Shape retrieval using angle-wise contour variance

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In this study, we propose a geometric feature set for 2D shape retrieval. Conventional Hough feature gives the edge locations along with angle and creates Hough table if there are multiple intersections at borders. In this paper, a statistical way to represent the relation of repeating contours at each angle around the shape centroid is presented. The main contribution of this paper is to use the standard deviation of repeating contours. We calculate the angle between the shape centroid and each point on the contour. For each integer angle value, three features were extracted: the number of contour repetitions, the average distance of the points at that angle to the centroid, and the standard deviation of the points at the same angle. Thus, a 2D image was represented by a constant sized matrix, regardless of its size. In the case of similarity between two images, instead of merging features within a single expression, the algorithm picked the feature with the highest similarity rate for that comparison. We tested the proposed method on MPEG-7, Kimia99, ETH-80 datasets for a benchmark with the state-of-the-art. It outperformed most of the recent methods in terms of retrieval rate.

Keywords: computer vision, pattern recognition, pattern matching, image matching, image recognition

1 Introduction

The growing number of archived images generated continuously by several acquisition devices has motivated the researchers to design complex algorithms for efficient and effective image retrieval. Image retrieval remains a challenging problem especially in computer vision, robot navigation, deep learning, and image segmentation. It is a fundamental task in image recognition to measure the similarity/distance between two images accurately through a given object query [1].

Textual annotations based image retrieval is mostly impractical due to the need for intensive manpower while dealing with large data warehouses. Furthermore, the shape is accepted as a promising and stable visual feature in image recognition and retrieval due to its discriminative strength [14]. Therefore, content-based image retrieval systems have gained great interest recently [5]. Identifying good shape descriptors and similarity measures are the primary concerns in related applications [6]. A successful algorithm should be able to deal with obstacles such as noise, occlusion, and distortion while keeping the essential characteristics of the image unchanged [7]. A great amount of shape comparison algorithms have been proposed [814]

and classified into two categories: (1) contour-based; and (2) region-based. Among the shape descriptors, the contour-based method has become more prominent in the last decade [1, 15]. This method represents a shape using the distribution of the spatial positions between the

contour points, and good performance has been noted. Distance sets [16], elastic matching [17], and symbolic representation [10] are also included in the contour-based methods. Only the information from the boundary of a shape is extracted while neglecting the rich information in the image region.

Affine distorted planar curve matching approach is exploited for occluded object recognition, and results show that it exhibits effectiveness more than the state-of-the-art partial curve matching methods [18]. A 2D shape descriptor based on a scalogram obtained from progressive filtering of a planar closed contour is presented [19]. Angular pattern, binary angular pattern, and a multiscale integration of both for shape retrieval are proposed for the efficient use of real-world shape retrieval applications [15]. Furthermore, another multi-scale feature integration framework using sequential backward selection algorithms with an angular pattern, and the binary angular pattern is described. Using the sequential backward selection algorithm improves the effectiveness of multi-scale feature integration [20]. A learning-based descriptor for both 2D and 3D image matching is demonstrated which summarizes the local features extracted from specific shapes to generate an integrated representation [21]. In Bag of Contour Fragments, an image is divided into contour fragments, which is, then independently described using a shape descriptor, and a compact image representation is generated by combining shape codes in the shape [22]. Fourier descriptors [2, 23] that incorporates the global and local features of the shape

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Fig. 1. Illustration of two shapes with same number of contour repetitions and average distance at the same angle

are relatively easier to implement, are also based on the theory of Fourier analysis. A robust two-stage image retrieval algorithm is proposed, including an irrelevant cluster filtering stage to remove irrelevant shapes from the candidate set, followed by a local-features-based matching and ranking method [24]. An object categorization framework is developed, which decomposes the input image into three visual cues: structure, texture, and shape [25]. In [26], the authors introduced the Hough transform. It represents a contour image by the distance of its edge points to the shape centroid along with the angle. The repeated contours are given in a table that contains the angle and distances of intersecting points to the axis. Also in [27], authors introduced a shape descriptor by examining how the patterns repeat along contours. In Fig. 1, let us assume that the red dash line and blue dotted line represent partial views of two different shapes. If we draw a hypothetical horizontal line from the centroid to the contour at θ , we see that it intersects the contour at three different points in both shapes. Moreover, the average distance from the centroid to the intersecting points of the red line and the intersecting points of the blue line is the same.

