RANKING AND CLASSIFICATION OF CRYPTOCURRENCY EXCHANGES USING THE METHODS OF A MULTIDIMENSIONAL COMPARATIVE ANALYSIS

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Abstract

Research background: The multidimensional assessment of the attractiveness of cryptocurrency exchanges seems to be an important issue, because the risk of the collapse of such an exchange or its use for illegal purposes is higher than in the case of traditional exchanges.

Purpose: The aim of the work is to create ranking and identify groups of cryptocurrency exchanges with a similar level of attractiveness.

Research methodology: 13 different composite indicators were considered. Finally, one of them was chosen as a representative according to the similarity of the obtained rankings. Clustering methods were used to identify groups of exchanges with a similar level of the constructed measure.

Result: The best according to the adopted criteria of rankings similarity was the taxonomic measure constructed using the standardized sum method with equal weights. Combining hierarchical clustering with the *k-means* algorithm allowed to improve the quality of clustering measured with the silhouette index.

Novelty: The originality of the paper lies in the use of different methods of a multidimensional comparative analysis on the cryptocurrency market.

Keywords: cryptocurrency exchanges, composite indicator, hierarchical clustering, *k-means*, linear ordering, weighting schemes, investments attractiveness

JEL classification: C02, C38, C63

Introduction

Composite indicators (known also as taxonomic measures or synthetic variables) are one of the tools that enable a comparison of objects described by many characteristics. The idea of using the multidimensional comparative analysis on capital markets was proposed by Łuniewska and Tarczyński (2006). Taxonomic measures of attractiveness of investments (TMAI) are used, among others, to choose and optimize the shares portfolios (Tarczyński, 2013; Tarczyński, 2014; Węgrzyn, 2014; Zielińska-Sitkiewicz 2015). This method was also used to create the rankings and classifications of traditional exchanges (Kompa, Witkowska, 2014, 2015, 2016). The idea of using TMAI to assess investment attractiveness on cryptocurrency exchanges was presented in the work of Kądziołka (2016), however, at that time, investments concerned mainly the bitcoin cryptocurrency. In the mentioned work the following characteristics of cryptocurrency exchanges were analysed: average trading volume, rate of return, risk of investment and percentage of days on which the investment was not available to the users. Currently, it is possible to invest in many various cryptocurrencies, and there are available additional characteristics of exchanges, including for example indicators related to security, transaction risk, asset diversification and liquidity. Therefore, there was proposed TMAI, which takes into account these characteristics of cryptocurrency exchanges. The problem of the multidimensional assessment of cryptocurrency exchanges was also discussed in the works of Kądziołka (2021b, 2021d), concerning the problem of the selection of the linear ordering method. In the mentioned works only the standardized sum method was used.

The multidimensional assessment of cryptocurrency exchanges is an important issue, because many of them are closed within a year from opening (Kądziołka, 2017, p. 110). However, there is associated a certain element of subjectivism with the selection of components (diagnostic variables) of the taxonomic measure. For example, compared to the work of Kądziołka (2016), the trading volume is here omitted. The ability to quickly sell and buy cryptocurrencies at the market price undoubtedly contributes to the increase in the attractiveness of the cryptocurrency exchange. To enable this, the exchange should be characterized by high trading activity and the buying and selling prices should not differ much from each other. In the case of traditional exchanges, the volume reflects the activity of traders, but for cryptocurrency exchanges it does not seem to be a fully reliable indicator representing the traders' activity. Many internet portals rank such exchanges according to the volume, and therefore the volume on such exchanges is often object to manipulation. One of the ways of volume manipulation is the so-called *wash trading*, i.e. the buying and selling of one asset by the same person or exchange platform. After

each such transaction, the volume is higher. Considering the above, the volume was omitted by the construction of taxonomic measures.

