Comparative study of forecasting approaches in monthly streamflow series from Brazilian hydroelectric plants using Extreme Learning Machines and Box & Jenkins models

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Abstract: Several activities regarding water resources management are dependent on accurate monthly streamflow forecasting, such as flood control, reservoir operation, water supply planning, hydropower generation, energy matrix planning, among others. Most of the literature is focused on propose, compare, and evaluate the forecasting models. However, the decision on forecasting approaches plays a significant role in such models' performance. In this paper, we are focused on investigating and confront the following forecasting approaches: i) use of a single model for the whole series (annual approach) versus using 12 models, each one responsible for predicting each month (monthly approach); ii) for multistep forecasting, the use of direct and recursive methods. The forecasting models addressed are the linear Autoregressive (AR) and Periodic Autoregressive (PAR) models, from the Box & Jenkins family, and the Extreme Learning Machines (ELM), an artificial neural network architecture. The computational analysis involves 20 time series associated with hydroelectric plants indicated that the monthly approach with the direct multistep method achieved the best overall performances, except for the cases in which the coefficient of variation is higher than two. In this case, the recursive approach stood out. Also, the ELM overcame the linear models in most cases.

Keywords: Streamflow series prediction; Multistep forecasting; Monthly approach; Box & Jenkins models; Extreme Learning Machines.

INTRODUCTION

Accurate streamflow predictions are crucial for several human activities related to the appropriate water management, such as water supply, navigation, disaster prevention, and flood control (Yang et al., 2017; Zhang et al., 2018). A relevant practical instance of this problem is the monthly streamflow series forecasting associated with hydroelectric plants, since the planning of both the energy matrix operation and for future generation capacity are directly linked to such predictions (CEPEL, 2020).

The aforementioned predictions are also related to crucial aspects of energetic planning, such as pricing strategies, expansion of the installed capacity, and the expected availability of energy for short and medium-term horizons (Honorato, et al., 2018; Siqueira et al., 2018; Zhang et al., 2018).

The most recent Hydropower Status Report from the International Hydropower Association (2020) reported that the world generated 4,306 TWh of electric energy employing hydroelectric plants in 2019. This amount represents the single most significant contribution from a renewable energy source in History (IHA, 2020). Hydropower is more stable than wind and solar energy since it is less vulnerable weather and climate conditions (Yang et al., 2020). In addition, the International Energy Agency (2012) estimated that hydroelectric production will double by 2050, reducing on a large scale the emissions of carbon dioxide (CO₂) and other pollutants from fossil fuel plants (de Souza Dias et al., 2018). The leaders in hydropower installed capacity are China (356.40 TWh), Brazil (109.06 TWh), United States (102.75 GW), and Canada (81.39 GW).

Undoubtedly, the search for the most suitable forecasting models stands out as the main investigated topic on monthly streamflow series prediction. However, there are crucial inter-

mediate steps that can decisively influence the performance of a model, such as data preprocessing, selection of the input variables, determination of their free parameters, and the forecasting approaches (Fendeková et al., 2014; Liu et al., 2018; Luo et al., 2019; Miedema, 2015; Siqueira et al., 2018). In this investigation, we discuss forecasting approaches considering two main aspects:

i) The choice for multistep prediction methodology: in multistep ahead forecasting, it is possible to estimate the future value of a series directly or recursively. The direct manner provides the output of a model on the horizon required. In the recursive case, the forecasting model always provides an estimate of the next value of the series (one step ahead), and this value is reinserted as input until achieving the final response in the desired horizon (Sorjamaa et al., 2007).

ii) The number of forecasting models used: in monthly streamflow series forecasting, it is possible to apply one model for the whole series (annual approach) or use a dedicated model for each month, totaling 12 models (monthly approach). Each model is specifically designed to handle each block of the series (Liu, 2018).

In Table 1, we present a summary of some relevant and recent studies on streamflow forecasting. Such investigations used databases from countries as China, Canada, Ecuador, Iraq, Mozambique, United States, Serbia, Norway, Turkey, Sri Lanka and Brazil (Arsenault and Côté, 2019; Hailegeorgis and Alfredsen, 2017; Siqueira et al., 2018; Stojković et al., 2017; Wei et al., 2013; Yang et al., 2017; Zhou et al., 2018; Zhu et al., 2016).

Some aspects of these studies can be highlighted. In relation to the number of forecasting models, most investigations use only one model to perform the prediction of the whole series.

Table 1. Summarization of important papers regarding streamflow series forecasting.

Authors/Year	Country	Number of models	Multistep approach	Prediction Method
Arsenault and Côté (2019)	Canada	1	Direct	Ensemble
Dilini et al. (2016)	Sri-Lanka	1	Recursive	NARX-ANN
Hailegeorgis and Alfredsen (2017)	Norway	1	Direct	Linear Regression
Hernandez-Ambato et al. (2017)	Ecuador	1	Recursive	ANN
Honorato et al. (2018)	Brazil	1	Direct	ANN and wavelet-ANN
Siqueira et al. (2014)	Brazil	12	Direct	Various ANN
Siqueira et al. (2018)	Brazil	12	Recursive	Various ANN
Stojković et al. (2017)	Serbia	1	Direct	Linear Regression
Tayyab et al. (2016)	China	1	Direct	ANN
Uamusse et al. (2015)	Mozambique	1	Direct	Fuzzy-ANN
Wei et al. (2013)	China	1	Direct	Wavelet-ANN
Yaseen et al. (2016)	Turkey/Iraq	1	Direct	Least Square SVR
Zhu et al. (2016)	China	1	Direct	SVR

The use of 12 forecasting models is explored in two cases, but a comparative evaluation between both possibilities is not considered. Regarding the multistep forecasting, the majority of papers addresses the direct instead of recursive approach (Hailegeorgis and Alfredsen, 2017; Tayyab et al., 2016; Yaseen et al., 2016). Therefore, the critical analyses in such works are limited to evaluate the performance of the forecasting models. To the best of our knowledge, the literature does not present a study regarding the forecasting approaches we have mentioned. With this investigation, we intend to fill this gap (IHA, 2020).

