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Fractional differential equations in National Sports Training in Colleges and Universities

Yan Li^{1†}¹ Physical Education Institute, Lianyungang Normal College, Lianyungang, 222000, China

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Abstract

To systematically understand the ability and technical mastery of football training of ethnic college students in colleges and universities, we use fractional differential equations to analyze the passing technique of college ethnic football. Research has found that establishing differential equation models in football can help us use mathematical ideas, methods, and knowledge to solve practical problems.

Keywords: fractional differential equations, colleges and universities, physical training, generalized path, line graph analysis, transfer coordination

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1 Introduction

In sports training in colleges and universities, people generally equate a team's offensive ability with the striker's level of 'one-to-one' [1]. In the fierce competition in football, the forward is an essential factor in winning, but the passing and receiving cooperation from the backcourt and midfielders is also essential.

2 Combination circuit for catching the ball

Network topology has been widely used in communication networks, computer networks, and operations research. Here we use line graph analysis in network theory to discuss passing and receiving in football [2]. We first define the generalized path

$$L(k_i) = \prod_{i=1}^n k_i \quad (1)$$

[†]Corresponding author.Email address: lygliyan2021@163.com

Here, k_i represents a quantity that affects the generalized path. In a football game, we mainly consider ‘the geometric distance between two points, the speed of the ball, the accuracy of passing, the ability to receive the ball, and the defense of the passing and receiving points.’ When multiple people pass and receive the ball multiple times, $L(k_i)$ forms a distance matrix, which follows the following rules

$$L^2(k_i) = L(k_i) * L(k_i) L^3(k_i) = L^2(k_i) * L(k_i) \cdots \quad (2)$$

We define

$$C * D = E = [e_{jk}]_{m \times n} \quad (3)$$

Among them $C = [c_{jk}]_{m \times l}$, $D = [d_{jk}]_{l \times n}$, $e_{jk} = \min_{c_{jl} \neq 0, d_{lk} \neq 0} \{c_{j1} * d_{1k}, c_{j2} * d_{2k}, \cdots, c_{jl} * d_{lk}\}$.

For example, $C = [1, 2, 3, 4, 0]$, $D = [0, 2, 1, 2, 0]^T$, $e_{jk} = \min_{c_{jl} \neq 0, d_{lk} \neq 0} \{0, 2, 4, 6, 0\}$.

We use Figure 1 to show the generalized path diagram of the pass when attacking. P_1 represents the node formed by the defender of the possession side and the goalkeeper. P_2, P_3 corresponds to midfielders and full-backs. P_4 stands for striker. P_5 refers to the goal. Then

$$L = \begin{bmatrix} 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad L^2 = [l_{jk}] = L * L = \begin{bmatrix} 0 & 0 & 6 & 3 & 6 \\ 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

Fig. 1 Generalized path diagram of passing on offense

The same

$$L^3 = L^2 * L = \begin{bmatrix} 0 & 0 & 0 & 7 & 4 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad L^4 = L^3 * L = \begin{bmatrix} 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad L^5 = L^4 * L = 0 \quad (5)$$

To find the shortest distance from P_j to P_k , we define

$$C \oplus D = [e_{jk}]_{m \times n} \quad (6)$$

among them

$$C = [a_{jk}]_{m \times n} D = [d_{jk}]_{m \times n}, e_{jk} = \begin{cases} 0, c_{jk} = d_{jk} = 0 \\ c_{jk}, c_{jk} > 0, d_{jk} = 0 \\ d_{jk}, c_{jk} = 0, d_{jk} > 0 \\ \min(c_{jk}, d_{jk}), c_{jk} \neq 0, d_{jk} \neq 0 \end{cases} \quad (7)$$

$$L_{\min} = L \oplus L^2 \oplus L^3 \oplus L^4 \oplus L^5 = \begin{bmatrix} 0 & 3 & 2 & 3 & 4 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = [1_{jk}]_{\min}$$

$[1_{jk}]_{\min}$ represents the shortest distance from P_j to P_k . The ultimate goal of passing and receiving is to get the (best) shot. There are many factors to consider when passing the ball. A node that is geometrically connected is not necessarily a valid point. From formula (1), it can be known that any factor other than the geometric distance is zero, and the generalized path is the point of leading and receiving the ball, which is the weak point [3]. From formula (2), we can know the importance of midfielders. Because P_1, P_5 is constant for the stadium. The value of $[1_{jk}]_{\min}$ mainly depends on the direction and size of P_2, P_3 .