There are many internet portals that allow to create rankings of cryptocurrency exchanges based on the value of one selected characteristic, e.g. trading volume (for example [bitcoincharts.com](https://www.bitcoincharts.com)). However, rankings based only on trading volume do not reflect the actual availability and ability of quickly selling or buying cryptocurrency at a price close to the market price, due to the frequent volume manipulations on these exchanges. Some portals introduced scoring systems that enable the creation of rankings of cryptocurrency exchanges based on different characteristics. Within each category, exchanges are awarded a certain number of points. The points are then summarized and normalized to values from the selected range, e.g. [0, 10]. These kinds of scoring systems were introduced, for example, by portals [coingecko.com](https://www.coingecko.com), [coinmarketcap.com](https://www.coinmarketcap.com) and [cryptocompare.com](https://www.cryptocompare.com). The scoring system operating on the portal [coingecko.com](https://www.coingecko.com) assigns to each exchange points from the set {1, 2, ..., 10}. The number of awarded points is a natural number, therefore there is no difference between the exchanges that obtained the same number of points. The [coinmarketcap.com](https://www.coinmarketcap.com) portal uses a scoring system based on indicators related to network traffic, liquidity and trading volume. The [cryptocompare.com](https://www.cryptocompare.com) portal uses many different characteristics within the scoring system, including for example the geographical location or the level of education of people managing the exchanges¹. There is implemented on the [cryptocompare.com](https://www.cryptocompare.com) portal a grading system, that assigns to each exchange a grade AA–F based on its total cumulative score.

The aim of work is to create rankings and identify groups of cryptocurrency exchanges with similar values of taxonomic measure of investment attractiveness. However, the construction of a taxonomic measure is not straightforward. It involves steps where an element of subjectivity is present, for example choice of the aggregation method and weighting scheme. Different weights of diagnostic variables may lead to different rankings. Different weighting schemes were compared and aggregation methods and the problem of selection of the final representative was discussed. After selection of the final taxonomic measure, a ranking of exchanges was created and there were identified groups of exchanges with similar levels of this measure. To achieve this goal, the *k-means* algorithm combined with the hierarchical clustering method was used. The initial number and values of clusters' centers for the *k-means* algorithm were determined using the hierarchical clustering method. All calculations were conducted using R software and publicly available data published on internet portals: www.coingecko.com, www.cryptocompare.com and www.coinmarketcap.com.

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¹ https://www.cryptocompare.com/media/35650785/cryptocompare_exchange_benchmarking_2019_06.pdf (3.12.2020).

1. Data and methods

By the construction of the taxonomic measure of investment attractiveness of cryptocurrency exchanges the following characteristics of exchanges were taken into account²:

1. Security Rank (x_1),
2. Liquidity indicator (x_2),
3. Number of different currencies that are available on the exchange (x_3); this variable is here named “Coins”,
4. Data provision indicator (x_4),
5. KYC/ Transaction Risk indicator (x_5),
6. Legal/ Reg indicator (x_6),
7. Existing time of an exchange (x_7).

The Security Rank indicator assesses the level of cybersecurity. It is estimated for cryptocurrency exchanges on the cer.live platform. This indicator is a certain aggregate measure that assesses the exposure of users and servers to threats from cyberspace. The higher the value of the indicator, the higher the level of safety of the exchange³.

The liquidity indicator is determined by the coinmarketcap.com portal. This indicator is an aggregate measure and takes into account various characteristics included in the so-called *order book*, such as order volume or distance from the average price. The higher the value of this indicator, the higher the liquidity of the cryptocurrency exchange, understood here as the possibility of quickly selling or buying cryptocurrencies at a price close to the market price⁴.

Data provision, KYC/Transaction Risk and Legal/Reg indicators are determined by the cryptocompare.com portal. Data provision indicator is an aggregate measure which takes into account various characteristics regarding the quality of services provided by the exchange, such as API's average response time (ms), ability to query historical trades, offering a web socket connection. The KYC/Transaction Risk indicator is an aggregate measure and takes into account various characteristics related to the anti-money laundering area, such as implementation of KYC (Know Your Customer) policies and monitoring of transactions, i.e., ability to identify and flag any suspicious flows of cryptocurrency that may come from illegal sources. Legal/Reg indicator is also an aggregate measure. It assesses some of the legal aspects of exchanges' functioning,

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² 80 exchanges were assessed, for which all of the characteristics were available. Some combinations of the presented diagnostic variables were used and described in the works of Kądziołka (2021b, 2021c, 2021d).