In relation to the forecasting models, there is a great appeal for the application of artificial neural networks (ANN) in streamflow forecasting (Hernandez-Ambato et al., 2017; Tayyab et al., 2016; Wei et al., 2013; Yang et al., 2017; Zhang et al., 2018), but linear regressions (Stojković et al., 2017), support vector regression (SVR) (Zhu et al., 2016), fuzzy-neural networks (Uamusse et al., 2015), mixed nonlinear autoregressive with exogenous input and ANN (NARX-ANN) (Dilini et al., 2016), and ensemble models (Arsenault et al., 2016) are still found.

The ANN are able to map the input/output patterns in a non-linear way, which confers an elevated approximation capability based on simple processing units, named artificial neurons (Araujo et al., 2020; Haykin, 2008). Previous studies on streamflow forecasting have shown the advantages of using such methods (Hernandez-Ambato et al., 2017; Tayyab et al., 2016). In this work, we use the Extreme Learning Machines (ELM) as a representative of the ANN. The ELM are feedforward architectures, with a high degree of similarity with the traditional Multilayer Perceptron (MLP). The main difference between them is the training process since, in ELM, it is merely and based on a closed-form solution (Miche et al., 2010). In many forecasting tasks, the ELM overcame other neural architectures and linear models (Silva et al., 2019; Siqueira et al., 2014; Siqueira et al., 2018).

In the current study, the database used to investigate the impact of the forecasting approaches are from Brazilian hydro-power plants. An important observation can be made: besides the country presents the 2nd largest hydropower installed capacity globally, around 60% of all electricity consumed comes from hydro sources (EPE, 2019). This is remarkable because it evidences the degree of dependence of this source (Hippel and McLeod, 1994). For instance, in the USA, hydro plants provide 27% of the electricity (Zhou et al., 2018).

In Brazil, the streamflow series forecasting is critical to the monthly programming of energy and water resources of the

National Interconnected System (SIN). For the development of such functions, The National Electric System Operator (ONS) has a chain of models optimizing the operation, addressing daily, weekly, and monthly average streamflow series (ONS, 2019). The monthly case is intended for short-term and medium-term planning (Maceira et al., 2002; 2004). The Monthly Program of Operation coordinated by ONS, the most used methodology for prediction is based on linear models of the Box & Jenkins family (Box et al., 2008), highlighting the Periodic Autoregressive (PAR) (CEPEL, 2020). In this case, 12 Autoregressive (AR) models are adjusted independently, and each one is optimized to provide the data for each of the 12 months of the year (Siqueira et al., 2014). For multistep ahead prediction, it is adopted the recursive approach (Sorjamaa et al., 2007).

As can be observed, there is a contrast among the forecasting approaches addressed in most papers from Table 1 and the Brazilian methodology regarding the monthly and annual approaches and the multistep prediction methods. In this investigation, we evaluate the impact of the forecasting approaches by comparing the performance of ELM, AR, and PAR models. Due to the intrinsic nonlinearity of the ANN, it could overcome the traditional linear models, which are still very used.

The work discusses in Section “METHODS” the forecasting approaches used, as well as the forecasting models: AR models, PAR models, and ELM; Section “DATA AND STUDY REGION” discusses the dataset addressed and some of its particularities; Section “RESULTS” shows the preprocessing of data, and the computational results, while Section “DISCUSSION” performs a critical analysis of the results. Conclusions and future works are in Section “CONCLUSIONS”.

METHODS

This section presents the approaches for monthly seasonal streamflow series forecasting: recursive and direct methods for horizons higher than 1 step ahead; the monthly and annual methodologies. Besides, the forecasting models used are described: AR model, PAR model, and ELM.

Direct and recursive predictions

There are two basic ways to make predictions for $P > 1$, namely, the direct and recursive methods (Hwang and Monn, 1991; Sorjamaa, et al., 2007). In the direct approach, the inputs (lags) in a given time t generate the sample x_{t+P} directly without

providing intermediate samples. The method follows Equation (1):

$$\hat{x}_{t+P} = G(x_t, x_{t-1}, \dots, x_{t-M+1}) \quad (1)$$

where $\hat{x}_{t+P}$ is the prediction P steps ahead, G represents the forecasting model (or the mapping associated with the model), x_t are the observed inputs, and M is the number of inputs (lags).

To exemplify the idea, consider a model with 2 inputs and $P = 12$. The prediction of the sample in $t = 14$ is $\hat{x}_{14} = G(x_1, x_2)$, being (x_1, x_2) the inputs, that is, the samples at $t = 1$ and $t = 2$.

In the recursive case, the forecasting model always provides a one-step ahead prediction ($P = 1$). If we are interested in multistep forecasting initially, the intermediate missing samples to obtain the estimated samples in the horizon P are predicted. Then, these predictions are used as inputs a step further, as if they are real values. Using the same example, we would have a set of predictions made as in Equation (2):

$$\begin{aligned} \hat{x}_3 &= F(x_1, x_2) \\ \hat{x}_4 &= F(x_2, \hat{x}_3) \\ \hat{x}_5 &= F(\hat{x}_3, \hat{x}_4) \\ &\dots \\ \hat{x}_{14} &= F(\hat{x}_{12}, \hat{x}_{13}) \end{aligned} \quad (2)$$

being F the respective mapping or forecasting model.

As can be noted, to predict the sample in $t = 14$, it is necessary to forecast initially $\hat{x}_3$, using real values (x_1, x_2) . After, we predict $\hat{x}_4$, but now using $\hat{x}_3$, a predicted data as input together with x_2 , an observed value. The process is repeated until $P=12$. Note that for $\hat{x}_{14}$, we used the predicted values for $t = 12$ and $t = 13$ as inputs of the model.

On the one hand, no intermediate samples are used in the direct method, preventing the propagation of errors from one step to another. Each predicted data presents an error component, which is propagated until the instant $t+P$, when using the recursive method. On the other hand, the correlation between the samples drops with the increase of the horizon. This behavior necessarily degrades the final response, justifying the use of the recursive method. These issues highlight the need to compare such approaches.

