A stochastic equivalence approach for an Ornstein-Uhlenbeck process driven power system dynamics

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Abstract

This paper develops a stochastic equivalence approach for an Ornstein-Uhlenbeck process-driven power system. The concept of stochastic equivalence coupled with stochastic differential rule plays the important role to develop the stochastic equivalence approach of this paper. This paper also develops the prediction theory of power system dynamics with the OU process as well.

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Additional Key Words and Phrases: stochastic equivalence approach, stochastic differential equation, the OU process, Kuramoto oscillator, power system dynamics.

1. INTRODUCTION

The stochastic differential equation (SDE) is caused by random term. This random term can be represented in form of white noise process or coloured noise process. For deterministic representation of the swing equation of a single machine-infinite bus (SMIB) system, see Kundur (1994). The white noise driven-Itô SDE model for a power system dynamics, see (Wang and Crow 2013; Hirpara and Sharma 2015). The white noise driven-Stratonovich SDE model for a power system dynamics and circuits and systems, see (Hirpara 2019; Patil and Sharma 2015) and the coloured noise driven-SDE model for a power system dynamics, see (Hänggi and Jung 1995; Patel and Sharma 2012; Patil and Sharma 2014; Hirpara and Sharma 2015; Guo and Shi 2017) and recent publication Verdejo et al. (2019) as well. The well-known Kuramoto model (Dörfler and Bullo 2010; Tönjes 2010; Simpson-Porco 2012; Schäfer et al. 2017; Supplemental material 2017), explaining essential mechanisms underlying synchronization, is very similar to the power system dynamics when neglecting inertia.

The problem becomes complex when the extended phase space approach increases the dimensionality of the augmented state vector with the Markovian property, see Patel and Sharma (2012). It becomes easier approaches that allow the Markovian property as well as reduce the dimensionality of the state vector and that is known as the stochastic equivalence approach, see Patil and Sharma (2014) for greater detail.

This paper is aimed to analyse power system dynamics with the stochastic equivalence approach. In contrast to Hirpara and Sharma (2015), we developed stochastic power system dynamics in the presence of an Ornstein-Uhlenbeck (OU) process with the stochastic equivalence approach and examine the effectiveness of the prediction equations with no observations or valueless observations. The Numerical simulations help to check stochastic equivalent SDE in MATLAB for a smaller correlation time.

The remainder of the paper is organized as follows. Section 2 describes mathematical preliminaries. Section 3 is about the stochastic equivalence approach for an OU driven power system dynamics and numerical simulations. Section 4 discusses concluding remarks.

2. MATHEMATICAL PRELIMINARIES

Consider the stochastic differential system described by

$$\dot{\mu}_t = A(\mu_t) + B(\mu_t)\eta_t, \quad (1)$$

where the input process η_t is a stationary process with zero mean, relatively smaller correlation time. The right-hand side $B(\mu_t)\eta_t$ of equation (1) has a multiplicative noise character. Two different SDEs can be regarded as the stochastically equivalent, if they are associated with the same Fokker-Planck equation. Importantly, the concept of stochastic equivalence is useful for the coloured noise-driven SDE, where the input noise process is a coloured noise process with a smaller correlation time. A simple calculation shows that the white noise-driven stochastic differential equation associated with the Fokker-Planck equation (Stratonovich 1963, p. 97) assumes the structure

$$\dot{\mu}_t = (S_1(\mu_t) - \frac{1}{2} \partial_{\mu_t} S_2(\mu_t)) + \sqrt{S_2(\mu_t)} w_0(t), \quad (2)$$

where the input process $w_0(t)$ is zero mean, stationary, Gaussian white noise process. Thanks to equation (4.180) of Stratonovich (1963), a special case of equation (4.180), i.e. the coefficients $S_1(\mu_t)$ and $S_2(\mu_t)$ for the stochastic differential system $\dot{\mu}_t = A(\mu_t) + B(\mu_t)\eta_t$, can be re-cast as

$$S_1(\mu_t) = A(\mu_t) + \frac{\gamma_1}{2} B(\mu_t) \partial_{\mu_t} B(\mu_t) + \gamma_2 B^2(\mu_t) \partial_{\mu_t} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right), \quad (3)$$

$$S_2(\mu_t) = \gamma_1 B^2(\mu_t) + 2\gamma_2 B^3(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right), \quad (4)$$

Where, $\gamma_1 = 2 \int_{-\infty}^0 R_{\eta\eta}(\tau) d\tau$, $\gamma_2 = \int_{-\infty}^0 \tau |R_{\eta\eta}(\tau)| d\tau$, and the autocorrelation

$R_{\eta\eta}(\tau) = E\eta_{t+\tau}\eta_t$. Note that $\partial_{\mu_t} = \frac{\partial}{\partial \mu_t}$ and $\partial_{\mu_t}^2 = \frac{\partial^2}{\partial \mu_t^2}$ are throughout the paper.