³ The methodology of the construction of a security rank indicator is described on the website: <https://cer.live/certified> (30.08.2020).

⁴ The methodology of the construction of a liquidity indicator is described on the website: <https://support.coinmarketcap.com/hc/en-us/articles/360043836931-Liquidity-Score-Market-Pair-Exchange-> (30.08.2020).

such as: legal exchange name, country risk rating, country cryptocurrency regulation, and insurance against losses⁵.

There is not universal method for constructing the TMAI. For this purpose, many different methods can be used. There are also different ways of data unification (e.g. normalization, standardization), which in consequence may lead to obtaining different rankings. Some of the methods for constructing taxonomic measures are presented, among others by Mastalerz-Kodzis, Pośpiech (2013), Dmitruk, Gawinecki (2017), Kukuła and Luty (2018). Used were the standardized sum method, Hellwig's method and TOPSIS procedure to construct taxonomic measures. These methods are described below.

1.1. Hellwig's method

The construction of a taxonomic measure using Hellwig's method involves the following steps (Bąk, 2018, pp. 21):

1. Standardization of the diagnostic variables according to the formula:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}$$

where x_{ij} is the value of j -th variable for i -th object, $\bar{x}_j$ – mean value of j -th variable, s_j – standard deviation of j -th variable, $i = 1, \dots, n, j = 1, \dots, m$.

2. Construction of the so-called development pattern, i.e. an abstract object with the “best” values of particular variables. Its coordinates are defined according to the formula:

$$z_{0j} = \begin{cases} \max_i \{z_{ij}\}, & \text{for stimulants} \\ \min_i \{z_{ij}\}, & \text{for destimulants} \end{cases}$$

A stimulant is a variable whose higher value indicates a higher level of the analysed phenomenon. A destimulant is a variable having a negative impact on this phenomenon (Młodak, 2006, p. 33). In the analysed case all diagnostic variables are stimulants.

3. Calculation for each analysed object with its distance from the pattern according to the formula:

$$d_{i0} = \sqrt{\sum_{j=1}^m (z_{ij} - z_{0j})^2}$$

⁵ The methodology of the construction of Data provision, KYC/Transaction Risk and Legal/Reg indicators is described on the website: www.cryptocompare.com/media/37072188/cryptocompare-exchange-benchmark-july-2020.pdf (3.12.2020).

4. Calculation of the synthetic variable according to the formula:

$$q_i = 1 - \frac{d_{i0}}{d_0}$$

where $d_0 = \bar{d}_0 + 2s_d$. The higher the value of this variable is the more attractive the object in term of adopted criteria is.

1.2. TOPSIS

In this method values of taxonomic measure are calculated according to the formula (Kukuła, Luty, 2018, pp. 187):

$$q_i = \frac{d_i^-}{d_i^- + d_i^+}$$

where:

$$d_i^+ = \sqrt{\sum_{j=1}^m (z_{ij} - z_j^+)^2}, \quad d_i^- = \sqrt{\sum_{j=1}^m (z_{ij} - z_j^-)^2}$$

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, \quad z_j^+ = \max_i \{z_{ij}\}, \quad z_j^- = \min_i \{z_{ij}\}$$

x_j – diagnostic variables in form of stimulants, $i = 1, \dots, n, j = 1, \dots, m$.

1.3. Standardized sum method

In this method values of taxonomic measure are calculated according to the formula:

$$TM_i = \sum_{j=1}^m w_j z_{ij}$$

where w_j is the weight of j -th variable, $i = 1, \dots, n, j = 1, \dots, m$ (here $n = 80, m = 7$, i.e. 80 exchanges and 7 diagnostic variables). In the case of stimulants, diagnostic variables are transformed according to the formula:

$$z_{ij} = \frac{x_{ij} - \min_i \{x_{ij}\}}{\max_i \{x_{ij}\} - \min_i \{x_{ij}\}}$$

In the analysed case six weighting schemes were considered: arbitrarily selected weights, equal weights, weights obtained with Papenbrock's method, weights based on coefficients of a variation of diagnostic variables, weights based on the importance of diagnostic variables and taxonomic measures obtained with a simulation method. The existing literature offers also many advanced methods for determining the weights of diagnostic variables, such as: PCA, mathematical programming, sensitivity analysis, multiple linear regression (Zhou, Ang, Poh, 2007; Becker, Saisana, Paruolo, Vandecasteele, 2017; Gan et al., 2017; Greco, Ishizka, Tasiou, Torrisi, 2019; Kuc-Czarnecka, 2019; Gomez-Limon, Arrizza, Guerrero-Baena, 2020). Each of the weighting schemes has its benefits and drawbacks. Below are the characterised applied weighting schemes.