In previous investigations, the monthly seasonal streamflow series forecasting is done considering up to 12 months ahead ($P = 12$). However, as the horizon rises, there is a loss in the models' predictive power, whether linear or not (Siqueira et al., 2014). It occurs due to the decrease in the correlation between samples temporally distant. According to Siqueira et al. (2018), there is a deterioration of the mean square error very evident with the growth of P . Therefore, is crucial to define the best multistep approach for streamflow forecasting.

Monthly and annual models

Monthly seasonal streamflow series related to hydroelectric plants can be modeled in two ways. The first is to consider a single model as the series generator. Therefore, only one adjustment is made and, regardless of which month belongs to the given data, it is predicted by a single model. This methodology is named annual approach (Siqueira et al., 2014; 2018).

For the monthly prediction, 12 different forecasting models are used, one for each month. According to Maceira et al. (2002), hydrological series with a seasonal component of up to one year presents periodic behavior related to statistical proper-

ties such as mean, variance, and correlation structure. This allows streamflow series to appear as natural candidates to be adjusted by monthly models.

Autoregressive model - AR

The usual way to predict future values of a time series is to perform a weighted combination of past observed samples (Tsay, 2005). For this, it is necessary to define a sufficiently representative model of the known data and estimate their respective free parameters, based on some previously established criterion (Siqueira et al., 2019).

The Autoregressive model of order p - AR (p) is part of the set of methods described by Box & Jenkins (Box et al., 2008) for stationary series. Its extensive use stems from its efficiency and relative ease in mathematical treatment (Box et al., 2008; Siqueira et al., 2018). The Equation that defines the AR(p) model is described in Equation (3):

$$\hat{x}_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \dots + \varphi_p x_{t-p} + a_t \quad (3)$$

where $x_{t-i}, i=1, \dots, p$ are the observed values, or delays, $\hat{x}_t$ is the predicted value for the time t , $\varphi_i, i=1, \dots, p$, are adjustable coefficients, and the term a_t is the random component or the error inherent in the forecasting process.

As can be noticed in Equation (3), the model does not have feedback on outgoing responses. Therefore, its optimal coefficients, towards a minimum mean square error (MSE), can be obtained analytically through the Yule-Walker equations (Box et al., 2008), presented in Equation (4):

$$\Phi_p = P_p^{-1} p_p \quad (4)$$

where $\Phi = [\varphi_1, \varphi_2, \dots, \varphi_p]$ is the vector of p coefficients that weight the entries and that define the order of the model.

The other terms of (4) correspond to the autocorrelation matrix and to the vector of estimates of autocorrelation functions of the entries, defined respectively as in Equation (5):

$$P_p = \begin{bmatrix} 1 & \rho_1 & \dots & \rho_{p-1} \\ \rho_1 & 1 & \dots & \rho_{p-2} \\ \dots & \dots & \dots & \dots \\ \rho_{p-1} & \rho_{p-2} & \dots & 1 \end{bmatrix} \quad p_p = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \dots \\ \rho_p \end{bmatrix} \quad (5)$$

where the autocorrelation coefficients are given by Equation (6):

$$\rho_i = \frac{E[(x_t - \mu)(x_{t+i} - \mu)]}{\sqrt{E[(x_t - \mu)^2]E[(x_{t+i} - \mu)^2]}} \quad (6)$$

being μ the sample average, and E is the Mathematical Expectation.

Note that such coefficients in (6) can be estimated using the method of moments (Box et al., 2008).

Periodic Autoregressive Model – PAR

Periodic Autoregressive models of order p_m – PAR(p_m) – are extensions of the AR model, addressed in time series that exhibit variations in their structure (Vecchia, 1985). The rationale for using the PAR is that some time series, such as seasonal hydrological ones, feature an autocorrelation structure connected to the

time delay between the samples and the period observed (Hippel and McLeod, 1994; Siqueira et al., 2019b). Besides, hydrological series with a periodicity of up to one year, as the monthly streamflow series, have a periodic behavior concerning their statistical properties like the mean, the variance, and correlation structure (Maceira et al., 2002). The mathematical expression of the model $PAR(p_m)$ is as in Equation (7):

$$\hat{x}_t = \phi_1^m x_{t-1} + \phi_2^m x_{t-2} + \dots + \phi_{p_m}^m x_{t-p_m} + a_t \quad (7)$$

where the higher index $m = 1, 2, \dots, 12$ means the month which the sample belongs.

In this manner, for each of the 12 months of the year, an AR model will be adjusted independently. For this, the terms of the Yule-Walker equations are rewritten for the periodic case in matrix form following Equation (8) (Belotti et al., 2020a):

$$\mathbf{P}_{p_m}^m = \begin{bmatrix} 1 & \rho_1^{m-1} & \dots & \rho_{p_m-1}^{m-1} \\ \rho_1^{m-1} & 1 & \dots & \rho_{p_m-2}^{m-2} \\ \dots & \dots & \dots & \dots \\ \rho_{p_m-1}^{m-1} & \rho_{p_m-2}^{m-2} & \dots & 1 \end{bmatrix} \quad \Phi_{p_m}^m = \begin{bmatrix} \phi_1^m \\ \phi_2^m \\ \dots \\ \phi_{p_m}^m \end{bmatrix} \quad \rho_{p_m}^m = \begin{bmatrix} \rho_1^m \\ \rho_2^m \\ \dots \\ \rho_{p_m}^m \end{bmatrix} \quad (8)$$

where $m = 1, 2, \dots, 12$ is the index of each month.

The optimal solution for calculating the $PAR(p_m)$ model's coefficients, in the minimal MSE sense, is given by Equation (9):

$$\Phi_{p_m}^m = (\mathbf{P}_{p_m}^m)^{-1} \rho_{p_m}^m \quad (9)$$

The PAR model can be directly used to predict and model the monthly streamflow series if they are stationary.

Extreme Learning Machines

Artificial neural networks are structures of distributed processing of information, consisting of simple processing units, called artificial neurons, with a high degree of interconnectivity (Haykin, 2008; Lizot et al., 2020; Siqueira et al., 2012a; Tang et al., 1991; Thesari et al., 2019; Wang, 2011). One striking feature of this methodology is the ability of learning (Kachba et al., 2020; Zhang et al., 2018). Usually, the learning process involves the adaptation of the efficiencies of the connections between neurons, represented as weights that balance the transmitted signals, aiming to bring the desired response in the output of the network (Atiya et al., 1999; Francelin et al., 1996; Yaseen et al., 2016).