Thus, both features give the same values even though the shapes are entirely different. The first contribution of this study is to use the standard deviation of repeating contours at each angle. Even though the average distance and number of repetitions are equal, the locations at which the contours repeat themselves at each angle are very distinguishing. Thus, it can be a distinctive feature for shape representation. We used the feature with the best result among the three instead of combining them to have a final decision. Each feature is effective in different types of shapes. Thus, combining all features to a single decision expression can suppress their contribution. In this paper, for each image, the most distinguishing feature is selected. The details about the proposed method are given in Section II. The proposed method is compared with the state-of-the-art methods for benchmarking by implementation on MPEG-7, Kimia99, and ETH-80 datasets and using dataset-specific measurement

metrics. The rest of this letter is organized as follows: Section II describes the proposed method. The experimental results are given in Section III. Section IV concludes the letter with remarks and future work.

2 Proposed method

A preprocessing step takes place, before the feature extraction stage. It includes making the images invariant to translation and scale. It is followed by extraction of features: number of contour repetitions; the average distance of contours to the centroid; and finally the standard deviation of the repeating contours. These three features are calculated for every integer value of 360. The detailed explanations for each feature are provided in this section.

Normalization against scaling and translation is conducted prior to feature extraction to obtain a more meaningful image representation. This study focuses on the contour rather than the binary image itself. Normalization against scale invariance is done before edge detection. It is achieved by setting the larger side of the shape to a fixed size among all the images in the dataset. This step is followed by edge detection. Thus, the input image becomes a contour shape. Let $p = \{p_1, p_2, \dots, p_m\}$ be the set of points on the contour with a length of m and $C = (C_x, C_y)$ be the shape centroid. Each point P in has its x and y components, representing the pixel coordinates of the point according to the left-top corner of the image. For translational invariance, we transform the set P to P' by subtracting C from every point. We handle the rotational invariance in the testing step.

$$P' = \{p_1 - C, p_2 - C, \dots, p_m - C\}. \quad (1)$$

Our feature set consists of three parameters. They are (1) the number of contour repetitions, (2) the average distance of the repeated contour points to the shape centroid, and (3) the standard deviation of the repeated contour points in terms of position. We extract these features for each angle value around the unit circle. In this section, the extraction of each feature and representation of each image is explained. The same procedure given below is applied to all images in datasets.

The first step is to calculate the angle between C and each point in set

$$\alpha = \text{mod} \left(\tan^{-1} \frac{p_{ky}}{p_{kx}}, 360 \right). \quad (2)$$

Since every convex shape is an enclosed contour, there will be at least one point $\alpha \in [1, 360]$. Even though the range of α is given, the same α value can be found for different points in P' according to (2). In Fig. 2, a sample image from the MPEG-7 dataset is shown. The large yellow point shows the centroid. We randomly chose four lines to illustrate contour repetition. Each line represents four different values of angle α . The white points are the locations where a line cuts the contour at that specific

Fig. 2. (a) – an original silhouette image from MPEG-7 dataset, and (b) – its contour representation with illustration of contour repetition at four different angles

angle. Depending on the shape, there might be a single intersection at some values of α , whereas there are also multiple intersections at some other values. The number of the intersections at a certain angle is equivalent to the number of repetitions that the contour makes at that angle. As a result, we create the first feature n_α , which is the number of contour repetitions at angle α . When we examine the lines in Fig. 2 counterclockwise, we observe that n_α is equal to 3, 1, 3, and 5, respectively.