Equations (3)-(4) are valid for the weakly coloured noise input process η_t . Equation (2) in conjunction with equations (3)-(4) assumes the structure

$$\begin{aligned} \dot{\mu}_t = & A(\mu_t) - \frac{\gamma_2}{2} B^2(\mu_t) \partial_{\mu_t} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right) - \frac{\gamma_2}{2} B^3(\mu_t) \partial_{\mu_t}^2 \left(\frac{A(\mu_t)}{B(\mu_t)} \right) \\ & + \sqrt{\gamma_1} B(\mu_t) \sqrt{1 + \frac{2\gamma_2}{\gamma_1} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right)} w_0(t), \end{aligned} \quad (5)$$

Note that equation (5) is *stochastically* equivalent to the SDE, i.e. $\dot{\mu}_t = A(\mu_t) + B(\mu_t)\eta_t$, since both are associated with the same Fokker-Planck equation. Equation (5) can be 're-formulated' in the Itô sense, i.e

$$\begin{aligned} d\mu_t = & \left(A(\mu_t) - \frac{\gamma_2}{2} B^2(\mu_t) \partial_{\mu_t} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right) - \frac{\gamma_2}{2} B^3(\mu_t) \partial_{\mu_t}^2 \left(\frac{A(\mu_t)}{B(\mu_t)} \right) \right) dt \\ & + \sqrt{\gamma_1} B(\mu_t) \sqrt{1 + \frac{2\gamma_2}{\gamma_1} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right)} dW_t, \end{aligned} \quad (6)$$

where

$$a(\mu_t) = A(\mu_t) - \frac{\gamma_2}{2} B^2(\mu_t) \partial_{\mu_t} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right) - \frac{\gamma_2}{2} B^3(x_t) \partial_{x_t}^2 \left(\frac{A(x_t)}{B(x_t)} \right), \quad (7)$$

$$b(\mu_t) = \sqrt{\gamma_1} B(\mu_t) \sqrt{1 + \frac{2\gamma_2}{\gamma_1} B(\mu_t) \partial_{x_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right)}. \quad (8)$$

Equation (6), the Itô stochastic differential equation, describes a rigorous interpretation of the SDE in contrast to equation (5).

The OU process is a Gauss-Markov process and satisfies the stochastic differential equation,

$$d\eta_t = -\frac{1}{\tau_{cor}} \eta_t dt + \frac{\sqrt{2N}}{\tau_{cor}} dW_t, \quad (9)$$

where the terms τ_{cor} and $\frac{\sqrt{2N}}{\tau_{cor}}$ has an interpretation as correlation time and the process noise coefficient. Furthermore, the autocorrelation $R_{\eta\eta}(\tau)$ of the OU

process, a stationary process, is $\frac{N}{\tau_{cor}} e^{-\frac{|\tau|}{\tau_{cor}}}$. This expression suggests that the

correlation time of the stationary coloured noise can be explained using the concept of the autocorrelation of the coloured noise process. For the OU process, a weakly coloured noise process, the terms γ_1, γ_2 of equations (3)-(4) become,

$$\gamma_1 = 2N, \gamma_2 = N\tau_{cor}. \quad (10)$$

From equation (10) and equations (7)-(8), we have the following system non-linearity $a(\mu_t)$ and process noise coefficient $b(\mu_t)$ for the OU process-driven stochastic differential system:

$$a(\mu_t) = A(\mu_t) - \frac{N\tau_{cor}}{2} B^2(\mu_t) \partial_{\mu_t} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right) - \frac{N\tau_{cor}}{2} B^3(\mu_t) \partial_{\mu_t}^2 \left(\frac{A(\mu_t)}{B(\mu_t)} \right), \quad (11)$$

$$b(\mu_t) = \sqrt{2N} B(\mu_t) \sqrt{1 + \tau_{cor} B(\mu_t) \partial_{\mu_t} \left(\frac{A(\mu_t)}{B(\mu_t)} \right)}. \quad (12)$$

For a coloured noise-driven SDE, the ‘scalar’ exact mean and variance evolutions, $d\langle\mu_t\rangle$ and dP_t are,

$$\begin{aligned}
d\langle\mu_t\rangle &= (A(\langle\mu_t\rangle) - \frac{\gamma_2}{2} B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&- \frac{\gamma_2}{2} B^3(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) + \frac{1}{2} P_t (\partial_{\langle\mu_t\rangle}^2 A(\langle\mu_t\rangle) - \frac{\gamma_2}{2} (B^3(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^4 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 7B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^3 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 5B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) + 10B(\langle\mu_t\rangle) (\partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle))^2 \partial_{\langle\mu_t\rangle}^2 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 2(\partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle))^3 \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) + 6B(\langle\mu_t\rangle) \\
&\times \partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right)) dt, \tag{13}
\end{aligned}$$