The prediction of streamflow series can be considered a nonlinear static mapping task (Belotti et al., 2020a; Siqueira et al., 2012b; Yaseen et al., 2017). The Multilayer Perceptron (MLP) is an architecture of ANN widely used in tackling this problem (Siqueira et al., 2014). For this, all the parameters of the layers of neurons of an MLP will need to be adjusted in a supervised manner (Haykin, 2008), through optimization algorithms using gradient vector information of the cost function based on the mean square error (Cybenkot, 1989).

However, in recent years, new proposals of ANN are gaining prominence. Inserted in this context are the ELM, feedforward networks, similar to the traditional MLPs and having a single hidden layer (Huang, et al., 2006a). What differentiates them is the process of adjustment of the weights, which in this case is quite simple (Feng et al., 2012; Mattos Neto et al., 2020; Siqueira and Luna, 2019). In an ELM, the neurons of the hidden

layer have weights generated randomly and independently. Also, the training is limited to the adaptation of the weights of the output layer. The optimal values of these coefficients are typically calculated in analytical form since the training involves the solution of a linear regression problem (Huang et al., 2012; Tadano et al., 2016). Therefore, it is not necessary to calculate derivatives of the cost function, perform the back-propagation of errors, or approximate the optimal solution iteratively. It brings a significant reduction of computational cost to adjust the network (Huang et al., 2006b).

The activations of the neurons of the hidden layer are given by Equation (10) (Mattos Neto et al., 2020, 2021; Ribeiro et al., 2020):

$$\mathbf{x}_n^h = \mathbf{f}^h(\mathbf{W}^h \mathbf{u}_n + \mathbf{b}), \quad (10)$$

being $\mathbf{u}_n = [u_n, u_{n-1}, \dots, u_{n-K+1}]^T$ the input signals, $\mathbf{W}^h \in \mathcal{R}^{N_h \times K}$ the coefficients of the hidden layer, $\mathbf{b}$ are the biases of the hidden neurons and $\mathbf{f}^h(\cdot) = (f_1^h(\cdot), f_2^h(\cdot), \dots, f_{N_h}^h(\cdot))$ their activation functions.

The outputs of the network are given Equation (11):

$$\mathbf{y} = \mathbf{W}^{\text{out}} \mathbf{x}_n^h, \quad (11)$$

where $\mathbf{W}^{\text{out}}$ is the output weight matrix.

The training process involves minimizing the mean square error between the output and the desired response $\mathbf{d}$. This linear regression task admits an analytical solution for linear output combiner's coefficients, based on the Moore-Penrose pseudo-inversion operator (Huang et al., 2006b), as in Equation (12):

$$\mathbf{W}^{\text{out}} = (\mathbf{X}_h^T \mathbf{X}_h)^{-1} \mathbf{X}_h^T \mathbf{d}, \quad (12)$$

where $(\mathbf{X}_h^T \mathbf{X}_h)^{-1} \mathbf{X}_h^T$ is the pseudoinverse of $\mathbf{X}_h \in \mathcal{R}^{T_s \times N_k}$, the matrix with the outputs of the intermediate layer, and T_s is the number of the training samples.

It should be noted that the adoption of the operator also allows calculating the weights with the minimum norm. According to Bartlett (1998), this is a decisive factor in maximizing the ability of network generalization.

Recent studies have shown that the ELM can achieve superior results compared to MLP in various forecasting tasks (Siqueira et al., 2014; Siqueira et al., 2018; Siqueira et al., 2019). Therefore, this work adopts such a method as a basic nonlinear model for comparative analysis.

Data and study region

This investigation considers the monthly seasonal streamflow series related to 20 Brazilian hydroelectric plants, which are located in different watersheds and rivers, as shown in Figure 1.

The monthly streamflow data regarding the plants are available on the ONS website (ONS, 2019). The samples are in cubic meters per second (m^3/s), ranging from January 1st. 1931 to December 31st. 2015, comprising 85 years, or 1020 months. In Table 2 one can notice the diversity of hydrological behavior of these plants, presenting distinct characteristics considering the mean (μ), standard deviation (σ), coefficient of variation ($\text{CV} = \mu/\sigma$), and basin area. Moreover, we provide the coordinates of each plant.

To exemplify the monthly behavior of the streamflows in Brazil, we present Table 3, showing the monthly mean and

Fig. 1. Location of the 20 Hydroelectric Plants associated with the streamflow series used.

Table 2. Averages, standard deviations, coefficient of variation, location (latitude and longitude), basin area and data length of the streamflow series used.

	Plant	Mean	St. deviation	CV	Coordinates	Basin area (km ²)
1.	14 de Julho	290.9284	257.1931	0.8840	29°S, -51.4°W	6
2.	Água Vermelha	2.0773e+03	1.2957e+03	0.6237	19.9°S, 50.3°W	647
3.	Belo Monte	8.0458e+03	7.7698e+03	0.9657	3.1°S, 51.8°W	478
4.	Boa Esperança	458.2059	241.7749	0.5277	6.4°S, 43.3°W	87.5
5.	Cana brava	856.3588	757.6024	0.8847	13.4°S, 48.1°W	139
6.	Chavantes	345.8422	211.9175	0.6128	23.1°S, 49.7°W	400
7.	Furnas	912.1225	613.5036	0.6726	20.7°S, 46.3°W	1440
8.	Gov. P. de Souza	19.8294	9.1280	0.4603	25.2°S, 48.8°W	16.3
9.	Itaipu	1.0324e+04	4.7691e+03	0.4619	25.4°S, 54.6°W	1350
10.	Itabepi	373.1902	428.7388	1.1488	15.9°S, 39.5°W	64.2
11.	Jirau	3.1098e+03	8.4791e+03	2.7266	9.2°S, 64.6°W	361.6
12.	Machadinho	745.4049	602.1645	0.8078	27.5°S, 51.7°W	56.7
13.	Paulo Afonso	2.6988e+03	2.0265e+03	0.7509	9.4°S, 38.2°W	630
14.	Santo Antônio	3.1573e+03	8.6085e+03	2.7265	8.7°S, 63.9°W	350
15.	Serra da mesa	764.9608	682.7458	0.8925	13.8°S, 48.3°W	1.784
16.	Sinop	933.0010	502.7034	0.5388	11.2°S, 55.4°W	337
17.	Sobradinho	1.8739e+03	1.4184e+03	0.7569	9.4°S, 40.8°W	4,214
18.	Teles Pires	2.2849e+03	1.5293e+03	0.6693	9.3°S, 56.7°W	150
19.	Três Irmãos	766.4373	470.5756	0.6140	20.6°S, 51.3°W	785
20.	Tucuruí	1.0935e+04	9.1823e+03	0.8397	3.8°S, 49.6°W	2850