1.4. Equal weights method

This method is based on the assumption, that all variables are of equal importance. In this case, equal weights are used i.e. $w_j = 1/m, j = 1, \dots, m$. It is one of the most often adopted methods, due to its simplicity. A taxonomic measure for which this weighting scheme was used is denoted TM_ew.

1.5. Method based on coefficients of variation

In this case, the weights of diagnostic variables are determined according to the formula:

$$w_j = \frac{V_j}{\sum_{k=1}^m V_k}$$

where V_j is the coefficient of variation of j -th variable, i.e. the ratio of the standard deviation to the mean (Dmitruk, Gawiniecki, 2017, p. 114). A taxonomic measure for which this weighting scheme was used is denoted TM_V.

1.6. Papenbrock's method

In this method, the weights are determined according to the dendrogram presenting correlation of diagnostic variables. As a measure of dissimilarity between variables the following formula was used: $d(X, Y) = 1 - |\rho_{XY}|$, where ρ_{XY} is the value of Spearman's correlation coefficient between the variables X and Y . The complete linkage method was used to linking clusters. In this method the distance between the clusters is equal to the greatest distance between any two objects belonging to different clusters (Stanisz, 2007, p. 120). To determine the weights of the variables, the method proposed by Papenbrock was used (2011, p. 47), in

which the weight at each next bisection of the tree is half the weight at the higher level. Figure 1 presents this weighting scheme in the analyzed case. A taxonomic measure for which this weighting scheme was used is denoted TM_P.

Figure 1. Weighting scheme based on Papenbrock's method

Source: own elaboration.

1.7. Rank – sum weights

This method may be used when the researcher can rank diagnostic variables in terms of their relative importance. In this situation each variable is weighted in proportion to its position in the rank order. Suppose that the researcher can rank diagnostic variables according to their importance and assign to each variable rank being an unique natural number from 1 to m , i. e. if $j \neq t$ then $r_j \neq r_t$. The weights are determined according to the formula (Jia, Fischer, Dyer, 1998, p. 91; Roszkowska, 2013, p. 20):

$$w_j = \frac{2(m+1-r_j)}{m(m+1)}$$

$j = 1, \dots, m$, r_j – rank of j -th variable. In the analysed case the following ranks of diagnostic variables were assumed: $r(x_2) = 1$, $r(x_1) = 2$, $r(x_4) = 3$, $r(x_5) = 4$, $r(x_7) = 5$, $r(x_6) = 6$, $r(x_3) = 7$. A taxonomic measure for which rank – sum weights method was used to establish the weights is denoted TM_RSW.

1.8. Simulation method for determining a composite indicator

This method consists of the following steps (Kądziołka, 2021a, p. 50):

1. Generate randomly s vectors of weights and determine s taxonomic measures based on the generated vectors of weights.
  2. Determine for values of each of the constructed taxonomic measures the mean and semi-standard deviation of Spearman's correlation coefficients with the values of other analysed taxonomic measures⁶.
  3. Determine a subset of the constructed measures such that for each taxonomic measure belonging to this subset there is no other taxonomic measure (among the initial set of s measures) with the higher mean of Spearman's correlation coefficients and lower or the same semi-standard deviation, or with the same mean of Spearman's correlation coefficients and lower semi-standard deviation.
  4. Select the final taxonomic measure from the reduced set based on the adopted criterion.
- In the analysed case all measures from the reduced set will be considered.

Figure 2. Mean and semi standard deviation of Spearman's correlation coefficients

Source: own elaboration.