$$\begin{aligned}
dP_t &= (2P_t (\partial_{\langle\mu_t\rangle} A(\langle\mu_t\rangle) - \frac{\gamma_2}{2} (4B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 2B(\langle\mu_t\rangle) (\partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle))^2 \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) + B^3(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^3 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right)) + \gamma_1 B^2(\langle\mu_t\rangle) \\
&\times (1 + \frac{2\gamma_2}{\gamma_1} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) + P_t (\gamma_2 B^3(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^3 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 6\gamma_2 B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 3\gamma_2 B^2(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ 6\gamma_2 B(\langle\mu_t\rangle) (\partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle))^2 \partial_{\langle\mu_t\rangle} \left(\frac{A(\langle\mu_t\rangle)}{B(\langle\mu_t\rangle)} \right) \\
&+ \gamma_1 B(\langle\mu_t\rangle) \partial_{\langle\mu_t\rangle}^2 B(\langle\mu_t\rangle) + \gamma_1 (\partial_{\langle\mu_t\rangle} B(\langle\mu_t\rangle))^2) dt. \tag{14}
\end{aligned}$$

Notably, the numerical coefficients of equations (13)-(14), approximate evolutions are attributed to the successive differentiation with respect to the term $\langle \mu_t \rangle$. Note

$$\text{that } \partial_{\langle \mu_t \rangle} = \frac{\partial}{\partial \langle \mu_t \rangle}, \partial_{\langle \mu_t \rangle}^2 = \frac{\partial^2}{\partial \langle \mu_t \rangle^2}, \partial_{\langle \mu_t \rangle}^3 = \frac{\partial^3}{\partial \langle \mu_t \rangle^3} \quad \text{and} \quad \partial_{\langle \mu_t \rangle}^4 = \frac{\partial^4}{\partial \langle \mu_t \rangle^4} \quad \text{are}$$

throughout the paper. For proof of equations (13)-(14), see the paper Patil and Sharma (2014).

Here, we state two special cases of evolution equations (13)-(14): (i) for the terms $\gamma_1 = 1$ and $\gamma_2 = 0$, equations (13)-(14) reduce to the *classical exact evolution equations*, see Eq. (4.159) of Jazwinski (1970, p. 137) (ii) the terms $\gamma_1 = 2N$ and $\gamma_2 = N\tau_{cor}$ lead to the exact evolution equations of the OU process-driven SDE.

3. POWER SYSTEM DYNAMICS AND NUMERICAL EXPERIMENTATIONS

In deterministic setting, the swing equation of a single machine-infinite bus (SMIB) system is given by the following second-order non-linear differential equation Kundur (1994):

$$M\ddot{\delta} + D\dot{\delta} + \frac{VE'_a}{X} \sin \delta = P_m. \quad (15)$$

Note that the terms $V, M, D, E'_a, \delta, X, P_m$ denotes the voltage magnitude of the infinite bus, combined inertia constant and the damping coefficient of the generator and turbine, the transient emf, the rotor angle of the generator, the total reactance, the input mechanical power respectively. After accounting random power fluctuations in the swing equation of the single machine-infinite bus system. we can get,

$$M\ddot{\delta} + D\dot{\delta} + \frac{VE'_a}{X} \sin \delta = P_m + \eta_t, \quad (16)$$

where η_t is the OU process. After accomplishing the phase space formulation, we have $\mu_1 = \delta_t, \mu_2 = \dot{\delta}_t$,

$$\dot{\mu}_1 = \mu_2 \quad (17)$$

$$\dot{\mu}_2 = -\frac{D}{M}\mu_2 - \frac{VE'_a}{M X} \sin \mu_1 + \frac{P_m}{M} + \frac{\eta_t}{M}. \quad (18)$$

Consider the contribution to the evolution of the phase variable $\delta_t = \mu_1$ coming from the damping term is considerably greater than the inertial term (Hänggi and Jung, 1995, p. 241), then the term $-\frac{D}{M}\mu_2 - \frac{VE'_a}{M X} \sin \mu_1 + \frac{P_m}{M} + \frac{\eta_t}{M}$ vanishes, i.e. If

we model a system without physical or virtual inertia, power system is best described as a Kuramoto oscillator (Dörfler and Bullo 2010; Tönjes 2010; Simpson-Porco 2012; Schäfer et al. 2017; Supplemental material 2017) with the equation of motion,

$$\dot{\delta}_t = -\frac{VE'_a}{D X} \sin \delta_t + \frac{P_m}{D} + \frac{1}{D} \eta_t, \quad (19)$$