standard deviation of the series from Furnas Plant. This seasonal behavior is similar to the other plants. The months from December to April, present a higher average, accompanied by the highest standard deviations. This is related to the rainfall periods throughout the year since the rainfall density in the summer is higher than in the winter (Siqueira et al., 2018).

RESULTS

Pre-processing

The linear prediction models of the Box & Jenkins methodology adopted herein assume that series are stationary (Nelson et al., 1999). Thus, before applying these models, it is

necessary to use a statistical transformation known as deseasonalization, to remove the seasonal component of the series. The standard procedure adopted in the literature is given by Equation (13) (Siqueira et al., 2020b; Souza et al., 2010):

$$z_{i,m} = \frac{x_{i,m} - \hat{\mu}_m}{\hat{\sigma}_m} \quad (13)$$

where $x_{i,m}$ represents the streamflow observed in months $m = 1, 2, \dots, 12$ of i -th year, $i = 1, 2, \dots, N_y$, being N_y the number of years composing the history, $\hat{\mu}_m$ is the historical average observed for the month m , and $\hat{\sigma}_m$ standard deviation.

Table 3. Monthly averages and standard deviations for Furnas series.

Month	$\hat{\mu}_m$	$\hat{\sigma}_m$
JAN	1.7492e+03	710.26
FEB	1.6247e+03	631.35
MAR	1.4490e+03	582.18
APR	0.9972e+03	345.83
MAY	0.7287e+03	227.23
JUN	0.6082e+03	236.96
JUL	0.4997e+03	150.33
AUG	0.4104e+03	120.36
SEP	0.4295e+03	219.21
OCT	0.5022e+03	220.81
NOV	0.7146e+03	297.53
DEC	1.2322e+03	451.27

It should be observed that the original series is transformed into a new approximately stationary series $\mathbf{z}_t$, with a zero mean and unit standard deviation (Siqueira and Luna, 2019). At the end of the forecasting process, the seasonal component is reinserted into the model's output response to allow a performance analysis in the real domain of the series.

Computational performances

In this work, we use AR, PAR, and ELM as forecasting models, which were previously described. The firsts were selected to represent the linear models, while the last was used to show a general behavior of an ANN.

Initially, the 85 years of the history of the series were divided into two sets: (1) test, consisting of 120 months, comprising the last 10 years; and (2) training, with the other available data, in a total of 75 years or 900 monthly observations. The performance analysis derives from the verification of the Mean Square Error (MSE), Mean Absolute Error (MAE), Correlation Coefficient (CC), and Nash–Sutcliffe Efficiency (NSE).

The inputs are inserted in the models considering the previous months. For example, if we would like to predict the data for October, and considering the first and second delays as inputs, the inputs are September and August. The same methodology is used for monthly and annual approaches.

According to McLeod (1994), the way the PAR model is used in this type of study considers some assumptions. The first is that the maximum number of inputs, or delays, is 6, that is, the order of the PAR model ranges from $p_m = 1$ to $p_m = 6$. The selection of the input variables is made based on partial autocorrelation function (PACF) (Siqueira et al, 2020a), with the restriction that these entries must be consecutive, admitting only significant intermediate values (Stedinger, 2001). This last condition is well detailed by Stedinger (2001), in which the author state that samples from temporarily distant months (for example, January and September) are not statistically related in hydrological series. We also adopted as forecasting horizons the values of $P = 1$ (next month), 3 (next season), 6 (next invert season), and 12 (the same month in the next year) steps ahead.

As for the prediction model being monthly or annually, the PAR and AR were used, respectively. In parallel, the ELM was applied according to these same paradigms: a unique model for the whole series (within the concept of the AR model) and 12 different networks, one for each month (as per PAR methodology). For the ELM, the selection of inputs was made through the wrapper with progressive selection (Kohavi and John, 1997; Mendes Junior et al., (2020); Siqueira et al., 2020a). The number of neurons in the intermediate layer was chosen

based on preliminary empirical tests. Besides, as described in Section “Direct and recursive predictions”, it was verified the impact of adopting the direct or the recursive approaches for $P > 1$ (Siqueira et al, 2014; Sorjamaa et al., 2007).

By combining the described procedures, we have four prediction approaches:

- monthly recursive (M-R) – with 12 models and recursive approach for $P > 1$;
- monthly direct (M-D) - 12 forecasting models, but with the use of direct multistep forecasting;
- recursive approach with a single annual model for the entire series (A-R);
- direct prediction with a single forecaster (A-D).

To exemplify the number of computational results, Tables 5 in Appendix A Section presents the performances of the forecasting models obtained for $P = 1$ step ahead. Results are presented for the real domain (MSE, MAE, CC, and NSE) and deseasonalized domain (MAE and MSE), i.e., after the series go through the process described in (13). It is possible to analyze the models' approximation and generalization capabilities in the deseasonalized case, since the data effectively used are in this domain. Regarding $P = 3, 6$, and 12 steps ahead we present Table 6, which contains the MSE reached by each model and forecasting approach.

As mentioned, we addressed the series associated with 20 plants, planned for 4 horizons, totaling 80 scenarios. One can notice in Tables 5 and 6 that the rows present the errors of a linear model and an ELM. Also, the results of the ELM are the average considering 30 independent experiments. The best performances in each case are highlighted in bold. The terms “MSEd” and “MAEd” are related to the deseasonalized space (Table 5).

The Friedman test was applied to verify the difference in the significance of the errors (Friedman, 1937; Luna and Ballini, 2011). The results found for the p -value were very close to zero, which indicates that changing the model and forecasting approach leads to significantly different results.

