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Ecological balance model of effective utilization of agricultural water resources based on fractional differential equations

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Abstract

Water is a key factor that controls the evolution of the ecosystem and restricts social and economic development. The rational allocation of water resources is the basis for the basin's sustainable use of water resources. The article uses fractional differential equations to discuss the increasingly prominent contradictions between supply and demand of agricultural water resources, serious pollution and overexploitation, the decline in groundwater level and water quality, low agricultural water efficiency, and serious waste from the perspective of sustainable development. Second, the article classifies the index system of the water resources carrying capacity evaluation model. Finally, model the resource balance of water supply and water demand in the irrigation area. The study found that the water quantity analysis and determination of the irrigation area will have important guiding significance for implementing the irrigation area's continued construction and water-saving transformation. The allocation and utilization of water resources in farmland irrigation areas will tend to be rationalized. This will greatly improve the water-saving efficiency of irrigation districts.

Keywords: fractional differential equation, available water, water demand, agricultural water use, water resources balance**AMS 2010 codes:** 39A10

1 Introduction

Nowadays, China's food security problem is outstanding. As the basic guarantee for increasing food production, the rational use and optimal allocation of water resources have become an important issue in agricultural food production [1]. This paper uses the water balance model to study and apply the specific problems of water resources in farmland irrigation areas. This can better guide the implementation of continuous construction and water-saving transformation in irrigation districts.

The irrigation area we selected is located in the Xinjiang Plain, with flat terrain and fertile soil. The total land area of the irrigation area is 90.73 km², and the planned irrigation area is 9073hm². The irrigation area was built in the 1970s, but the project fell into disrepair for a long time and suffered serious damage. The

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supporting facilities of the project are imperfect, and the leakage is serious. The average irrigation water utilization coefficient is only 0.4. The water productivity is only 0.35 kg/m^3 . The current water availability and crop water requirements of farmland irrigation areas are relatively large. From a long-term perspective, the irrigation area has great water-saving potential [2]. We should first analyze and calculate the water resources balance in the irrigation area. Formulate water-saving measures based on the irrigation area's conditions, and incorporate the water-saving transformation and continued construction of supporting projects in the irrigation area into the new schedule. In this way, farmland irrigation districts can be built into representative water-saving and high-efficiency medium-scale irrigation districts.

2 Analysis and application of water resources balance model in the farmland irrigation area

The supply and demand of water resources are contradictory and mutually restrictive. Coordinating the water requirements of various sectors of social and economic development in the production and life of the national economy is an important way to solve this contradiction [3]. The utilization rate of water resources in the irrigation area is the key to saving water and energy in the irrigation area. The water balance analysis and calculation of the irrigation area itself is an effective method to evaluate the utilization rate of water resources. The article puts the economy first as the primary condition and adopts supply based on demand [4]. There are no industrial water requirements in this irrigation area, and only agricultural water and domestic water needs are considered.

2.1 Water supply of Songhua River

The water available for farmland irrigation is KG , which is the water available for the Songhua River Water Source Project [5]. The available water quantity of surface water in this irrigation area is defined as KGB . The available water quantity of groundwater is defined as KGX . The relationship between the three is as follows:

$$KG = KGB + KGX \quad (1)$$

Defining the available surface water KGB is the water volume of the Songhua River at the head of the irrigation area. According to the existing groundwater data in the irrigation area, the available groundwater is defined as the recoverable amount of groundwater determined by the recharge of groundwater.

Under normal circumstances, the recharge of groundwater is equal to the sum of the local precipitation and the groundwater recharge from the side seepage in the front of the mountain. The groundwater replenishment in this area is mainly the replenishment GRB of Songhua River irrigation infiltration [6]. The relationship between the Irrigation Infiltration Replenishment GRB of the Jiseonghua River and the water volume of the Jiseonghua River can be calculated by the following formula:

$$GRB = \gamma(1 - \eta_{\text{channel}})KGB + (1 - \eta_{\text{field}})\eta_{\text{channel}}KGB \quad (2)$$

η_{channel} is the water utilization coefficient of the canal system at all levels in the irrigation area. η_{field} is the field water utilization coefficient at the end of the irrigation area. γ is the infiltration replenishment coefficient of the canal system at all levels in the irrigation area. The first term at the right end of the formula Eq. (2) is the infiltration replenishment amount of the canal system at all levels in the irrigation area [7]. The second item is the last level of field water infiltration and replenishment in the irrigation area. η_{channel} is related to the anti-seepage structure of the canals at all levels in the irrigation area. γ is related to the amount of drainage from the last-level canal to the field in the irrigation area. The exploitable coefficient of groundwater is taken as 0.6. The recoverable amount KGX of groundwater is:

$$KGX = 0.6(GRB + 0.95) \quad (3)$$

Select reasonable parameters for formula (3) according to the existing specific conditions in the irrigation area. To calculate the groundwater recharge of different values from small to large [8]. At the same time, the corresponding mineable amount KGX will also be calculated. Draw the two into Figure 1.