The process of the construction of the taxonomic measure involving this method is presented below in detail. First, there were generated $s = 1,000$ weights vectors $(w_1^r, \dots, w_7^r)$ and 1,000 taxonomic measures were created: $TM_i^r = \sum_{j=1}^m w_j^r z_{ij}$, $r = 1, \dots, s$. Then, for the values

⁶ A semi-standard deviation was used that incorporates only the negative deviations from the mean value. Deviations above the target are a positive phenomenon. The higher the value of Spearman's correlation coefficient, the more similar were the rankings.

of all pairs of the taxonomic measures, the matrix of Spearman's correlation coefficients was created. It was symmetric matrix with a dimension of $1,000 \times 1,000$. Figure 2 presents the mean and semi-standard deviation of Spearman's correlation coefficients for the analysed measures.

Then, a subset of taxonomic measures, described in step 3 of the procedure, was created. Figure 3 shows the reduced set of taxonomic measures. The labels represent the identification numbers of the measures.

Figure 3. Reduced set of taxonomic measures

Source: own elaboration.

Table 1. Structure of weights of the analysed taxonomic measures

	w1	w2	w3	w4	w5	w6	w7
TM_ew	1/7	1/7	1/7	1/7	1/7	1/7	1/7
TM_arb	0.2000	0.200	0.120	0.120	0.1200	0.120	0.120
TM_211	0.1220	0.139	0.139	0.175	0.1400	0.175	0.110
TM_325	0.1710	0.130	0.186	0.108	0.1130	0.168	0.124
TM_333	0.1620	0.152	0.128	0.149	0.1740	0.130	0.105
TM_701	0.1910	0.081	0.179	0.147	0.1000	0.149	0.153
TM_918	0.1580	0.106	0.175	0.122	0.1460	0.147	0.146
TM_940	0.0740	0.079	0.262	0.150	0.1280	0.153	0.154
TM_P	0.0625	0.125	0.250	0.125	0.0625	0.250	0.125
TM_V	0.1370	0.170	0.246	0.099	0.0920	0.111	0.145
TM_RSW	0.2140	0.250	0.036	0.179	0.1430	0.071	0.107

Source: own elaboration.

Table 1 shows the weights' structures of the analysed taxonomic measures created with the use of the standardized sum method. A taxonomic measure with arbitrarily set weights

is denoted TM_arb . It was assumed in the case of arbitrarily selected weights that the most important characteristics are liquidity and security of the exchange.

2. Selection of the final taxonomic measure

Spearman's correlation coefficients for the values of the analyzed taxonomic measures were in the range $[0.841, 0.994]$. It suggested a high consistency of the ordering of exchanges in the obtained rankings. However, differences were observed in the positions of the exchanges. This situation is presented with the example of exchanges' rankings according to the values of the measures TM_P i TM_RSW (Figure 4).

Figure 4. Positions of exchanges according to the values of TM_P and TM_RSW

Source: own elaboration.

The question arises, which ranking should be selected as the solution of the problem of linear ordering of exchanges and according to which criteria the selection should be done? There is no universal solution, how to select the final representative. This issue was discussed in the works of Kukula and Luty (2015) and Kądziołka (2021b, 2021d). Kukula and Luty (2015, p. 223) proposed to select this measure, for which the function u_p , where:

$$u_p = \frac{1}{k-1} \sum_{q=1, q \neq p}^k m_{pq}$$

reaches the maximal value. In this formula k is the number of analysed rankings, $p, q = 1, \dots, k$ – indexes of analysed rankings, m_{pq} is the measure of the similarity of rankings p and q , determined according to the following formula:

$$m_{pq} = 1 - \frac{2 \sum_{i=1}^n |c_{ip} - c_{iq}|}{n^2 - z}$$

c_{ip} – position of i -th object in p -th ranking, c_{iq} – position of i -th object in q -th ranking, n – number of objects, $z = 1$, if n is an odd number, $z = 0$, if n is an even number. Kądziołka (2021b) proposed a procedure for selecting the final representative based on the discrimination ability of taxonomic measures and the similarity of the obtained rankings. Kądziołka (2021d) proposed an aggregate measure for assessing the correctness of linear ordering methods. Each of these methods has its own logic and their results can differ from each other. There is an open question, which of them should be used to choose the final linear ordering method?