Table 4 presents, on the left side, the forecasting approach that achieved the smaller MSE in the test set, considering all plants. To $P = 1$, note that the only possible analysis is related to the annual (“A”) or monthly methods (“M”). On the right side of the table, we depict how many times each approach presented the best performance. The use of **bold**, *italics*, and ~~crossed-out~~ is to facilitate the visualization. In addition, Figure 2 shows the same information separated by the horizon.

Figure 3 presents the boxplot graphic of the MSE in the real domain to the test set of Furnas series, regarding the 30 independent simulations of the ELM to $P = 1$ and $P = 12$ with direct and recursive approaches. The acronyms “JAN” to “DEC” means the dispersion of each month, “MONT” is the total dispersion of monthly approach, and “AN” the dispersion of the annual approach. Note that the final response of a neural network rarely converges to the same result because of the random initialization of the weights (Haykin, 2008).

We highlight three aspects: *i*) the degree of dispersion of each month follows the monthly average, which means the higher the average, the higher the dispersion; *ii*) in the direct approach, the total dispersion of the results regarding the monthly and annual models are similar (right side of the figure); *iii*) the dispersion of the recursive approach is higher than the direct approach.

It can also be observed a higher dispersion from December to March, following the mean (see Table 3). Although the graphs are not shown for all the scenarios, the general behavior was similar for all plants.

Table 4. Number of best performances achieved by each combined approach (M-D / M-R/ A-D/ A-R) considering each plant and $P > 1$ forecasting horizons. For $P = 1$ we compare just the annual (A) and monthly (M) approaches.

Plant	Steps ahead				Best performance			
	$P = 1$	$P = 3$	$P = 6$	$P = 12$	M-D	M-R	A-D	A-R
14 de Julho	M	M-D	M-D	M-R	2	1	-	-
Água Vermelha	M	A-R	M-D	M-D	2	-	-	1
Belo Monte	M	M-D	A-R	A-D	1	-	1	1
Boa Esperança	M	M-R	M-D	M-R	1	2	-	-
Cana brava	M	M-D	M-D	M-D	3	-	-	-
Chavantes	A	A-R	M-D	M-D	2	-	-	1
Furnas	M	A-R	M-D	M-D	2	-	-	1
Gov. P. de Souza	M	A-D	M-D	M-D	2	-	1	-
Itaipu	M	M-D	A-R	M-D	2	-	-	1
Itapebi	M	M-R	A-D	A-R	-	1	1	1
Jirau	A	M-R	M-R	M-R	-	3	-	-
Machadinho	M	M-D	M-D	M-D	3	-	-	-
Paulo Afonso	M	M-D	M-D	M-D	3	-	-	-
Santo Antônio	A	M-R	M-R	M-R	-	3	-	-
Serra da Mesa	M	M-D	M-D	A-D	2	-	1	-
Sinop	A	A-D	M-R	M-R	-	2	1	-
Sobradinho	M	M-D	M-D	M-D	3	-	-	-
Teles Pires	A	A-R	M-D	M-R	1	1	-	1
Três Irmãos	A	A-D	A-R	M-R	-	1	1	1
Tucuruí	M	M-D	A-R	M-D	2	-	-	1

Fig. 2. Number of best results found by each forecasting approach (M-D, M-R, A-D, A-R) by forecasting horizon.

DISCUSSION

As mentioned in the previous section, Table 5 (Appendix A) presents the results achieved by the forecasting models (AR, PAR, and ELM) considering each forecasting approach (M-R, M-D, A-R, A-D) and $P = 1$. Table 4 displays a comparison among only the forecasting approaches, while Table 6 (Appendix A) depicts the MSE for $P = 3, 6$, and 12 steps ahead.

The performances reached allow other relevant considerations beyond the analysis of the forecasting approaches. An equivalence was not observed between the best performance based on the MSE in the real and deseasonalized domain in many scenarios. This fact highlights the need to develop different techniques of removing the seasonal component to favor the output responses before reinserting the seasonal component. In the same way, when comparing the performances by different error metrics, it can be noticed that, in some cases, the best MSE is different from the smallest MAE, NSE, or CC. As the function minimized in the optimization and calculation of the coefficients is based on the MSE, we adopted this metric in the

real domain to determine the best performances (Belotti et al., 2020a; Siqueira et al., 2014).

In Table 4 and Figure 2, we present which forecasting approach presented the smallest MSE, considering 20 hydroelectric plants and four forecasting horizons ($P = 1, 3, 6$, and 12), totaling 80 scenarios. The analysis of the results evinces that to $P = 1$, the monthly approach achieved the best performance in 14 out of 20 cases (70%), while the annual model in 6. For $P > 1$ (60 scenarios), 31 cases were in favor of monthly + direct approach (M-D), 14 for monthly + recursive method (M-R), 6 for annual + direct approach (A-D), and 9 to annual + recursive methodology (A-R). In addition, only in 4 cases, the same combination achieved the best results for some plant, depicting how challenging the task is.

Comparing only the monthly and annual approaches, the performances revealed that the monthly approach is more suitable to the task, as the methodology achieved the lowest errors in 59 cases (73.75%), regarding all horizons. This is a significant result since many studies involving monthly streamflow series predictions use one forecasting model for the whole series

Fig. 3. Boxplot graphic of ELM results for Furnas (a) $P = 1$, (b) $P = 12$ - direct approach - and (c) $P = 12$ -recursive approach. “MONT” is the total dispersion of the monthly approach, and “AN” the dispersion of the annual approach.

(annual approach), as discussed in the Introduction. Indeed, the statistical behavior regarding each month presents a particular characteristic in terms of mean and standard deviation. Therefore, the forecasting models can model a more well-behaved dataset if we separate them according to their similarities. It is an indication that this kind of approach for seasonal series, in general, should be considered, even if the seasonality is not monthly. Regarding the multistep forecasting ($P > 1$), the direct method stood out since it performed better in 37 out of 60 cases.

