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STRESS-STRAIN RELATION LAWS FOR CONCRETE AND STEEL REINFORCEMENT USED IN NON-LINEAR STATIC ANALYTICAL STUDIES OF THE MOMENT RESISTING REINFORCED CONCRETE (RC) FRAME MODELS

BY

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Abstract. Following the previous analytical studies performed with ATENA software for a series of RC moment resisting frame models, it were used in the pre-processing stage the stress-strain relation laws for concrete and steel reinforcement. These mathematical and graphical relations represent a necessity in the current conditions of numerical analysis and imply a correct knowledge of the deformation mode of the „reinforced concrete” which is a composite material. Thus, it is desired through this research paper the theoretical exposition of: equivalent uniaxial law for concrete, biaxial compressive failure and tensile failure consideration laws for concrete, bilinear with hardening law for steel reinforcement, cycling steel reinforcement model and steel reinforcement bond model. Finally, it will be possible to validate the correctness of the analytical RC frame systems through the experimental results of the optimal RC frame model after seismic platform testing.

Keywords: Stress-strain laws, concrete, steel reinforcement, ATENA software, RC frame system.

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1. Stress-Strain Relations for Concrete

1.1. Equivalent Uniaxial Law

In analytical studies performed with ATENA software (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c), „the nonlinear behavior of concrete in the biaxial stress state is described by means of the so-called effective stress σ_c^{ef} and the equivalent uniaxial strain ε^{eq} (Equation 1). The effective stress is in most cases a principal stress” (Cervenka *et al.*, 2012), (Bi *et al.*, 2020).

$$\varepsilon^{eq} = \frac{\sigma_{ci}}{E_{ci}} \quad (1)$$

„The complete equivalent uniaxial stress-strain diagram for concrete is represented in Fig. 1 and the material state number are used in the results of the nonlinear analysis to indicate the state of damage of concrete” (Cervenka *et al.*, 2012), (Dong *et al.*, 2021).

Fig. 1 – „Uniaxial stress-strain law for concrete” (Cervenka *et al.*, 2012)

1.2. Biaxial Compressive Failure

„A biaxial stress failure criterion according to Kupfer *et al.* (Kupfer *et al.*, 1969) is used for consideration of concrete degradation of the moment resisting RC frame systems in analytical studies (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c) performed with ATENA software (see Fig. 2). In the compression-compression stress state, the failure function is” (Cervenka *et al.*, 2012) (Equation 2):

$$f_c'^{ef} = \frac{1+3.65a}{(1+a)^2} \cdot f_{c,a}' = \frac{\sigma_{c1}}{\sigma_{c2}} \quad (2)$$

„where: σ_{c1} , σ_{c2} are the principal stresses in concrete and f_c' is the uniaxial cylinder strength (Shiming & Yupu, 2013), (Zhou *et al.*, 2020). In the biaxial stress state, the strength of concrete is predicted under the assumption of a proportional stress path” (Cervenka *et al.*, 2012).

„In the tension-compression state, the failure function continues linearly from the point $\sigma_{c1}=0$, $\sigma_{c2}=f_c'$ into the tension-compression region with the linearly decreasing strength” (Cervenka *et al.*, 2012) (Equation 3):

$$f_c'^{ef} = f_c' r_{ec}, \quad r_{ec} = (1 + 5.3278 \frac{\sigma_{c1}}{f_c'}), \quad 1.0 \geq r_{ec} \geq 0.9 \quad (3)$$

„where: r_{ec} is the reduction factor of the compressive strength in the principal direction 2 due to the tensile stress in the principal direction 1” (Cervenka *et al.*, 2012).

1.3. Tensile Failure

„In the tension-tension state, the tensile strength is constant and equal to the uniaxial tensile strength f_t' . In the tension-compression state, the tensile strength is reduced by the relation” (Cervenka *et al.*, 2012) (Equation 4):

$$f_t'^{ef} = f_t' r_{et} \quad (4)$$

„where: r_{et} (see Equation 5 and Equation 6) is the reduction factor of the tensile strength in the direction 1 due to the compressive stress in the direction 2. The reduction function has one of the following forms (see Fig. 3)” (Cervenka *et al.*, 2012). Also, „mesh-size dependency and prediction of dynamic tensile failure are solved by nonlocal model”, as specified in Xiangzhen *et al.* (Xiangzhen *et al.*, 2019), (Xiangzhen *et al.*, 2020).

Fig. 2 – „Biaxial failure function for concrete” (Cervenka *et al.*, 2012), (Kupfer *et al.*, 1969) (see Table 1).