Thus, the above equation can be recast by utilizing a more convenient notation by choosing the state variable notation μ_t for the rotor angle δ_t . Thus,

$$\dot{\mu}_t = A(\mu_t, t) + B(\mu_t, t)\eta_t, \quad (20)$$

where

$$A(\mu_t, t) = -\frac{VE'_a}{D X} \sin \mu_t + \frac{P_m}{D}, \quad B(\mu_t, t) = B(t) = \frac{1}{D}.$$

Note that for a non-linear time varying system. The input argument of the right-hand side of equation (20) involves the time variable t as well. From equation (20) and equation (6) of the paper, we get

$$d\mu_t = \left(-\frac{VE'_a}{D X} \sin \mu_t \left(1 + \frac{\gamma_2}{2D^2}\right) + \frac{P_m}{D}\right)dt + \frac{\sqrt{\gamma_1}}{D} \sqrt{1 - \frac{2\gamma_2}{\gamma_1} \frac{VE'_a}{D X} \cos \mu_t} dW_t, \quad (21)$$

After combining equations (13)-(14) with equation (20). As a result of these, we have the following system of prediction equations for the power system driven by the OU process,

$$d\langle \mu_t \rangle = \left(\frac{VE'_a}{D X} \sin \langle \mu_t \rangle \left(1 + \frac{N\tau_{cor}}{2D^2}\right) \left(\frac{P_t}{2} - 1\right) + \frac{P_m}{D}\right)dt, \quad (22)$$

$$dP_t = \left(P_t \frac{VE'_a}{D X} \cos \langle \mu_t \rangle \left(-2 + \frac{N\tau_{cor}}{D^2}\right) - \frac{5}{2} N\tau_{cor} \frac{VE'_a}{D^3 X} \cos \langle \mu_t \rangle + \frac{2N}{D^2}\right)dt. \quad (23)$$

Note that equation (20) and equation (21) are stochastically equivalent and suggest qualitative characteristics of the rotor angle of the machine. For numerical experimentations of equations (20)-(21), the initial conditions and system parameters for power system dynamics are the following Hirpara and Sharma (2015):

$$\begin{aligned} V &= 1.0 \text{ pu}, E'_a = 1.2 \text{ pu}, D = 5 \text{ pu} / \text{rad} / \text{sec}, \\ X'_d &= 0.15 \text{ pu}, X_l = 0.1 \text{ pu}, X = 0.25 \text{ pu}, \tau_{cor} = 0.1 \\ P_m &= 1.0 \text{ pu}, \langle \mu(0) \rangle = 1 \text{ rad}, P(0) = 0 \text{ rad}^2, N = 0.05 \end{aligned}$$

In first case, we consider that the correlation time τ_{cor} is smaller than second case. Note that the solid line trajectory (-) of figure 1 demonstrates the numerical simulation of equation (20) and the dotted line (...) of figure 1 denotes the numerical simulation of equation (21) of the paper. The terms $\gamma_1 = 2N$ and $\gamma_2 = N\tau_{cor}$ corresponding to correlation time $\tau_{cor} = 0.1$ become $\gamma_1 = 0.1$ and $\gamma_2 = 0.005$. Figure 1 suggested that qualitative characteristics of both SDEs conforms each other for a smaller correlation time. The trajectories of both SDEs are coinciding with each other. Thus, both SDEs are stochastically equivalent for the OU process with smaller correlation time. Figures 2 and 3 demonstrate numerical simulation of equation (22), mean trajectory and equation (23), variance trajectory with smaller correlation time.

For second case, we simulated equations (20) and (21) by choosing larger correlation time $\tau_{cor} = 1$ other parameters are same as first case. The terms $\gamma_1 = 0.1$ and $\gamma_2 = 0.05$ for larger correlation time.

Figure 1: a comparison between trajectories

Figure 2: a comparison between trajectories

Figure 3: a variance trajectory

Figure 4: a comparison between trajectories

Figure 5: a comparison between trajectories

Figure 6: a variance trajectory

The graphical notations are same as first case. Figure 4 reveals that the difference between the state trajectories resulting from the SDEs stated in equation (20) and equation (21) are larger. Figures 5 and 6 demonstrate numerical simulation of equation (22), mean trajectory and equation (23), variance trajectory with larger correlation time. Thus, the numerical simulation suggests that both SDEs, equation (20) and equation (21), are stochastically equivalent for the input coloured noise process with ‘smaller correlation time’.

4. CONCLUSION

The main achievement of this paper is to develop prediction algorithm for the OU process-driven SDE. It is shown that for a smaller correlation time, the trajectories of SDEs stated in equation (20) and equation (21) are coinciding to each other. Thus, both SDEs are stochastically equivalent to each other and for larger correlation time, difference between both trajectories are larger. In this paper we have also developed conditional mean and variance equation for power system dynamics with stochastic equivalence approach. For future direction, in this paper we have considered no observation or value less observation. For available observation, we were required to develop filtering algorithm for the OU process driven power system dynamics.

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