Analyzing the combination of the two approaches, it is clear that monthly + direct (M-D) method achieved most of the best results, even if we observe each forecasting horizon P (see Figure 2). Moreover, considering only the highest horizons ($P = 6$ and 12), this behavior is more remarkable, inasmuch

such combination presented the best MSE 12 out of 20 times for $P = 6$, and 10 out of 20 times regarding $P = 12$.

However, we highlight an interesting finding. To Jirau and Santo Antonio plants, the *M-R* approach was the best in all multistep scenarios, which only occurs to them. A careful look in Table 2 reveals that these plants present the coefficient of variation higher than 2, indicating that the standard deviation is more the double of the value of the mean. It means that these are challenging scenarios, with a high variance of water throughout the year. As this behavior is not the most common, the recursive approach can be more suitable. Besides, to $P = 1$, the annual approach achieved the lowest error in both cases, which corroborates the previous statement.

Fig. 4. Best predictions achieved for Furnas series (a) $P = 1$, (b) $P = 3$, (c) $P = 6$, and (d) $P = 12$.

Tables 5 and 6 also allow a comparative discussion about the performance of the ELM, and AR or PAR, for all forecasting approaches. We observed that in 65 out of 80 scenarios examined (81.25%) the ELM reached smaller errors than linear models. As can be seen, the ELM overcame the respective linear approaches in most cases.

Another analysis regarding such tables is to compare the forecasting capability of the ELM and the linear models, considering the 4 combined forecasting approach (A-R, A-D, M-R, M-D), for all 20 plants, and $P = 3, 6$, and 12 steps ahead (totaling 240 possibilities). The ELM presented better results in 165 cases, while the AR and PAR in 75. For $P = 1$, in which we only can compare the annual and monthly approaches, the ELM overcame the linear models in 28 out of 40 cases. These remarks show consistent gains in using nonlinear methods for solving this problem.

The computational results allow us to advocate the use of the ELM instead of the linear models. Being an ANN architecture, the ELM can model the space in a nonlinear way, while the linear models weigh the inputs to provide predictions. Besides, ELM are universal approximator capable of generalizing its output response. The only comparative drawback is that the linear models are adjusted using a closed-form solution (Yule-Walker equations), while the ELM must be adjusted for different amounts of internal neurons, as well as its final response is not deterministic. Therefore, we must run the algorithm at least 30 times. However, a single simulation presents computational cost similar to a linear model (Moore-Penrose Operator), which grants competitiveness to the method. This is not observed for other classic ANN, as the MLP.

Hydropower reservoir operation and management depend on many variables, such as precipitation, evaporation, hydrological and climate conditions, water availability, among others (Yang et al., 2015). Reservoir control is a complex task and can be supported by accurate daily inflow information (Yang et al., 2016; Yang et al., 2020). In this sense, the ELM can be a candidate methodology to deal with inflow data.

Finally, especially for the Brazilian case, the results achieved are essential since its electric sector privileges applying the recursive approach in multistep forecasting. Also, the use of ANN instead of linear models must be encouraged. It seems to be clear that the general method adopted by the Brazilian Electric Sector should be revised, except concerning the monthly approach.

Figure 4 allows for visual appreciation of the best execution regarding the ELM and the respective best prediction approach in real space. As can be noted, the output response tends to deteriorate as the horizon grows.

CONCLUSIONS

Accurate monthly streamflow predictions are essential for water resources management for decisions related to the operation of the energy matrix of a country, the planning of hydropower generation, flood control, and water supply, among others. In this sense, this work performed an investigation on the use of forecasting approaches techniques for monthly streamflow series. We focused on analyzing the use of monthly versus annual approaches, as well as the adoption of the direct versus recursive methods to multistep prediction. The forecasting models used were the AR – (annual model) and PAR – (monthly model), and ELM neural networks. The ELM were selected since they present good results in prediction tasks, overcoming well-known architectures, such as the MLP and traditional linear models.

In this sense, the main findings were:

- The computational results indicated that the use of the direct approach led to most of the best performances;
- The monthly method overcame to the annual methodology in most cases;
- The monthly recursive prediction achieved the best errors to all multistep tasks, to the series in which the coefficient of variation is higher than 2;
- The ELM have reached the best overall performances in comparison to AR and PAR. It also presented a low computational cost to be adjusted;
- The Brazilian Electric Sector uses the PAR model with the recursive approach, but such methodology achieved the best performance in less than 17% of the cases.

Future works can be developed using series from other countries and hydrologic plants. These approaches should also be investigated in weekly or daily hydrological series. Also, other input selection methods can be addressed (Belotti, et al. 2020b; Siqueira et al., 2020c), such as bio-inspired metaheuristics (Puchta et al., 2016; Puchta et al. 2020; Santana et al., 2019; Santos et al., 2018).

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APENDIX

In this section, we present in Table 5 all results achieved by the models for each series related to the hydroelectric plants and prediction approach to $P = 1$ step ahead. The acronym “A” means annual approach and “M” monthly approach.

Table 5. Performance metrics of forecasting models (ELM, AR, PAR) considering the forecasting approaches proposed for $P = 1$.