The second feature of this method is the average distance between the points at angle α and shape centroid C . The distance from the centroid to contour points and the angle information has originally been used in the literature [26], but the contour repetition was not considered. Let $P'_\alpha = \{p'_{\alpha 1}, p'_{\alpha 2}, \dots, p'_{\alpha n_\alpha}\}$ be the set of points at angle α . The average distance is

$$d_\alpha = \frac{1}{n_\alpha} \sum_{n=1}^{n_\alpha} \left(\sqrt{(p'_{\alpha iy})^2 + (p'_{\alpha ix})^2} \right). \quad (3)$$

The first feature tells about the contour curvature, whereas the second feature tells the relation between the set p_α and C . Assume that, we have two entirely different shapes. When we extract the number of contour repetitions and the average distance of the points to the centroid, there is a possibility of getting the same results. This is an undesired outcome. Ideally, a feature set must be unique to the specific shape as illustrated in Fig. 1. Let us assume that the red dash line and blue dotted lines represent partial views of two different shapes. We extract the first two features for both shapes at $\alpha = 0$, which is represented by the green arrow. We can observe that both shapes intersect with the line three times. Thus, the first feature is the same for both shapes. The red line and blue lines cut the green one at horizontal coordinates of (1.0, 1.8, 4.2) and (2.1, 2.4, 2.8) respectively. When we calculate the average distance, both lines have a distance of 2.43 pixels to the centroid.

Therefore, a more distinct feature is required to represent the shapes. As shown in Fig. 1, the location of the intersecting points of the red line has large diversity among themselves, whereas the intersecting points on the blue line are very close to each other. Therefore, the standard deviation of the intersecting points at an angle presents us with how close or far they are located from each other.

In Fig. 1, the standard deviations of the points on the red line and blue line are 1.8 and 0.35 respectively. We calculate the standard deviation of the set as in (4). Thus, we extract the distribution of the points of the same set.

$$\sigma^\alpha = \sigma(p'_{\alpha 1}, p'_{\alpha 2}, \dots, p'_{\alpha N_{x1}}). \quad (4)$$

The last feature is extracted by applying the procedure given above throughout the image for all values of α . We represent the image with matrix $\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{359}]^\top$ where each element $\mathbf{s}[n_x, d_x, \sigma_\alpha]$ is a vector. It is an incrementally sorted matrix. In the final stage, all elements in each feature space are normalized with the maximum value of the same space among the dataset. To compare two shapes with matrixes $\mathbf{A}$ and $\mathbf{B}$, a meaningful distance calculation scheme is required. In a column-wise manner, we use the Manhattan distance to calculate the distance of two images in terms of each feature space separately as follows

$$\Delta^n = \sum_{i=0}^3 59 |a_i^n - b_i^n|, \quad (5)$$

$$\Delta^d = \sum_{i=0}^3 59 |a_i^d - b_i^d|, \quad (6)$$

$$\Delta^\sigma = \sum_{i=0}^3 59 |a_i^\sigma - b_i^\sigma|. \quad (7)$$

In (5), (6) and (7) $\mathbf{a} = [a_i^n, a_i^d, a_i^\sigma]$, and $\mathbf{b} = [b_i^n, b_i^d, b_i^\sigma]$ are the i^{th} row of $\mathbf{A}$ and $\mathbf{B}$. Equation (5) computes the absolute difference between the same feature columns of two shapes. If the matrix $\mathbf{A}$ represents a rotated version of the image that is also represented by matrix $\mathbf{B}$, the distances $\Delta_i^n, \Delta_i^d, \Delta_i^\sigma$ would be nonzero although they should be zero. This problem will occur, since we do not handle rotational invariance during the preprocessing stage. Since the unit of number of rows is same as the rotation degree, rotation of β degrees in the shape will cause β iterations of vertical rotational shift in its corresponding matrix. Thus, rotational invariance is achieved by keeping $\mathbf{A}$ stationary but rotating $\mathbf{B}$ for 360 times one by one. After each iteration, (5) is repeated. At the end of one full cycle, we obtain a distance matrix with

Fig. 3. Examples of image types (a) – standard deviation, (b) – number of contour repetition, (c) – and average distance feature are distinct at