3 Best offensive intensity

The best-generalized path gives us the whereabouts of the ball in the pass. But how to form the best path is primarily affected by the defensive team's defensive lineup. Let us illustrate this point with a simple diagram [4]. The three U_1, U_2, U_3 secant lines in Figure 2 below represent the three defense lines of the defender. And define

Fig. 2 The equivalent model of a football field pass

$$U_i = \sum P_k P_{k+1} \quad (8)$$

$$U_1 = 5, U_2 = 5, U_3 = 4$$

We introduce the concept of generalized attack intensity and define the following formula. The attack intensity in football refers to the attack intensity, the reliability of passing and receiving the ball, and the degree of threat to the defense [5].

$$f_{jk} \leftrightarrow P_{jk} \quad (9)$$

A linear model can represent the generalized attack intensity secant

$$\min_{f_{jk}} U = \sum f_{jk} \quad (10)$$

The minimum cut is equivalent to the maximum flow, so the generalized attack intensity cut is

$$\min_{f_{jk}} U = \sum f_{jk} - \sum f_{jk} \begin{cases} U, j \neq s \\ 0, j \neq s, t \\ -U, j = t \end{cases} \quad (11)$$

s, t represents a confident start and endpoint, respectively. Through the secant analysis, we know two ways to increase the attack intensity: one is to increase the attack line, and the other is to reduce the path to the attack point P_5 . This is the value that forms the minimum secant U_3 to form the maximum attack intensity [6].

4 Unconstrained dynamic equations

We can write the dynamic equations of the model by the Kane method

$$F_j + F_j^* = 0 \quad (12)$$

We replace the extra force acting on the system with an equal force system. Then the generalized active force F_j can be written as

$$F_j = \sum_{K=1}^{15} \left(\frac{\partial V_k}{\partial \dot{x}_j} F_k + \frac{\partial \omega_k}{\partial \dot{x}_j} M_k \right) \quad (13)$$

F_k and M_k represent the ‘equivalent’ force and force matrix acting on the link B_k . The line of action of F_k passes through the center of the link. $G_k \cdot V_k$ and ω_k are respectively called the velocity and angular velocity through the point G_k in the inertial reference frame. $\frac{\partial V_k}{\partial \dot{x}_j}$ and $\frac{\partial \omega_k}{\partial \dot{x}_j}$ are respectively referred to as ‘change rate of position deviation’ and ‘change rate of azimuth deviation.’ M_k is

$$M_k = M_k^{ext} + \sum M_k^l \quad (14)$$

M_k is the moment produced by the external force acting on B_k . M_k^l is the force applied by B_l to B_k . It is generally caused by force B_l exerted on B_k . Therefore, rewriting (14) has

$$M_k = F_j^{ext} + F_j^{int} \quad (15)$$

among them

$$F_j^{ext} = \sum_{K=1}^{15} \left(\frac{\partial V_k}{\partial \dot{x}_j} F_k + \frac{\partial \omega_k}{\partial \dot{x}_j} M_k^{ext} \right) \quad (16)$$

$$F_j^{int} = \sum_{K=1}^{15} \left(\sum \frac{\partial \omega_k}{\partial \dot{x}_j} M_k \right) \quad (17)$$

Due to

$$\frac{\partial \omega_k}{\partial \dot{x}_j} = \omega_{ijk} n_{oi}, \quad \frac{\partial V_k}{\partial \dot{x}_j} = v_{ijk} n_{oi} \quad (18)$$

Equation (17) can be rewritten as

$$F_j^{ext} = v_{kjk} F_{ki} + \omega_{kjk} M_{ki}^{ext} \quad (19)$$

Where F_{ki} and M_{ki}^{ext} are the components of F_k and M_k^{ext} in the n_{oi} direction, respectively [7]. Let's analyze the generalized inertial force. From Eq. (14), we can get:

$$F_j^* = \sum_{k=1}^{15} \left(\frac{\partial V_k}{\partial \dot{x}_j} F_k^* + \frac{\partial \omega_k}{\partial \dot{x}_j} M_k^* \right) \quad (20)$$

Among them

$$F_j^* = -m_k a_k = -m_k (\dot{v}_{ijk} \ddot{x}_j + \dot{v}_{ijk} \dot{x}_j) n_{oi}, \quad T_k^* = -I_k \cdot a_k - \omega_k \times I_k \cdot \omega_k$$