Fig. 1 The relationship between groundwater recharge and extractable amount

2.2 Corresponding water demand for living and irrigation in the irrigation area

2.2.1 Rural domestic water and water demand in the irrigation area

According to the water quota, the rural domestic water consumption NS is defined as the product of the population R and the quota D . The formula for rural domestic water consumption is $NS = 365RD$. The water sources of rural domestic water include surface water and groundwater [9]. The actual water demand can be estimated to the water source engineering office's water supply based on the planned domestic water consumption in the irrigation area. Since the water supply ratios of different water sources are different, we define a_s , a_g as the water supply ratio of surface water sources and underground wells. The corresponding water utilization coefficient of the two is defined as η_s , η_g . Calculate the actual water demand NSX in the irrigation area as:

$$NSX = a_s NS / \eta_s + a_g NS / \eta_g \quad (4)$$

2.2.2 Water requirement for irrigation

The planned annual total irrigation area of this irrigation district is 9073 hm^2 . These are all for water-lifting irrigation. The water source of the farmland irrigation area comes from the Songhua River. According to the irrigation system design calculation, the planned net irrigation quota $d = 6090 \text{ m}^3/\text{hm}^2$ in the irrigation area [10]. The planned irrigation area is F . According to the calculation formula of the net irrigation water amount, the net irrigation water amount $JG = dF$ is obtained, and the water demand GSX is:

$$GSX = a_s JG / \eta_s + a_g JG / \eta_g \quad (5)$$

In summary, the total water demand ZX in the irrigation area is the sum of the above two water demands, namely:

$$ZX = NSX + GSX \quad (6)$$

The water requirements for the two items in the above article can be supplied by surface water and underground well water. The total water demand ZX specifically includes the surface water demand ZXB and the groundwater demand ZXX . The above four indicators are closely related and intricate [11]. The two indicators of surface water and groundwater demand are closely related to the indicators of the two water sources. The key point of the four indicators is that groundwater's available water quantity (exploitable quantity) should be determined according to the available water quantity of surface water (water quantity of the Songhua River). Therefore, it is

one of the best and most effective methods to determine the irrigation area's water supply and demand through analyzing and calculating the water supply and demand balance model when there are multiple water sources in the irrigation area.

2.3 Differential equation difference scheme construction

Due to the circulation of water at the bends of the big river, the sediment transport rate is linked to time to simulate the formation and evolution of river beaches and ridges. In a relatively ideal state, the time-fractional differential equation is used to describe the movement process of quicksand during the formation of the river beach [12]. We give a numerical simulation. It is further organized into a form that is convenient for discussion:

$$(1 + 2\gamma_2)\eta_m^1 - \gamma_2(\eta_{m+1}^1 + \eta_{m-1}^1) = \gamma_1(\eta_m^{1-p} - \eta_m^{-p}) + \eta_m^0 + \rho_m^1 \quad (7)$$

$$(1 + 2\gamma_2)\eta_m^{n+1} - \gamma_2(\eta_{m+1}^{n+1} + \eta_{m-1}^{n+1}) = \gamma_1(\eta_m^{n+1-p} - \eta_m^{n-p}) + \sum_{k=0}^{n-1} (\omega_k - \omega_{k+1})\eta_m^{n-k} + \omega_n\eta_m^0 + \rho_m^{n+1} \quad (8)$$

Here, the approximate solution of the different format (7) and formula (8) is $\bar{\eta}_m^n$, and the error is $e_m^n = \eta_m^n - \bar{\eta}_m^n$.

