A new subclass of meromorphic functions with positive coefficients defining by linear operator

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Abstract

In this paper, we introduce and study a new class $\sigma, (\alpha, \lambda)$ of meromorphic univalent functions defined in $E = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\} = E \setminus \{0\}$. We obtain coefficient inequalities, distortion theorems, extreme points, closure theorems, radius of convexity estimates and integral operators. Finally, we obtained neighbourhood result for the class $\sigma_p(\gamma, \lambda)$.

Mathematics Subject Classification 2010: 30C45.

Keywords: meromorphic functions, analytic functions, neighborhood.

1. INTRODUCTION

Let Σ denote the class of meromorphic functions of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \quad (a_n \geq 0) \quad (1)$$

which are analytic in the punctured unit disc $E = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\}$. A function $f \in \Sigma$ is meromorphic starlike of order α ($0 \leq \alpha < 1$) if

$$-Re \left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha, \quad (z \in E). \quad (2)$$

The class of all such functions is denoted by $\Sigma^*(\alpha)$. A function $f \in \Sigma$ is meromorphic convex of order α ($0 \leq \alpha < 1$) if

$$-Re \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \quad (z \in E). \quad (3)$$

The class of such functions is denoted by $\Sigma_k^*(\alpha)$. The class $\Sigma(\alpha)$ and $\Sigma_k^*(\alpha)$ were introduced and studied by Clunie [1], Pommerenke [6], Miller [4] and Mogra et al. [5]. Let Σ_p be the class of functions $f \in \Sigma$ with $a_n \geq 0$. The subclass of Σ_p consisting of starlike function of order α is denoted by $\Sigma_p^*(\alpha)$. For a function $f(z) \in \Sigma$, Frasin

and Darus [2] defined an operator $I^k : \Sigma \rightarrow \Sigma$ as follows

$$\begin{aligned} I^0 f(z) &= f(z) \\ I^1 f(z) &= zf'(z) + \frac{2}{z} \\ I^2 f(z) &= z(I^1 f(z))' + \frac{2}{z} \end{aligned}$$

and for $k = 1, 2, 3, \dots$, we have

$$\begin{aligned} I^k f(z) &= z(I^{k-1} f(z))' + \frac{2}{z} \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} n^k a_n z^k \quad (k \in N_0 = N \cup \{0\}, z \in E). \end{aligned}$$

Definition 1.1. Let $0 \leq \alpha < 1$. Further let $f(z) \in \Sigma_p$ be given by (1), $0 \leq \lambda < 1$. The class $\sigma_p(\alpha, \lambda)$ is defined by

$$\sigma_p(\alpha, \lambda) = \left\{ f \in \Sigma_p : \operatorname{Re} \left[\frac{zf'(z)}{(\lambda - 1)f(z) + \lambda zf'(z)} \right] > \alpha \right\}. \quad (4)$$

Clearly $\sigma_p(\alpha, 0)$ reduces to $\Sigma_p^*(\alpha)$.

In this paper, we obtain the coefficient inequalities, growth and distortion inequalities, as well as closure results for the class $\sigma_p(\alpha, \lambda)$. Properties of certain integral operators and neighbourhood properties are also discussed for the class. The subclass of Σ_p consisting of starlike functions of order α is denoted by $\Sigma_p^*(\alpha)$.

2. COEFFICIENTS INEQUALITIES

Our first theorem gives a necessary and sufficient condition for a function f to be in the class $\Sigma_p(\lambda, \alpha)$.