In this paper, as the final representative will be chosen this measure, for which the mean value of Spearman's correlation coefficients will reach the maximal value. The mean of Spearman's correlation coefficients was defined according to the formula (Kądziołka, 2021c, p. 136):

$$Mean\_CS_i = \frac{1}{k-1} \sum_{j=1, j \neq i}^k \rho_{ij}$$

where ρ_{ij} is Spearman's correlation coefficient between the values of i -th and j -th taxonomic measure, k is the number of analysed taxonomic measures (here: $k = 13$).

Table 2. Values of the *Mean_CS* for analysed taxonomic measures

Taxonomic measure	Mean_CS
TM_ew	0.97407
TM_325	0.97248
TM_918	0.97159
TM_211	0.97042
TM_333	0.96594
TM_arb	0.96591
TM_701	0.96579
TOPSIS	0.96132
TM_940	0.95776
Hellwig's method	0.95211
TM_V	0.94730
TM_P	0.93088
TM_RSW	0.92564

Source: own elaboration

Table 2 presents values of the *Mean_CS* for the analysed taxonomic measures. In the analysed case, the highest value of this indicator was obtained for *TM_ew*, which was chosen as the final representative.

3. Ranking and classification of cryptocurrency exchanges

Cryptocurrency exchanges were ordered from the best to the worst according to the values of *TM_ew* (Table 3). Then, there were identified groups of exchanges with similar values of the final composite indicator. In many works dealing with the issue of identifying groups of objects with similar levels of taxonomic measure, the objects were divided into four groups according to the mean and standard deviation of the value of synthetic variable (e.g. Roeske-Słomka, 2003, p. 81; Kompa, Witkowska, 2015, p. 218–219; Kompa, Witkowska, 2016, p. 30; Malina, 2020, p. 146; Skica, Rodzinka, Zaremba, 2020, p. 304). However, the division into four groups is not always optimal. In this study hierarchical clustering was used to determine the number of groups of similar exchanges. The complete linkage method was used to linking clusters. Figure 5 presents the obtained dendrogram. The analysed exchanges were divided into three groups. The place of the division of the dendrogram is marked by a dashed line.

Figure 5. Division of exchanges according to the dendrogram

Source: own elaboration.

The quality of clustering was assessed with the so-called *silhouette index*⁷. The mean silhouette index was equal to 0.57. This result suggested the good quality of the clustering (Prus, Król, 2017, p. 187). However, silhouette indicators for some of the exchanges were negative numbers. It suggested, that these exchanges were placed into the wrong groups. The number of groups and clusters' centers obtained with hierarchical clustering were then used in the *k-means* algorithm. Optimizing the results of hierarchical clustering with the *k-means* algorithm⁸ allowed solving the mentioned problem of negative silhouette indexes for some exchanges and thus obtaining a higher mean silhouette index, which finally was equal to 0.6. Table 3 presents the values of TM_ew and information about groups obtained with the combined algorithm⁹.

Table 3. Ranking and classification of cryptocurrency exchanges

Rank	Cryptocurrency exchange	TM_ew	Group	Rank	Cryptocurrency exchange	TM_ew	Group
1	2	3	4	5	6	7	8
1	HitBTC	0.71959	1	41	BTCBOX	0.37357	2
2	Binance	0.68446	1	42	BitMax	0.37124	2
3	Coinbase Pro	0.65242	1	43	P2PB2B	0.36969	2
4	OKEx	0.63912	1	44	Graviex	0.36494	2
5	Gate.io	0.63910	1	45	CoinBene	0.36202	2
6	Bitfinex	0.63261	1	46	Exrates	0.35924	2
7	Kraken	0.63246	1	47	BtcTurk PRO	0.35672	2
8	Huobi Global	0.61552	1	48	BTCMarkets	0.35600	2
9	Gemini	0.59628	1	49	BTC-Alpha	0.35309	2
10	Bittrex	0.59045	1	50	Lbank	0.34985	2
11	KuCoin	0.58153	1	51	ZBG	0.34923	2
12	Liquid	0.58135	1	52	Digifinex	0.34802	2
13	Poloniex	0.56564	1	53	BitMart	0.34786	2
14	Binance US	0.55460	1	54	CoinEx	0.34683	2
15	Bitstamp	0.52580	1	55	Coinone	0.34669	2
16	itBit	0.52210	1	56	Bleutrade	0.33790	2
17	Bitcoin.com	0.51980	1	57	Bitso	0.33305	2
18	bitFlyer	0.49067	2	58	CoinTiger	0.32464	2

⁷ The construction of this index is described in detail in the work of Rousseeuw (1986).