The inertial dyadic I_k is given by

$$I_k = I_{mnk} n_{om} n_{on} \quad (21)$$

Among them, the value of element I_{mnk} related to n_{om} and n_{mnk} is related. It has the same value as n_{kr} and n_{ks} related element I_{rsk}^k ($k = 1, 2, 3; m, n, r, s, k$). O represents the origin of the reference coordinate of the inertial reference system [8]. The expression is as follows:

$$I_{mnk} = n_{om} n_{on} n_{ks} n_{rsk}^k \quad (22)$$

$$S_0 K_{mr} = n_{om} n_{kr}, \quad S_0 K_{ns} = n_{on} n_{ks}, \quad I_{mnk} = S_0 K_{mr} S_0 K_{ns} I_{rsk} \quad (23)$$

Organized

$$\begin{aligned} T_k^* &= -(I_{mnk} n_{om} n_{on}) [(\dot{v}_{ijk} \ddot{x}_j + \omega_{ijk} \dot{x}_j) n_{oi}] - (\omega_{ijk} \dot{x}_j n_{oi}) \times I_{mnk} n_{om} n_{on} \omega_{ijk} \dot{x}_j n_{oi} \\ T_k^* &= -[I_{mnk} (\dot{\omega}_{njk} \ddot{x}_j + \omega_{njk} \dot{x}_j) + e_{iqm} \omega_{ipk} \omega_{njk} \dot{x}_p \dot{x}_j I_{qnk}] n_{om} \end{aligned} \quad (24)$$

Among them

$$T_k^* = \sum_{k=1}^{15} \left\{ m_k v_{ijk} (\dot{v}_{ipk} \ddot{x}_j + \dot{v}_{ipk} \dot{x}_j) + \omega_{ijk} [I_{ink} (\omega_{njk} \dot{x}_p + \dot{\omega}_{njk} \dot{x}_p) + e_{mqi} \omega_{mpk} \omega_{nhk} \omega_{nhk} \dot{x}_p \dot{x}_h I_{qnk}] \right\}$$

i, m, n, q and p are summed from 1 to 3. Both j and h are summed from 1 to 40. The kinetic equation (14) can be expressed in the following concise form

$$A_{jp} \ddot{x}_p = f_j + F_j^{ext} + F_j^{int} \quad (25)$$

Where p is summed from 1 to 40. A_{jp} and f_j are given by

$$A_{jp} = \sum_{k=1}^{15} [m_k v_{ijk} v_{ipk} + I_{ink} \omega_{ijk} \omega_{njk} \dot{x}_p] \quad (26)$$

$$f_j = \sum_{k=1}^{15} [m_k v_{ijk} \dot{v}_{ipk} \dot{x}_p + I_{ink} \omega_{ijk} \dot{\omega}_{njk} \dot{x}_p + e_{mqi} I_{qnk} \omega_{mpk} \omega_{nhk} \dot{x}_p \dot{x}_h] \quad (27)$$

Equation (24) forms 40 second-order nonlinear differential equations. Use the Runge-Kutta method to find numerical solutions to these equations. We can find 40 generalized coordinates. If some equations of these generalized coordinates are known, a part of the Eq. (24) becomes an algebraic equation for finding the sum of unknown forces [9]. All the coefficients in the equation can be obtained by calculating the four coefficient matrices of ω_{ijk} , v_{ijk} , $\dot{v}_{ijk} \dot{x}_j$ and $\dot{\omega}_{ijk} \dot{x}_j$. So we can solve the dynamic equation on the computer.

5 Constrained dynamic equations

To obtain the dynamic equation of the constraint system, we only need to multiply the left side of the dynamic equation of the original system by transposing the coefficient matrix of the constraint equation [10]. Therefore, from Eq. (24), the dynamic equation of the multi-rigid body mechanical model of the human body under constraints can be obtained:

$$\begin{aligned} a(F_j + F_j^*) &= 0 \\ aA_{jp}\ddot{x}_p &= a(f_j + F_j^{ext} + F_j^{int}) \end{aligned} \quad (28)$$

Where a is the transposition of the coefficient matrix of the constraint equation. In the formula, A_{jp} , f_j , F_j^{ext} and F_j^{int} are determined by formulas (18–21).