THEOREM 2.1. Let $f(z) \in \Sigma_p$ be given by (1). Then $f \in \sigma_p(\lambda, \alpha)$ if and only if

$$\sum_{n=1}^{\infty} n^k [n + \alpha - \alpha\lambda(1+n)] a_n \leq 1 - \alpha. \quad (5)$$

PROOF. If $f \in \sigma_p(\lambda, \alpha)$ then

$$\operatorname{Re} \left\{ \frac{z(I^k f(z))'}{(\lambda - 1)(I^k f(z)) + \lambda z(I^k f(z))'} \right\} = \operatorname{Re} \left\{ \frac{-1 + \sum_{n=1}^{\infty} n^{k+1} a_n z^{n+1}}{-1 + \sum_{n=1}^{\infty} (\lambda - 1 + \lambda n) n^k a_n z^{n+1}} \right\} > \alpha.$$

By letting $z \rightarrow 1^-$, we have $\left\{ \frac{-1 + \sum_{n=1}^{\infty} n^{k+1} a_n}{-1 + \sum_{n=1}^{\infty} n^k (\lambda - 1 + \lambda n) a_n} \right\} > \alpha$.

This shows that (5) holds.

Conversely assume that (5) holds. Since

$Re(w) > \alpha$ if and only if $|w - 1| < |w + 1 - 2\alpha|$.

It is sufficient to show that

$$\left| \frac{z(I^k f(z))' - [(\lambda - 1)I^k f(z) + \lambda z(I^k f(z))']}{z(I^k f(z))' + (1 - 2\alpha)[(\lambda - 1)I^k f(z) + \lambda z(I^k f(z))']} \right| < 1.$$

Using (5), we see that

$$\begin{aligned} & \left| \frac{z(I^k f(z))' - [(\lambda - 1)I^k f(z) + \lambda z(I^k f(z))']}{z(I^k f(z))' + (1 - 2\alpha)[(\lambda - 1)I^k f(z) + \lambda z(I^k f(z))']} \right| \\ &= \left| \frac{\sum_{n=1}^{\infty} n^k (1 - \lambda)(n + 1) a_n z^{n+1}}{-2(1 - \alpha) + \sum_{n=1}^{\infty} n^k [n(1 + (1 - 2\alpha)\lambda) + (1 - 2\alpha)(\lambda - 1)] a_n z^{n+1}} \right| \\ &\leq \frac{\sum_{n=1}^{\infty} n^k (1 - \lambda)(n + 1) a_n}{2(1 - \alpha) - \sum_{n=1}^{\infty} n^k [n(1 + (1 - 2\alpha)\lambda) + (1 - 2\alpha)(\lambda - 1)] a_n} \\ &\leq 1. \end{aligned}$$

Thus we have $f \in \sigma_p(\lambda, \alpha)$.

COROLLARY 2.2. If $f \in \sigma_p(\lambda, \alpha)$ then

$$a_n \leq \frac{(1 - \alpha)}{n^k [n + \alpha - \alpha\lambda(1 + n)]}.$$

PROOF. The result is sharp for the functions $F_n(z)$ given by

$$F_n(z) = \frac{1}{z} + \frac{(1 - \alpha)}{n^k [n + \alpha - \alpha\lambda(1 + n)]} z^n, \quad n = 1, 2, 3, \dots$$

3. GROWTH AND DISTORTION THEOREM

THEOREM 3.1. If $f \in \sigma_p(\lambda, \alpha)$, then

$$\frac{1}{r} - \frac{(1 - \alpha)}{(1 + \alpha - 2\alpha\lambda)} r \leq |f(z)| \leq \frac{1}{r} + \frac{(1 - \alpha)}{(1 + \alpha - 2\alpha\lambda)} r. \quad (6)$$

The result is sharp for the function

$$f(z) = \frac{1}{z} + \frac{1-\alpha}{1+\alpha-2\alpha\lambda} z. \quad (7)$$

PROOF. Since $f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n$, we have

$$|f(z)| \leq \frac{1}{r} + \sum_{n=1}^{\infty} a_n r^n \leq \frac{1}{r} + r \sum_{n=1}^{\infty} a_n$$

since,

$$\sum_{n=1}^{\infty} a_n \leq \frac{1-\alpha}{1+\alpha-2\alpha\lambda}.$$

Using this, we have

$$|f(z)| \leq \frac{1}{r} + \frac{1-\alpha}{1+\alpha-2\alpha\lambda} r.$$

Similarly

$$|f(z)| \geq \frac{1}{r} - \frac{1-\alpha}{1+\alpha-2\alpha\lambda} r.$$

The result is sharp for $f(z) = \frac{1}{z} + \frac{1-\alpha}{1+\alpha-2\alpha\lambda} z$.