Fig. 4. Retrieval results of all classes of MPEG-7 images using proposed method: (a) – apple, bottle, car, children classic, device2, fountain, prescar, truck, (b) – pocket, crown, frog, bell, glass, cup, device9, horse, device6, ray, device8, device1, camel, misk, rat, (c) – spoon heart, elephant, fly, bone, (e) – watch, hat, dog, fork, stef

each row is $\mathbf{d} = [\Delta_i^n, \Delta_i^d, \Delta_i^\sigma]$. The minimum value of each column is selected and it gives the result of distance calculation for all features. Multi-feature based studies use distance fusion method with the corresponding weighing coefficient of each distance [28]. Although weighting them might increase the accuracy, it may hold down the contribution of each feature. We observed that the standard deviation of points is very distinct if the shape has unique outlines and fluctuations as shown in Fig. 3 (a). The number of repetitions is distinct if the shape has major repetitions without edge fluctuations as in Fig. 3(b), whereas average distance is robust in shapes in which the height and width sizes are highly diverged illustrated in Fig. 3(c).

Instead of a single distance expression, which is a linear combination of all features, we used the distances separately to obtain three results and selected the best of three. A comparison is made between a query matrix and all the other matrixes in the dataset in terms of three features. A recognition or retrieval score depending on the evaluation metric is returned for each feature. We select the feature, with the largest score as shown in (8)

$$\varphi = \max[\varphi^n, \varphi^d, \varphi^\sigma], \quad (8)$$

where φ is the final selected score, φ^n , and φ^σ are the scores of number of contour repetitions, average distance and standard deviation, respectively.

Even though the selected result of an image is independent of the other images of the same class, there is a consistency between the selected feature and shape class. In the MPEG-7 dataset, 56 classes out of 70 showed 100% consistency on feature selection. It means that the same feature provides the best outcome for all the objects in the same class even though they have variations. Although different features are selected among the objects in the remaining 14 classes, there is still a strong bias on one feature in each class. Therefore, the contribution of the proposed features is not suppressed by a combination of the features.

3 Experimental results

This section presents the performance of the proposed method. To test its performance, we compared our method with the recent state-of-the-art. We demonstrate the retrieval and recognition performance of the proposed method by conducting experiments on three widely used shape datasets: MPEG-7 [29], Kimia99 [30] and ETH-80 [31]. All the datasets are composed of various shape categories. All categories contain high amount of intra-class variations due to scale, translation, rotation and large deformations. MPEG-7 dataset contains 1400 shape samples from 70 different classes. Each class consists of 20 shapes with variations. The benchmarking metric for this dataset is the retrieval performance and is measured by bulls eye rating in which every shape is used as query and the number of similar images that belong to the same class were counted in the top 40 matches [29].

Since the maximum number of correct matches for a single query image is 20, the total number of all possible correct matches is 28,000. The image retrieval results of the proposed method on categories of objects in the MPEG-7 dataset are shown in Fig. 4. The x-axis and y-axis denote the image classes and retrieval numbers out of 40, respectively. The retrieval results show that use of standard deviation along with selection of best feature retrieves complex objects such as chopper, octopus, camel etc. efficiently. In Fig. 4, red, green, blue, yellow and black lines represent the bulls eye rating that are 100%, above

Table 1. Bull’s eye scores of the state of the art methods for MPEG-7 datasheets(%)

Method	Bulls eye score
ECCobj2D (32)	54.56
ARH+HVH+Zernike (33)	63.91
MCF (2)	67.57
MDM (5)	69.19
d-TASS (19)	76.92
DBCSR (34)	84.17
AICD(18)	84.26
SC + DP (35)	86.80
HSC (36)	87.31
AP, BAP using SFS (15)	87.40
AP, BAP using SBS (20)	87.97
ASC (12)	88.30
CBTA (1)	89.45
HF (37)	89.66
Shape Vocabulary (21)	90.41
Our Method	90.43
CBTA+SC (1)	93.65
AIR (38)	93.67
TEM (39)	94.64

90%, above 80%, above 50% and below that, respectively. In 50 out of 70 classes, bulls eye rating is more than 90%. The only class with below 50% rate is sea snake. We compared the proposed method with 17 recent state-of-the-art studies. The bulls eye rating of the proposed method and state-of-the-art methods are compared in Tab. 1.

