 Test results and analysis

3.1 Analysis of failure form and test process

The six specimens all failed under bending and had good flexibility, and the failure process was similar. Taking the test piece PPECC5 as an example, the two faces perpendicular to the horizontal load direction are the front faces, namely, face 1 and face 2. The other two sides are sides. The force in the pushing direction of the actuator is positive, and the force in the opposite direction is negative [7]. When the horizontal force direction is positive, the upper face 1 of the column and the lower face 2 are under tension. The specimen is elastic before the horizontal repeated load reaches 45 kN. When the positive horizontal load reaches 45 kN, and the negative horizontal load reaches 50 kN, horizontal cracks appear in the front tension area of the upper and lower ends of the column. With the increase of the displacement of the column top, new cracks continue to appear in the front pull area of the upper and lower ends of the column, and the cracks have further expanded. At the same time, horizontal cracks also appeared on both sides of the upper and lower ends of the column [8]. When the displacement of the top of the column reaches 30 mm, the concrete at the upper and lower ends of the column will spall in a large area, and the angle steel and steel plate hoops will be exposed, and the angle steel will appear to be buckled under compression. The load-bearing capacity of the component drops rapidly, and the specimen is destroyed.

3.2 Hysteresis curve

The load-displacement hysteresis curve of the specimen is shown in Figure 4. The following characteristics can be seen from the hysteresis curve:

Fig. 4 Specimen load-displacement hysteresis curve

When the horizontal load is less than the cracking load, the specimen is elastic. As the load increases, the load-displacement curve gradually deviates from the straight line, and there is a certain residual deformation when unloading [9]. After the load reaches the yield load, the stiffness of the test piece gradually decreases, and the decreased amplitude increases with the increase of the number of cycles.

3.3 Skeleton curve

Figure 5 shows the skeleton curves of six test pieces. It can be seen that the deformation capacity of the specimens with the same hooping ratio decreases, but the horizontal bearing capacity gradually increases [10].

Fig. 5 Skeleton curves of six specimens

4 The calculation formula for normal section bearing capacity of large eccentric angle steel-concrete columns

The analysis of the previous test process shows that the damage of the test piece occurs at the end of the test piece, and the ultimate flexural bearing capacity of the damaged section can be obtained by Eq. (1):

$$M_u^t = P_u(L/2) + N\Delta_m/2 \quad (1)$$

As shown in Table 2, L is the clear height of the test column and N is the axial force exerted on the test piece [11]. The M_u^t obtained from the test is shown in Table 2. The assumption of flat section is still valid, and the ultimate compressive strain of concrete when reaching the limit state can still be taken as 0.0033. The concrete stress pattern in the compression zone is equivalent to a rectangular stress pattern. The limit height x_b of the compression zone can be calculated according to formula (2):

$$x_b = \frac{0.8(h - a_s)}{1 + \frac{f_y}{0.0033E_{ss}}} \quad (2)$$

a_s is the distance from the inner edge of the tension angle to the tension edge. f_y is the yield strength of the tension angle steel; E_{ss} is the elastic modulus of the angle steel. Figure 6 shows the cross-sectional stress when the full cross-section of the large-eccentric column under compression and tension angle steel yields. In the figure b is the width of the column section.

Fig. 6 Sectional stress when the full section of both compression and tension angle steel yields

Table 2 Comparison of test results and calculated results

Specimen number	P_u/kN	Δ_m/mm	N/kN	$M_u^t/(\text{kN}\cdot\text{m})$	$M_u^c/(\text{kN}\cdot\text{m})$	M_u^c/M_u^t
PPECC1	104.54	15.95	458	55.92	51.53	0.92
PPECC3	97.89	19.53	458	53.42	51.53	0.96
PPECC4	114.25	15.06	565	61.38	55.52	0.90
PPECC5	121.85	16.38	565	65.55	55.52	0.85
PPECC6	106.66	14.89	565	57.54	55.52	0.96

We did not consider the effect of section strain gradient on improving the concrete compressive strength of eccentrically compressed members. The equivalent rectangular concrete compressive strength is still taken as f_c , and the effect of different eccentricity e_a is not considered in the bearing capacity formula. The bearing capacity of the section shown in Figure 6 can be calculated according to the following formula:

$$\begin{cases} N = f_c b x + f_y' A_{ss}' - f_y A_{ss} \\ N(\eta e_0 + \frac{h}{2} - a_{ss}) = f_c b x (h - a_{ss} - \frac{x}{2}) + f_y' A_{ss}' (h - a_{ss} - a_{ss}') \end{cases} \quad (3)$$

The x calculation by Eq. (3) must satisfy the calculation $x < x_b$, x_b according to Eq. (2). At the same time:

$$x \geq \frac{0.8a'_s}{1 - \frac{f_v}{0.0033E_{ss}}} \quad (4)$$

When the condition of formula (4) cannot be satisfied, the compression angle steel cannot reach the full-section yield in the limit state. There may be two situations where the section is yielded, or the whole section is not [12]. When the outer edge of the compression angle steel yields, the concrete height in the compression zone is

$$x = \frac{0.8c}{1 - \frac{f_v}{0.0033E_{ss}}} \quad (5)$$

c is the thickness of the angle steel protective layer.