THEOREM 3.2. If $f \in \sigma_p(\alpha, \lambda)$ then

$$\frac{1}{r^2} - \frac{1-\alpha}{1+\alpha-2\alpha\lambda} \leq |f'(z)| \leq \frac{1}{r^2} + \frac{1-\alpha}{1+\alpha-2\alpha\lambda} \quad (|z| = r).$$

The result is sharp for the function given by (7).

4. CLOSURE THEOREMS

Let the functions $F_k(z)$ be given by

$$F_k(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,k} z^n, \quad k = 1, 2, 3, \dots \quad (8)$$

We shall prove the following closure theorems for the class $\sigma_p(\lambda, \alpha)$.

THEOREM 4.1. Let the function defined by (8) be in the class $\sigma_p(\lambda, \alpha)$ for every $k = 1, 2, 3, \dots$. Then the function $f(z)$ defined by

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \quad (a_n \geq 0)$$

belongs to the class $\sigma_p(\lambda, \alpha)$, where $a_n = \frac{1}{m} \sum_{k=1}^m a_{n,k}$ ($n = 1, 2, \dots$).

PROOF. Since $F_k(z) \in \sigma_p(\lambda, \alpha)$, it follows from Theorem 2.1 that,

$$\sum_{n=1}^{\infty} n^k [n + \alpha - \alpha\lambda(1+n)] a_{n,k} \leq 1 - \alpha, \text{ for all } k = 1, 2, \dots, m. \quad (9)$$

Hence

$$\begin{aligned} & \sum_{n=1}^{\infty} n^k [n + \alpha - \alpha\lambda(1+n)] a_n \\ &= \sum_{n=1}^{\infty} n^k [n + \alpha - \alpha\lambda(1+n)] \left(\frac{1}{m} \sum_{k=1}^m a_{n,k} \right) \\ &= \frac{1}{m} \sum_{n=1}^{\infty} \left(\sum_{k=1}^m n^k [n + \alpha - \alpha\lambda(1+n)] a_{n,k} \right) \\ &\leq 1 - \alpha. \end{aligned}$$

By Theorem 2.1, we have $f(z) \in \sigma_p(\lambda, \alpha)$.

THEOREM 4.2. The class $\sigma_p(\alpha, \lambda)$ is closed under convex linear combination.

PROOF. Let the function $F_k(z)$ given by (8) be in the class $\sigma_p(\alpha, \lambda)$. Then it is enough to show that the function

$$H(z) = \lambda F_1(z) + (1 - \lambda) F_2(z), \quad (0 \leq \lambda \leq 1)$$

is also in the class $\sigma_p(\alpha, \lambda)$. Since for $(0 \leq \lambda \leq 1)$,

$$H(z) = \frac{1}{z} + \sum_{n=1}^{\infty} [\lambda a_{n,1} + (1 - \lambda) a_{n,2}] z^n.$$

We observe that

$$\begin{aligned} & \sum_{n=1}^{\infty} \{n^k [n + \alpha - \alpha\lambda(1+n)]\} [\lambda a_{n,1} + (1 - \lambda) a_{n,2}] \\ &= \lambda \sum_{n=1}^{\infty} \{n^k [n + \alpha - \alpha\lambda(1+n)]\} a_{n,1} + (1 - \lambda) \sum_{n=1}^{\infty} \{n^k [n + \alpha - \alpha\lambda(1+n)]\} a_{n,2} \\ &\leq 1 - \alpha. \end{aligned}$$

By Theorem 2.1, we have $H(z) \in \sigma_p(\alpha, \lambda)$.