Fig. 3 – „Tension-compression failure function for concrete” (Cervenka *et al.*, 2012) (see Table 1).

$$r_{et} = 1 - 0.95 \frac{\sigma_{c2}}{f'_c} \quad (5)$$

$$r_{et} = \frac{A + (A-1)B}{AB}, \quad B = Kx + A, \quad x = \frac{\sigma_{c2}}{f'_c} \quad (6)$$

„The relation in Equation 5 is the linear decrease of the tensile strength and Equation 6 is the hyperbolic decrease” (see Fig. 3) (Cervenka *et al.*, 2012).

2. Stress-Strain Laws for Steel Reinforcement

2.1. Bilinear with Hardening Law

„The conventional stress-strain relationship of steel bar with a bilinear curve is widely used in numerical models because of its simplicity” (Wang *et al.*, 2019). Thus, „the bilinear with hardening law is accepted as shown in Fig. 4. The initial elastic part has the elastic modulus of steel reinforcement E_s . The second line represents the plasticity of the steel with hardening and its slope is the hardening modulus E_{sh} . Limit strain ε_{lim} represents limited ductility of steel reinforcement” (Cervenka *et al.*, 2012).

Fig. 4 – „The bilinear with hardening stress-strain law for Bst 500 S longitudinal reinforcement” (Cervenka *et al.*, 2012), (ATENA software, 2015) (see Table 1).

In these conditions, the longitudinal steel reinforcement deformation in the structure of the lateral elements (RC beams and RC columns) of the moment resisting RC frame models it was considered in analytical studies performed with ATENA software (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c) according to bilinear with hardening stress-strain law (Fig. 4).

2.2. Cycling Steel Reinforcement Model

„To ensure the serviceability of reinforced concrete structures, deflection control is an important design objective. Excessive concrete cracking and excessive deformation are one of the most common causes of damage and result in large annual cost to the construction industry” (Xu *et al.*, 2016). In these conditions, it is necessary to use a suitable cycling steel reinforcement model, to avoid these negative effects. In numerical analysis developed by Sococol *et al.* (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c) with ATENA software, „the reinforcing steel stress-strain behavior is described by the nonlinear model of Menegotto and Pinto. Thus, in ATENA software, this model is extended to account of the isotropic hardening due to an arbitrary hardening law that can be specified for steel reinforcement. The stress in the cycling model is calculated according to the following expression (Equation 7)” (Cervenka *et al.*, 2012):

$$\sigma = (\sigma_0 - \sigma_r) \sigma^* + \sigma_r \quad (7)$$

where:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{1/R}}, \quad \varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_0 - \varepsilon_r}, \quad R = R_0 - \frac{c_1 \xi}{c_2 + \xi} \quad (8)$$

„where: R_0 , c_1 and c_2 are experimentally determined parameters. The Fig. 5 shows the meaning of strain values ε_r , ε_0 , ξ and stress values σ_r and σ_0 . These values changes for each cycle. The values with the subscript „r” indicate the point where the cycle started and the subscript „0” indicates the theoretical yield point that would be reached during the unloading if the response would not have been modified by the hysteretic behavior. During the calculation of this point, the material stress-strain law is considered” (Cervenka *et al.*, 2012) (Equation 9):

$$\sigma^* = f_R(\varepsilon_{eq}), \quad \varepsilon_{eq} = \sum_{i=1}^{N_{inc}} |\Delta \varepsilon_{eq}^i| \quad (9)$$

Fig. 5 – „Cycling steel reinforcement model based on Menegotto and Pinto”
(Cervenka *et al.*, 2012), (ATENA software, 2015).

Table 1

The mean values concrete strength and steel reinforcement strength used in non-linear static analysis (ATENA software, 2015), (Cervenka *et al.*, 2012) (see Fig. 1 – Fig. 3 for concrete and Fig. 4 - Fig. 5 for longitudinal steel reinforcement)

Material name	Elastic modulus E [MPa]	Poisson's ratio μ	Tensile strength f_t [MPa]	Compressive strength f_c [MPa]
C20/25 concrete strength class	3.000E+04	0.200	2.200E+00	-2.800E+01

Material name	Type	Elastic modulus E [MPa]	σ_y [MPa]	σ_t [MPa]	ϵ_{lim}
Bst 500 S	Bilinear with Hardening	2.000E+05	575.000	632.500	0.060

2.3. Steel Reinforcement Bond Model

„The basic property of the steel reinforcement bond model is the bond-slip relationship. This relationship defines the bond strength (cohesion) τ_b (see Equation 10 – Equation 13) depending on the value of current slip between steel reinforcement and surrounding concrete” (Cervenka *et al.*, 2012).