6 Application examples

The election here is one of the ten classic World Cup events-a-goal matches between Argentina and the Netherlands. The picture is the eight sports characteristic pictures from the attack from the middle of the ball to the goal. Through image reconstruction, a panoramic view of the whole process is obtained [11]. From the reconstructed panorama, the combination line of passing and receiving for the offensive midfielder is obtained as shown in Figure 3:

Fig. 3 Midfielder pass and catch combination line

According to the offensive strength, the offensive midfielder's adequate passing directions, namely, to the right and pass back. The defense of the three forwards on the offensive side is successful for the defense. Still, the defensive guards are too densely positioned to create opportunities for the proper offensive back [12]. Although a '6-3 situation is formed in the local area, there is unnecessary defensive overlap, resulting in insufficient defensive space. At this time, the three strikers of the offensive side are effectively blocked, and the goalkeeper's proper position is not a fatal threat to the defensive side.

Figures 4 and 5 are the characteristic pictures of the 'back bow' on the pole in the shooting action using the improved dynamic model and the original model [13]. It can be seen that the hip movement, which is the crucial part of the technical movement, more realistically describes the characteristics of the movement by increasing the degree of freedom.

Fig. 4 Improved dynamics model

Fig. 5 Original model

7 Conclusion

The essential physical fitness and technical level of a football player are essential. It is an integral part of the technical level of the entire team. However, the most considerable charm of football is the ‘acting’ of talented players and the unpredictable results. The team’s tactical style and performance are also the most attractive. Research has found that establishing differential equation models in football can help us use mathematical ideas, methods, and knowledge to solve practical problems.

References

- [1] Harabagiu, N. The motor training of senior volleyball players by applying the Data-Volley programme. *Știința Culturii Fizice.*, 2019; 2(34), 108-114.
- [2] Ustaoglu, G., Atala, M. H., & Gümüş, K. Ç. Evaluation of Oral Health-Related Quality of Life Among Dentistry Students. *Van Medical Journal.*, 2020; 27(2), 190-196.
- [3] Sági, M., Domjan, A., Fehérné Kiss, A., & Pintér, S. Is kinect training superior to conventional balance training for healthy older adults to improve postural control?. *Games for health journal.*, 2019; 8(1), 41-48.
- [4] Lungu, A. J., Swinkels, W., Claesen, L., Tu, P., Egger, J., & Chen, X. A review on the applications of virtual reality, augmented reality and mixed reality in surgical simulation: an extension to different kinds of surgery. *Expert Review of Medical Devices.*, 2021; 18(1), 47-62.
- [5] Giusti, A., Colombaro, I., Garra, R., Garrappa, R., Polito, F., Popolizio, M., & Mainardi, F. A practical guide to Prabhakar fractional calculus. *Fractional Calculus and Applied Analysis.*, 2020; 23(1), 9-54.
- [6] Lieu Tran, T. B., Törngren, M., Nguyen, H. D., Paulen, R., Gleason, N. W., & Duong, T. H. Trends in preparing

- cyber-physical systems engineers. *Cyber-Physical Systems.*, 2019; 5(2), 65-91.
- [7] Wang, C., Yang, G., Papanastasiou, G., Zhang, H., Rodrigues, J. J., & de Albuquerque, V. H. C. Industrial cyber-physical systems-based cloud IoT edge for federated heterogeneous distillation. *IEEE Transactions on Industrial Informatics.*, 2020; 17(8), 5511-5521.
- [8] Cumino, C., Pavignano, M., Spreafico, M. L., & Zich, U. Geometry to Build Models, Models to Visualize Geometry. *Digital Experiences in Mathematics Education.*, 2021; 7(1), 149-166.
- [9] Koratala, A., & Bejjanki, H. Point-of-care ultrasound for the nephrologist: emphysematous pyelonephritis versus staghorn calculus. *Clinical and experimental nephrology.*, 2019; 23(10), 1257-1258.
- [10] Mishra, S. U. C. H. A. N. A., Mishra, L. N., Mishra, R. K., & Patnaik, S. R. I. K. A. N. T. A. Some applications of fractional calculus in technological development. *Journal of Fractional Calculus and Applications.*, 2019; 10(1), 228-235.
- [11] Zjavka, L. PV power intra-day predictions using PDE models of polynomial networks based on operational calculus. *IET Renewable Power Generation*, 2020; 14(8), 1405-1412.
- [12] Iglesias Martínez, M., Antonino-Daviu, J., de Córdoba, P. & Conejero, J. Higher-Order Spectral Analysis of Stray Flux Signals for Faults Detection in Induction Motors. *Applied Mathematics and Nonlinear Sciences.*, 2020;5(2) 1-14.
- [13] de Assis, R., Pazim, R., Malavazi, M., Petry, P., de Assis, L. & Venturino, E. A Mathematical Model to describe the herd behaviour considering group defense. *Applied Mathematics and Nonlinear Sciences.*, 2020; 5(1) 11-24.