THEOREM 4.3. Let $F_0(z) = 1$ and

$$F_n(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{1 - \alpha}{n^k [n + \alpha - \alpha\lambda(1+n)]} z^n, \text{ for } n = 1, 2, \dots.$$

Then $f(z) \in \sigma_p(\alpha, \lambda)$ if and only if $f(z)$ can be expressed in the form $f(z) = \sum_{n=0}^{\infty} \lambda_n F_n(z)$, where $\lambda_n \geq 0$ and $\sum_{n=0}^{\infty} \lambda_n = 1$.

PROOF. Let

$$\begin{aligned} f(z) &= \sum_{n=0}^{\infty} \lambda_n F_n(z) \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} \frac{\lambda_n(1-\alpha)}{n^k[n+\alpha-\alpha\lambda(1+n)]} z^n. \end{aligned}$$

Then

$$\begin{aligned} &\sum_{n=1}^{\infty} \frac{\lambda_n(1-\alpha)}{n^k[n+\alpha-\alpha\lambda(1+n)]} \frac{n^k[n+\alpha-\alpha\lambda(1+n)]}{(1-\alpha)} \\ &= \sum_{n=1}^{\infty} \lambda_n = 1 - \lambda_0 \leq 1. \end{aligned}$$

By Theorem 2.1, we have $H(z) \in \sigma_p(\alpha, \lambda)$.

Conversely, let $f(z) \in \sigma_p(\alpha, \lambda)$. From Theorem 2.1, we have

$$a_n \leq \frac{1-\alpha}{n^k[n+\alpha-\alpha\lambda(1+n)]} \text{ for } n = 1, 2, \dots.$$

We may take $\lambda_n = \frac{n^k[n+\alpha-\alpha\lambda(1+n)]}{1-\alpha} a_n$ for $n = 1, 2, \dots$ and $\lambda_0 = 1 - \sum_{n=1}^{\infty} \lambda_n$.

Then $f(z) = \sum_{n=0}^{\infty} \lambda_n F_n(z)$.

5. RADIUS OF MEROMORPHIC STARLIKENESS AND MEROMORPHIC CONVEXITY

THEOREM 5.1. Let the function f be in the class $\sigma_p(\alpha, \lambda)$. Then f is meromorphically starlike of order ρ ($0 \leq \rho < 1$) in $|z| < r_1(\alpha, \lambda, \rho)$, where

$$r_1(\alpha, \lambda, \rho) = \inf_{n \geq 1} \left[\frac{(1-\rho)(1-\alpha)}{n^k(n+2-\rho)[n+\alpha-\alpha\lambda(1+n)]} \right]^{\frac{1}{n+1}}. \quad (10)$$

PROOF. Let $f(z)$ is in $\sigma_p(\alpha, \lambda)$. Then by Theorem 2.1, we have

$$n^k[n+\alpha-\alpha\lambda(1+n)] a_n \leq (1-\alpha). \quad (11)$$

It is sufficient to show that

$$\left| 1 + \frac{zf'(z)}{f(z)} \right| \leq 1 - \rho. \quad (12)$$

Or equivalently

$$\left| 1 + \frac{zf'(z)}{f(z)} \right| = \left| \frac{\sum_{n=1}^{\infty} (n+1)a_n z^n}{\frac{1}{z} - \sum_{n=1}^{\infty} a_n z^n} \right| \leq 1 - \rho.$$

or

$$\sum_{n=1}^{\infty} \frac{(n+2-\rho)}{1-\rho} a_n |z|^{n+1} \leq 1, \text{ for } 0 \leq \rho < 1 \text{ and } |z| < r_1(\alpha, \lambda, \rho).$$

By theorem 2.1, (12) will be true if

$$\left(\frac{n+2-\rho}{1-\rho} \right) |z|^{n+1} \leq \frac{1-\alpha}{n^k [n+\alpha-\alpha\lambda(1+n)]}$$

or if

$$|z| \leq \left[\frac{1-\alpha}{(n+2-\rho)\{n^k[n+\alpha-\alpha\lambda(1+n)]\}} \right]^{\frac{1}{n+1}}, n \geq 1.$$

This completes the proof of Theorem.