Also, „bond stresses between steel reinforcement and concrete play a crucial role in ensuring reliable force transfer from the reinforcement to surrounding concrete” (Long *et al.*, 2020), (Sabău, 2020). Thus, it was considered appropriate to use the bond-slip model according to CEB-FIP model code 1990 (CEB-FIP, 1990) (see Fig. 6) found in ATENA software (ATENA software, 2015) for the series of analytical studies regarding the consideration of the optimal RC frame experimental model performed by Sococol *et al.* (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c). In this case, „the law is generated based on the concrete compressive strength, reinforcement diameter and reinforcement type. The important parameters are also the confinement conditions and the quality of concrete casting” (Cervenka *et al.*, 2012).

The conditions for considering the transverse reinforcement mode of the structural elements in RC moment resisting frame models can be studied in research papers conducted by Sococol *et al.*

Fig. 6 – „Bond-slip law by CEB-FIP model code 1990” (CEB-FIP, 1990), (Cervenka *et al.*, 2012), (ATENA software, 2015).

$$\tau_b = \tau_{\max} \left(\frac{s}{s_1} \right)^\alpha, \quad 0 \leq s \leq s_1 \quad (10)$$

$$\tau_b = \tau_{\max}, \quad s_1 < s \leq s_2 \quad (11)$$

$$\tau_b = \tau_{\max} - (\tau_{\max} - \tau_f) \left(\frac{s - s_2}{s_3 - s_2} \right), \quad s_2 \leq s \leq s_3 \quad (12)$$

$$\tau_b = \tau_f, \quad s_3 < s \quad (13)$$

3. Graphic Results of the Stress-Strain Laws for Concrete and Steel Reinforcement in ATENA software

According to analytical studies conducted by Sococol et al. (Sococol *et al.*, 2020 a), (Sococol *et al.*, 2020 b), (Sococol *et al.*, 2020 c) with ATENA software (ATENA software, 2015) considering stress-strain relations for concrete and steel reinforcement from current research paper, it can be graphically observed the cracking mode of K_5 RC frame model (see Fig. 7) (Sococol *et al.*, 2020 c). The analytical results will superpose with the experimental results after seismic platform test of the optimal moment resisting RC frame model.

Fig. 7 – K_5 moment resisting RC frame system (Sococol *et al.*, 2020 c): (a) „Representation of the discrete system with lateral load consideration in the longitudinal direction”; (b) „Pushover loading consideration”; (c) „Steel reinforcement carcass”; (d), (e) „Structural mesh discretization” (Bitencourt *et al.*, 2018); (f) „Principal Fracture Strains Max (PFSM) representation”.

4. Conclusions

The necessity for knowledge of stress-strain relation laws for concrete and steel reinforcement in analytical studies of the RC frame structures is highly important. The validation of the final results for numerical analysis are closely related to the correct consideration of stress-strain material laws.

In the case of the numerical studies specified in Sococol *et al.*, ATENA software was used with stress-strain laws implementation in the chapters of this research article.

Thus, in the final stage of experimental study, it will be verified the results obtained for the optimal RC moment resisting frame model (which will be tested on the seismic platform) with the existing analytical results.

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RELAȚIILE TENSIUNE-DEFORMAȚIE SPECIFICĂ PENTRU BETON ȘI
ARMĂTURĂ UTILIZATE ÎN STUDIILE ANALITICE STATICE NELINIARE A
MODELELOR SEISMO-REZISTENTE TIP CADRU DE BETON ARMAT

(Rezumat)

În urma calculului analitic efectuat cu programul ATENA pentru o serie de trame (modele) tip cadru de beton armat în studiile de cercetare anterioare, s-au utilizat în etapa de pre-procesare relații tensiune-deformație specifică corespunzătoare betonului și armăturii. Aceste relații cu reprezentare matematică și grafică reprezintă o necesitate în condițiile actuale de analiză numerică și implică o cunoaștere corectă a modului de deformare a materialului compozit „beton armat”. Astfel, prin prezenta lucrare de cercetare, se dorește expunerea teoretică a: equivalent uniaxial law for concrete, biaxial compressive failure and tensile failure consideration laws for concrete, bilinear with hardening law for steel reinforcement, cycling steel reinforcement model and steel reinforcement bond model. În final, prin intermediul rezultatelor experimentale a modelului optim tip cadru de beton armat ce urmează să fie încercat pe platforma seismică se va putea valida corectitudinea modelelor analitice.