Theorem 1. *The difference scheme (7) and formula (8) are unconditionally stable.*

Prove that $\omega_k = (k+1)^{1-a} - k^{1-a}$, so there is

$$1 = \omega_0 > \omega_1 > \cdots > \omega_k > \omega_{k+1} > \cdots > 0$$

The error equation is obtained from the different format (7) and formula (8):

$$(1 + 2\gamma_2)\varepsilon_m^1 - \gamma_2(\varepsilon_{m+1}^1 + \varepsilon_{m-1}^1) = \gamma_1(\varepsilon_m^{1-p} - \varepsilon_m^{-p}) + \varepsilon_m^0 \quad (9)$$

$$(1 + 2\gamma_2)\varepsilon_m^{n+1} - \gamma_2(\varepsilon_{m+1}^{n+1} + \varepsilon_{m-1}^{n+1}) = \gamma_1(\varepsilon_m^{n+1-p} - \varepsilon_m^{n-p}) + \sum_{k=0}^{n-1} (\omega_k - \omega_{k+1})\varepsilon_m^{n-k} + \omega_n\varepsilon_m^0 \quad (10)$$

Let $|\varepsilon_j^k| = \max_{0 \leq i \leq M} |\varepsilon_i^k|$, $k = 0, 1, 2, \dots, N$. Then when $n = 1$, by formula (9): $\varepsilon_m^n = 0$, $n = -p, -p+1, \dots, -1$; $m = 1, 2, \dots, M-1$. From equation (9):

$$|\varepsilon_j^1| \leq |(1 + 2\gamma_2)\varepsilon_j^1 - \gamma_2(\varepsilon_{j+1}^1 + \varepsilon_{j-1}^1)| \leq \gamma_1|\varepsilon_j^{1-p} - \varepsilon_j^{-p}| + |\varepsilon_j^0| = (1 + 2\gamma_1)|\varepsilon_j^0| \quad (11)$$

Assuming that when $n \leq k$, $|\varepsilon_j^k| \leq (1 + 2\gamma_1)^k |\varepsilon_j^0|$, is established, when $n = k+1$, by formula (11):

$$|\varepsilon_j^{k+1}| \leq |(1 + 2\gamma_1)^k \varepsilon_j^{k+1} - \gamma_2(\varepsilon_{j+1}^{k+1} + \varepsilon_{j-1}^{k+1})| \leq \gamma_1|\varepsilon_j^{k+1-p} - \varepsilon_j^{k-p}| + \sum_{i=0}^{k-1} (\omega_i - \omega_{i+1})|\varepsilon_j^{k-i}| + \omega_k|\varepsilon_j^0|$$

Because there is $|\varepsilon_j^{k+1-p}| \leq |\varepsilon_j^k|$, $|\varepsilon_j^{k-p}| \leq |\varepsilon_j^k|$, $|\varepsilon_j^{k-i}| \leq |\varepsilon_j^k|$, the above formula is less than or equal to

$$2\gamma_1|\varepsilon_j^k| + |\varepsilon_j^k| \sum_{i=0}^{k-1} (\omega_i - \omega_{i+1}) + \omega_k|\varepsilon_j^0| = 2\gamma_1|\varepsilon_j^k| + |\varepsilon_j^k|(\omega_0 - \omega_k) + \omega_k|\varepsilon_j^0|$$

Taking into account the conditions that ω_k satisfy, the above formula is equal to

$$(1 + 2\gamma_1)|\varepsilon_j^k| \leq (1 + 2\gamma_1)^{k+1} |\varepsilon_j^0| \quad (12)$$

Here, the maximum value of k is $\left[\frac{T}{\Delta t}\right]$, which proves that the difference scheme (7) and formula (8) are unconditionally stable. Record the error as $\varepsilon_m^n = \eta(x_m, t_n) - \eta_m^n$ here. According to (11): $\varepsilon_m^n = 0$, $n = -p, -p+1, \dots, -1, 0, m = 1, 2, \dots, M-1$.

According to formula (7), formula (8) and local truncation error calculation:

$$(1 + 2\gamma_2) e_m^1 - \gamma_2 (e_{m+1}^1 + e_{m-1}^1) = \gamma_1 (e_m^{1-p} - e_m^{-p}) + \tau^a \Gamma(2-a) R_m^1 \quad (13)$$

$$\gamma_1 (e_m^{n+1-p} - e_m^{n-p}) + \sum_{k=0}^{n-1} (\omega_k - \omega_{k+1}) e_m^{n-k} + \tau^a \Gamma(2-a) R_m^{n+1} \quad (14)$$

R_m^{n+1} is the local truncation error. According to Taylor's formula and the integral median theorem, we get: there is a constant $C > 0$ such that

$$|R_m^n| \leq C(\tau + h^2), \quad m = 1, 2, \dots, M; \quad n = 1, 2, \dots, N.$$

Theorem 2. Difference scheme (7) and formula (8) converge unconditionally.

Proof. Use mathematical induction.