THEOREM 5.2. Let the function $f(z)$ be in the class $\sigma_p(\alpha, \lambda)$. Then f is meromorphically convex of order ρ ($0 \leq \rho < 1$) in $|z| < r_2(\alpha, \lambda, \rho)$, where

$$r_2(\alpha, \lambda, \rho) = \inf_{n \geq 1} \left[\frac{(1-\rho)\{n^k[n+\alpha-\alpha\lambda(n+1)]\}}{n(n+2-\rho)(1-\alpha)} \right]^{\frac{1}{n+1}}, n \geq 1. \quad (13)$$

PROOF. Let $f(z)$ be in $\sigma_p(\alpha, \lambda)$. Then by Theorem 2.1, we have

$$\sum_{n=1}^{\infty} n^k [n+\alpha-\alpha\lambda(1+n)] a_n \leq (1-\alpha). \quad (14)$$

It is sufficient to show that

$$\left| 2 + \frac{zf''(z)}{f'(z)} \right| \leq 1 - \delta.$$

for $|z| < r_2 = r_2(\alpha, \lambda, \rho)$, where $r_2(\alpha, \lambda, \rho)$ is specified in the statement of the Theorem. Then

$$\left| 2 + \frac{zf''(z)}{f'(z)} \right| = \left| \frac{\sum_{n=1}^{\infty} n(n+1)a_n z^{n-1}}{\frac{1}{z} + \sum_{n=1}^{\infty} na_n z^{n-1}} \right| \leq \frac{\sum_{n=1}^{\infty} n(n+1)a_n |z|^{n-1}}{1 - \sum_{n=1}^{\infty} na_n |z|^{n-1}}.$$

This will be bounded by $(1-\rho)$ if

$$\sum_{n=1}^{\infty} \frac{n(n+2-\delta)}{1-\delta} a_n |z|^{n+1} \leq 1, \quad (15)$$

By (14), it follows that (15) is true if

$$\frac{n(n+2-\rho)}{1-\rho}|z|^{n+1} \leq \frac{n^k[n+\alpha-\alpha\lambda(1+n)]}{1-\alpha}, \quad n \geq 1$$

or

$$|z| \leq \left[\frac{(1-\rho)\{n^k[n+\alpha-\alpha\lambda(1+n)]\}}{n(n+2-\rho)(1-\alpha)} \right]^{\frac{1}{n+1}}, \quad n \geq 1. \quad (16)$$

This completes the proof of Theorem.

6. INTEGRAL OPERATORS

In this section, we consider integral transform of functions in the class $\sigma_p(\alpha, \lambda)$.

THEOREM 6.1. Let the function $f(z)$ given by (1) be in $\sigma_p(\alpha, \lambda)$. Then the integral operator

$$F(z) = c \int_0^1 u^c f(uz) du \quad (0 \leq u \leq 1, 0 < c < \infty)$$

is in $\sigma_p(\delta, \lambda)$, where

$$\delta = \frac{(c+2)(1+\alpha-2\alpha\lambda) - c(1-\alpha)}{c(1-\alpha)(1-2\lambda) + (1+\alpha)(1-2\lambda)(c+2)}.$$

The result is sharp for the function

$$f(z) = \frac{1}{z} + \frac{1-\alpha}{(1+\alpha-2\alpha\lambda)}z.$$

PROOF. Let $f(z) \in \sigma_p(\alpha, \lambda)$. Then

$$\begin{aligned} F(z) &= c \int_0^1 u^c f(uz) du \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} \frac{c}{c+n+1} a_n z^n. \end{aligned}$$

It is sufficient to show that

$$\sum_{n=1}^{\infty} \frac{c\{n^k[n+\delta-\delta\lambda(1+n)]\}}{(c+n+1)(1-\delta)} a_n \leq 1. \quad (17)$$

Since $f \in \sigma_p(\alpha, \lambda)$, we have

$$\sum_{n=1}^{\infty} \frac{n^k[n+\alpha-\alpha\lambda(1+n)]}{(1-\alpha)} a_n \leq 1. \quad (18)$$