The proposed method can achieve 90. 43% retrieval rate. We see that the proposed method outperforms most of the recent studies. The retrieval performance of contour repetition, average distance, standard deviation and uniform Euclidean distance are, 59. 6%, 69. 7%, 75. 1% and 75. 2%, respectively.

The proposed feature selection method increases the performance of the system significantly. The second experiment is conducted on the Kimia99 shape dataset [30]. It contains variations such as occlusions and distorted parts that change the outer contour of the original shape. This dataset is relatively smaller. It consists of 99 shapes in nine different classes in total. Each class contains 11 shapes. The retrieval accuracy is calculated by counting the overall results of the top 10 retrievals from the same class; excluding the query shape [37]. The best possible value is 99, which is summed up by the consistency of all 99-query samples. We compared our method to eight recent methods. The top 10 consistency results of several methods are compared in Tab. 2. According to the results given in Tab. 2, the proposed method shows comparable performance in the first 8 levels results to the state-of-the-art. In the 9th and 10th levels, the retrieval rate of the

proposed method is below the majority of the studies yet still can outperform some others. The reason is that the dataset is small and objects of each class have large deformations. Thus, information to represent a class shape might not be sufficient.

Table 2. The retrieval results of state-of-the-art methods for KIMIA99 dataset

Lit	1	2	3	4	5	6	7	8	9	10
[34]	91	81	73	75	63	57	51	44	35	30
[34]	99	99	97	97	97	98	93	90	79	53
[37]	99	99	99	99	98	99	99	96	95	88
[32]	99	95	98	97	94	92	91	80	74	50
[23]	97	96	96	97	88	87	83	74	68	57
[24]	99	99	99	99	99	99	99	99	99	99
[1]	99	99	99	99	98	98	97	95	95	75
[14]	96	94	94	87	88	82	80	70	62	55
our	99	95	98	97	94	92	91	80	74	50

This dataset contains both colored and binary versions of the same shapes. Only the binary section is considered. There are eight classes of objects with 10 objects in each class in the ETH-80 dataset [31]. For each object, there are 41 binary images with deformations in terms of rotation, translation, and size. The commonly used test method for this dataset in literature is leave-one-object-out cross- validation.

Table 3. Recognition rate of state-of-the-art methods for eth- 80 dataset (%)

Method	Recognition rate
IDSC + Morphology (40)	88.04
HF (37)	88.72
HF + SC (37)	89.73
Robust symbolic (10)	90.28
Kernel-edit (41)	91.33
BCF (22)	91.49
MCV (25)	97.13
Our Method	99.85

We spare one object as a query and compare it with the remaining 79 objects in the set. Comparison is successful if the object is labeled with the correct class. This procedure is repeated for every object in the dataset. The recognition rate is the average of 80 objects. We compared our method with eight recent state-of-the-art studies and recognition results are presented in Tab. 3. It shows that the proposed method outperforms all of the state-of-the-art methods. The retrieval performances of contour repetition, average distance, standard deviation, and uniform Euclidean distance are, 65. 7%, 82. 3%, 96. 6%, and 84. 8%, respectively. The methods in Tab. 3 are based on binary images. In the literature, there are also other studies

which are color based [31] and principal component analysis (PCA) based [31] showing lower performance than the methods given in Tab. 3.

4 Conclusion

In this paper, we have proposed a new contour-based feature set for 2D image representation. Its performance is compared with state-of-the-art on commonly used datasets. The proposed method can represent contour images robustly. It is observed that our feature selection method gives the best result, followed by standard deviation feature, average distance, and lastly number of contour repetitions. Also, the standard deviation of repeated contour points is more robust than the other two features.

A drawback of the method is the time consumption. Unlike the feature extraction stage, the testing stage consumes much time since the distance between any two shapes is calculated 360 times to remove the rotational variance of the feature set. Thus, we aim to conduct further research to minimize time consumption.

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