Let $|\varepsilon_j^k| = \max_{0 \leq i \leq M} |\varepsilon_i^k|$, $k = 0, 1, 2, \dots, N$. When $n = 1$, by formula (7):

$$|e_j^1| \leq |(1 + 2\gamma_2) e_j^1 - \gamma_2 (e_{j+1}^1 + e_{j-1}^1)| \leq |\gamma_1 (e_j^{1-p} - e_j^{-p}) + \tau^a \Gamma(2-a) R_j^1| \leq C_1 (1 + 2\gamma_1) = \frac{1}{\omega_0} C_1 (1 + 2\gamma_1)$$

Here $C_1 = \tau^a \Gamma(2-a)(\tau + h^2)$

Assuming that when $n \leq k$, $|e_j^k| \leq \frac{1}{\omega_k} C_1 (1 + 2\gamma_1)^k$, is true, then when $n \leq k+1$, by formula (8):

$$\begin{aligned} |e_j^{k+1}| &\leq |(1 + 2\gamma_2) e_j^{k+1} - \gamma_2 (e_{j+1}^{k+1} + e_{j-1}^{k+1})| \\ &\leq |\gamma_1 (e_j^{k+1-p} - e_j^{k-p})| + |e_j^k| \sum_{i=0}^{k-1} (\omega_i - \omega_{i+1}) + \tau^a \Gamma(2-a) |R_m^{n+1}| \\ &\leq 2\gamma_1 \frac{1}{\omega_k} C_1 (1 + 2\gamma_1)^k + \frac{1}{\omega_k} C_1 (1 + 2\gamma_1)^k (\omega_0 - \omega_k) + \tau^a \Gamma(2-a) C (\tau + h^2) \\ &\leq 2\gamma_1 \frac{1}{\omega_k} C_1 (1 + 2\gamma_1)^k + \frac{1}{\omega_k} C_1 (1 + 2\gamma_1)^k - C_1 (1 + 2\gamma_1)^k + C_1 \\ &\leq \frac{1}{\omega_{k+1}} C_1 (1 + 2\gamma_1)^{k+1} \end{aligned}$$

In addition, the maximum value of k is $\left[\frac{T}{\Delta t}\right]$, so the difference scheme (7) and equation (8) converge unconditionally.

3 Analysis and calculation of water resources balance in the farmland irrigation area

The calculation of the water diversion from the Songhua River in this irrigation area can be based on the water resources balance analysis model [13]. We make the total water available to meet the total water demand in the irrigation area to achieve high-efficiency water saving. This can achieve the purpose of rational use of water resources. With the increase of the water volume of the Tisonghua River and the corresponding increase in the supply of groundwater, the water balance analysis result is a surplus of water. When the water volume of the Tisonghua River decreases, the groundwater replenishment will also decrease accordingly, and the water balance analysis result is that the water volume is insufficient. A conclusion can be drawn based on the above

Fig. 2 The relationship between the available water supply, water demand, and the water volume of the Songhua River

analysis. There is an optimal balance for the amount of water in the Jacques Songhua River. The water supply and water demand of this irrigation area are related to the water diversion volume of the Songhua River [14]. After the simulation analysis and calculation of specific data, this paper draws the relationship between the water supply, the water demand, and the Jacquard River's water volume (Figure 2).

According to the above theory and specific calculation and analysis, it can be seen that it is more reasonable and feasible to raise the Songhua River water volume to 83.21 million m³ in the farmland irrigation area. See Table 1 for detailed results after water resources balance in this irrigation area.

Table 1 Water resources balance analysis results for the planning level year (2020) of farmland irrigation districts

Classification	Water demand/10,000 m ³			Water supply volume/ 10,000 m ³	Remaining water shortage/ 10,000 m ³
	Rural people and animals	Agricultural irrigation	Total		
Surface water		8596	8595	8589	−6.4
Groundwater	0.32	267.37	267.69	267.69	0
Total	0.32	8863.1	8863.1	8856.73	−6.4

4 Conclusion

The article analyzes and calculates the balance of water resources in farmland irrigation areas. The analysis and determination of the water volume of the irrigation area will have important guiding significance for the implementation of the continuous construction and water-saving transformation of the irrigation area. The article uses a water resource balance model to analyze the relationship between water supply and water demand in the region and find the best point of water balance. In this way, a reasonable amount of water supply can be determined so that the irrigation area can achieve the long-term goal of water-saving and energy-saving. The irrigation area is located in the arid and semiarid areas of China. It will be the next specific goal to analyze the water consumption balance of the irrigation area while achieving the balance of water supply and demand. Its research and application have put forward new requirements for the modern management of irrigation districts. This is also a new task that we will continue to study and realize in the future.

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