Note that (17) is satisfied if

$$\frac{c[n + \delta - \delta\lambda(1+n)]}{(c+n+1)(1-\delta)} \leq \frac{[n + \alpha - \alpha\lambda(1+n)]}{(1-\alpha)}.$$

Solving for δ , we have

$$\delta \leq \frac{(c+n+1)[n + \alpha - \alpha\lambda(1+n)] - cn(1-\alpha)}{c(1-\alpha)[1-\lambda(1+n)] + [n + \alpha - \alpha\lambda(1+n)](c+n+1)} = G(n).$$

A simple computation will show that $G(n)$ is increasing and $G(n) \geq G(1)$. Using this, the result follows.

For the choice of $\lambda = 0$, we have the following result of Uralegaddi and Gangi [8].

Remark 6.2. Let the function $f(z)$ defined by (1) be in $\Sigma_p^*(\alpha)$. Then integral operator

$$F(z) = c \int_0^1 u^c f(uz) du \quad (0 < u \leq 1, 0 < c < \infty)$$

is in $\Sigma_p^*(\alpha)$, where $\delta = \frac{1+\alpha+c\alpha}{1+\alpha+c}$.

The result is sharp for the function

$$f(z) = \frac{1}{z} + \frac{1-\alpha}{1+\alpha}z.$$

7. NEIGHBOURHOODS FOR THE CLASS $\Sigma_p^\gamma(\alpha, \lambda)$

In this section, we determine the neighbourhood for the class $\Sigma_p^\gamma(\alpha, \lambda)$, which we define as follows.

Definition 7.1. A function $f \in \Sigma_p$ is said to be in the class $\sigma_p^\gamma(\alpha, \lambda)$ if there exists a function $g \in \Sigma_p^\gamma(\alpha, \lambda)$ such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - \gamma, \quad (z \in E, 0 \leq \gamma < 1). \quad (19)$$

Following the earlier works on neighbourhoods of analytic functions by Goodman [3] and Ruscheweyh [7], we define the δ -neighbourhood of function $f \in \Sigma_p$ by

$$N_\delta(f) = \left\{ g \in \Sigma_p : g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \text{ and } \sum_{n=1}^{\infty} n|a_n - b_n| \leq \delta \right\}. \quad (20)$$

THEOREM 7.2. If $g \in \sigma_p(\alpha, \lambda)$ and

$$\gamma = 1 - \frac{\delta(1 + \alpha - 2\alpha\lambda)}{2\alpha - 2\alpha\lambda} \quad (21)$$

then $N_\delta(g) \subset \sigma_p^\gamma(\alpha, \lambda)$.

PROOF. Let $f \in N_\delta(g)$. Then we find from (20) that

$$\sum_{n=1}^{\infty} n|a_n - b_n| \leq \delta, \quad (22)$$

which implies the coefficient inequality

$$\sum_{n=1}^{\infty} |a_n - b_n| \geq \delta \quad (n \in N). \quad (23)$$

Since $g \in \sigma_p(\alpha, \lambda)$, we have

$$\sum_{n=1}^{\infty} b_n \leq \frac{1 - \alpha}{1 + \alpha - 2\alpha\lambda}. \quad (24)$$

So that

$$\begin{aligned} \left| \frac{f(z)}{g(z)} - 1 \right| &< \frac{\sum_{n=1}^{\infty} |a_n - b_n|}{1 - \sum_{n=1}^{\infty} b_n} \\ &= \frac{\delta(1 + \alpha - 2\alpha\lambda)}{2\alpha - 2\alpha\lambda} \\ &= 1 - \gamma \end{aligned}$$

provided γ is given by (21). Hence by definition, $f \in \sigma_p^\gamma(\alpha, \lambda)$.

ACKNOWLEDGEMENTS

The authors express their sincere thanks to the esteemed referee(s) for their careful readings, valuable suggestions and comments, which helped them to improve the presentation of the paper.

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