⁸ The existing literature offers many variants of the *k-means* algorithm. Lloyds algorithm was used and is described among others by Morissette and Chartier (2013).

⁹ There was also considered a division of exchanges into 6 groups, but the mean silhouette index for the combined algorithm was lower than in the case of 3 groups and for hierarchical clustering, there was the problem with the negative values of the silhouette index for some of the exchanges. Hierarchical clustering (Ward's method) combined with the *k-means* algorithm was also used in the work of Kądziołka (2021b). Although there were some differences between the sets of diagnostic variables in these works, the divisions of exchanges into three groups were quite similar.

1	2	3	4	5	6	7	8
19	OKCoin	0.48431	2	59	CoinJar Exchange	0.32390	2
20	FTX	0.45809	2	60	GoPax	0.31636	3
21	Bitbank	0.45405	2	61	Bitpanda Pro	0.30068	3
22	CEX.IO	0.45217	2	62	Kuna Exchange	0.29914	3
23	AAX	0.45114	2	63	Korbit	0.29222	3
24	TheRockTrading	0.44396	2	64	eToroX	0.29065	3
25	Coincheck	0.44186	2	65	LakeBTC	0.28414	3
26	Coinfield	0.43831	2	66	Coindeal	0.28336	3
27	Independent Reserve	0.43804	2	67	Livecoin	0.27835	3
28	Coinfloor	0.43319	2	68	Coinfalcon	0.26797	3
29	Coinsbit	0.42450	2	69	Lykke	0.26755	3
30	Upbit	0.41359	2	70	Bitkub	0.26133	3
31	ZB	0.40305	2	71	FatBTC	0.25292	3
32	Luno	0.39975	2	72	CoinAll	0.24045	3
33	BitBay	0.39546	2	73	BCEX	0.23925	3
34	BW.com	0.39474	2	74	EXX	0.23699	3
35	STEX	0.39073	2	75	Bgogo	0.22789	3
36	EXMO	0.38858	2	76	Tidex	0.20308	3
37	DSX Global	0.38061	2	77	TokensNet	0.18729	3
38	Currency.com	0.38057	2	78	Catex	0.18650	3
39	BigONE	0.37531	2	79	CBX	0.13905	3
40	Zaif	0.37475	2	80	BITEXBOOK	0.13195	3

Source: own elaboration.

Figure 6. Mean values of TM_ew and its components for particular groups

Source: own evaluation.

Figure 6 presents the mean values of TM_{ew} and its components for particular groups. Exchanges from the first group were characterized by the highest mean value of taxonomic measure and all of its components. Exchanges from the third group were characterized by the lowest mean value of TM_{ew} and its components.

Concluding remarks

A multidimensional assessment of the attractiveness of cryptocurrency exchanges seems to be more comprehensive and reliable than one-dimensional analyzes based, for example, only on the trading volume, which is often manipulated on such exchanges. There is no universal method for constructing a taxonomic measure. Different taxonomic measures may produce different rankings and the question arises, which ranking should be selected as the solution of the problem of linear ordering of objects? A method for selecting the final synthetic variable was used and based on the measure of the consistency of exchanges' ordering. To identify groups of exchanges with similar levels of the TMAI's value hierarchical clustering combined with the *k-means* algorithm was applied. Hierarchical clustering makes it possible to divide the objects into different numbers of groups according to the dendrogram. In the analysed case, combining hierarchical clustering with the *k-means* algorithm allowed to improve the quality of clustering.

Taxonomic measures can be used to rank objects described by many characteristics. This approach enables identifying groups of objects similar in terms of aggregate value. Finally, it seems worthwhile to use additionally nonlinear ordering methods to recognize patterns and relationships in diagnostic variables.

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