$$\frac{0.0033}{\varepsilon'_y} = \frac{x/0.8}{(x/0.8 - c - d)} \quad (6)$$

The stress of the inner edge of the compressed angle steel is σ'_{ss} :

$$\frac{f'_y}{\sigma'_{ss}} = \frac{x/0.8 - c - d}{x/0.8 - a'_s} \quad (7)$$

At this time, the section bearing capacity can be calculated according to formula (8):

$$\begin{cases} N = f_c b x + f'_y A'_{ss} + \sigma'_{ss} (A'_{ss} - A'^{y'}_{ss}) + \frac{1}{2} (f'_y - \sigma'_{ss}) (A'_{ss} - A'^{y'}_{ss}) - f_y A_{ss} \\ N (\eta e_0 + \frac{h}{2} - a_{ss}) = f_c b x (h - a_{ss} - \frac{x}{2}) + f'_y A'^{y'}_{ss} (h - a_{ss} - c - g) + \sigma'_{ss} (A'_{ss} - A'^{y'}_{ss}) \cdot \\ [h - a_{ss} - c - d - \frac{1}{2} (a'_s - c - d)] + \frac{1}{2} (f'_y - \sigma'_{ss}) (A'_{ss} - A'^{y'}_{ss}) \cdot [h - a_{ss} - c - d - \frac{1}{3} (a'_s - c - d)] \end{cases} \quad (8)$$

The formula $A'^{y'}_{ss}$ is the area of the pressure angle steel reaching the yield part. g is the distance from the area's center of gravity where the pressure angle steel reaches the yield part to the outer edge of the pressure angle steel. From Eq. (8) x needs to satisfy the following equation:

$$\frac{0.8c}{1 - \frac{f_v}{0.0033E_{ss}}} < x < \frac{0.8a'_s}{1 - \frac{f_v}{0.0033E_{ss}}} \quad (9)$$

When x does not satisfy the formula (9), it means that the full section of the compression angle steel has not yielded. This situation in which all the tight angles have not yielded does not fully play the role of the tight angles. This situation should be avoided when designing.

The flexural bearing capacity of the section of the 6 test columns obtained according to the above calculation formula is shown in Table 2. Compared with the test result M^t_u , the average value of M^c_u is the mean square error, and the coefficient of variation is. The formula calculation result is conservative.

5 The calculation formula for normal section bearing capacity of small eccentric angle steel-concrete columns

When $x > x_b$ calculated by the formula (3) shows that the full section of the tension angle steel fails to reach the yield strength in the ultimate state. The yield strength should be calculated based on small eccentric compression members. Tensile angle steel may show partial yielding or not yielding at all. When the outer edge of the tension angle steel yields, the concrete height x of the compression zone is:

$$x = \frac{0.8(h - c)}{1 + \frac{f_y}{0.0033E_{ss}}} \quad (10)$$

d_1 is the length of the tension angle steel to yield. The strain indication shows that:

$$\frac{0.0033}{\varepsilon_y} = \frac{x/0.8}{(x/0.8 - c - d_1)} \quad (11)$$

The stress of the inner edge of the tension angle steel is σ_{ss} :

$$\frac{f_y}{\sigma_{ss}} = \frac{x/0.8 - c - d_1}{x/0.8 - a_s} \quad (12)$$

At this time, the section bearing capacity can be calculated according to formula (13):

$$\begin{cases} N = f_c b x + f_y' A_{ss}' + \sigma_{ss}' (A_{ss}' - A_{ss}^y) - \frac{1}{2} (f_y - \sigma_{ss}) (A_{ss} - A_{ss}^y) - f_y A_{ss}^y \\ N \left(\eta e_0 + \frac{h}{2} - a_{ss}' \right) = f_c b x \left(a_{ss}' - \frac{x}{2} \right) + f_y A_{ss}^y (h - a_{ss}' - c - g_1) + \sigma_{ss}' (A_{ss} - A_{ss}^y) \cdot \\ \left[h - a_{ss}' - c - d - \frac{1}{2} (a_s - c - d) \right] + \frac{1}{2} (f_y - \sigma_{ss}) (A_{ss} - A_{ss}^y) \\ \left[h - a_{ss}' - c - d - \frac{1}{3} (a_s - c - d) \right] \end{cases} \quad (13)$$

A_{ss}^y is the area of the tensioned angle that reaches the yielding part. Equation (13) x needs to satisfy the following equation:

$$\frac{0.8(h - a_s)}{1 + \frac{f_y}{0.0033E_{ss}}} < x < \frac{0.8(h - c)}{1 + \frac{f_y}{0.0033E_{ss}}} \quad (14)$$

When x does not satisfy the formula (14), it means that the full section of the tension angle steel has not yielded. This situation in which all the tension angles have not yielded also does not fully play the role of the tension angles. This situation should also be avoided when designing.

6 Conclusion

We have completed the test of six-angle steel concrete columns with steel plate hoops under the horizontal low-cycle reciprocating load and obtained the test column's failure form and hysteresis curve. The hysteretic curve extracts the skeleton curve of six-angle steel-concrete columns. The specimens all suffered a large eccentric bending failure. The force performance of the normal section of the angle steel-concrete column is discussed based on experimental research. At the same time, the formula for calculating the bearing capacity of the normal section of a large eccentrically loaded angle steel-concrete column is given. The paper introduces the calculation formula for the bearing capacity of the normal section of a small eccentric load angle steel-concrete column.

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