14 de Julho							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	5.3326e+04	161.5092	0.8673	0.7096	0.3019	0.5497
	ELM	4.3241e+04	148.6923	0.7278	0.6613	0.5593	0.7736
A	AR	5.5159e+04	166.6962	0.8837	0.7243	0.2779	0.5307
	ELM	5.1055e+04	160.6210	0.8428	0.7085	0.4210	0.6494
Água Vermelha							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	4.3997e+05	408.8729	0.4511	0.4997	0.7235	0.8509
	ELM	3.9971e+05	384.3615	0.3862	0.4652	0.8432	0.9246
A	AR	4.4489e+05	399.4328	0.4069	0.4757	0.7204	0.8506
	ELM	4.1441e+05	393.1916	0.3984	0.4795	0.7772	0.8818
Belo Monte							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	5.3423e+06	1.3941e+03	0.4017	0.4931	0.8998	0.9496
	ELM	3.8327e+06	1.1944e+03	0.2985	0.4168	0.9704	0.9881
A	AR	4.3492e+06	1.2654e+03	0.3544	0.4571	0.9184	0.9593
	ELM	4.4398e+06	1.2720e+03	0.3537	0.4577	0.9212	0.9614
Boa Esperança							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.4121e+04	71.4677	0.7093	0.5471	0.6000	0.7759
	ELM	1.2490e+04	66.2336	0.4875	0.4658	0.7669	0.8550
A	AR	1.4053e+04	67.3870	0.5186	0.4700	0.6019	0.7767
	ELM	1.4210e+04	69.1320	0.5274	0.4838	0.6307	0.7950
Cana Brava							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.1543e+05	202.8349	0.3610	0.4704	0.6344	0.8026
	ELM	9.1808e+04	183.5940	0.2504	0.3889	0.7754	0.9186
A	AR	1.1573e+05	198.9409	0.2878	0.4135	0.6335	0.8086
	ELM	1.1747e+05	200.1862	0.2880	0.4121	0.6810	0.8339
Chavantes							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	2.3674e+04	113.4355	0.7749	0.6533	0.4593	0.6787
	ELM	2.3654e+04	115.5243	0.7619	0.6589	0.7983	0.9356
A	AR	2.1597e+04	110.7855	0.7145	0.6394	0.8068	0.8122
	ELM	2.2255e+04	113.6232	0.7464	0.6598	0.5261	0.7297
Furnas							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.2006e+05	209.0800	0.4412	0.4952	0.6764	0.8227
	ELM	9.9480e+04	184.0606	0.3634	0.4406	0.7604	0.8902
A	AR	1.2129e+05	209.4246	0.4239	0.4903	0.6731	0.8217
	ELM	1.2582e+05	212.0106	0.4476	0.5039	0.7234	0.8506
Governador Parigot de Souza							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	5.9493e+01	5.3670	0.9344	0.6629	0.3808	0.6175
	ELM	5.2425e+01	5.2915	0.8464	0.6579	0.5248	0.7616
A	AR	5.5849e+01	5.2846	0.8905	0.6535	0.4187	0.6472
	ELM	5.3587e+01	5.1945	0.8712	0.6469	0.4805	0.6969

Itaipu							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	8.0053e+06	2.1757e+03	0.6720	0.6414	0.6418	0.8035
	ELM	7.3875e+06	2.0895e+03	0.6243	0.6211	0.7218	0.9176
A	AR	8.0463e+06	2.1940e+03	0.6675	0.6473	0.6400	0.8019
	ELM	7.7824e+06	2.2032e+03	0.6487	0.6528	0.6687	0.8184
Itapebi							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	4.5869e+04	115.8852	0.2530	0.3545	0.3682	0.6187
	ELM	4.2565e+04	117.4965	0.2389	0.3603	0.4376	0.7825
A	AR	5.1484e+04	117.1778	0.2628	0.3601	0.2909	0.5898
	ELM	5.5106e+04	125.7625	0.2822	0.3896	0.3596	0.6272
Jirau							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.0058e+07	2.5281e+03	0.2976	0.4304	0.9356	0.9680
	ELM	1.7680e+08	1.7262e+04	22.7350	2.2557	0.8880	0.9659
A	AR	7.3184e+06	2.1866e+03	0.1730	0.3447	0.9531	0.9763
	ELM	2.4834e+07	3.3871e+03	0.5184	0.5269	0.9563	0.9782
Machadinho							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	2.7715e+05	383.5990	0.8038	0.6974	0.3548	0.5960
	ELM	2.2160e+05	347.4806	0.6720	0.6453	0.5494	0.7323
A	AR	2.7595e+05	384.7618	0.8139	0.7032	0.3576	0.5981
	ELM	2.5237e+05	369.3114	0.7812	0.6853	0.4737	0.6930
Paulo Afonso							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	6.6143e+05	510.3236	0.2791	0.4079	0.7172	0.8488
	ELM	5.3050e+05	462.2640	0.2271	0.3696	0.7590	0.8932
A	AR	7.0020e+05	515.4807	0.2792	0.3975	0.7006	0.8451
	ELM	7.1794e+05	520.0039	0.2880	0.4030	0.7270	0.8624
Santo Antônio							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.0509e+07	2.5910e+03	0.3004	0.4335	0.9347	0.9675
	ELM	2.1189e+07	2.8507e+03	0.5437	0.5943	0.8635	0.9600
A	AR	7.6372e+06	2.2339e+03	0.1755	0.3474	0.9525	0.9760
	ELM	1.8867e+07	3.1510e+03	0.3943	0.4865	0.9562	0.9780
Serra da Mesa							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	1.2626e+05	218.3400	4.3961	0.8725	0.5121	0.7404
	ELM	8.0598e+04	168.5827	0.2808	0.4070	0.7867	0.9031
A	AR	9.5682e+04	181.0763	0.3016	0.4241	0.6303	0.8095
	ELM	9.3308e+04	176.7058	0.2938	0.4144	0.6960	0.8461
Sinop							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	5.9140e+04	165.5104	2.6936	0.9247	0.7912	0.8897
	ELM	4.4480e+04	137.4640	0.5854	0.5713	0.8510	0.9250
A	AR	4.2297e+04	130.2503	0.5575	0.5453	0.8507	0.9226
	ELM	4.0825e+04	130.9792	0.5408	0.5437	0.8616	0.9288
Sobradinho							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	3.3295e+05	403.3243	1.3450	0.6209	0.6135	0.7919
	ELM	2.3812e+05	314.5585	0.1961	0.3538	0.7510	0.9069
A	AR	2.8660e+05	347.3967	0.2374	0.3799	0.6673	0.8234
	ELM	3.1888e+05	366.4942	0.2547	0.3967	0.6870	0.8438
Teles Pires							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	3.2643e+05	378.2157	1.6069	0.6955	0.8873	0.9424
	ELM	2.4714e+05	319.2759	0.3600	0.4404	0.8906	0.9780
A	AR	2.4383e+05	314.3994	0.3452	0.4328	0.9158	0.9581
	ELM	2.3616e+05	312.9234	0.3433	0.4400	0.9246	0.9725
Três Irmãos							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	7.5827e+04	193.0740	0.5769	0.5915	0.6274	0.7945
	ELM	6.9420e+04	183.5618	0.5392	0.5646	0.7315	0.8448
A	AR	7.5786e+04	194.4370	0.5523	0.5864	0.6276	0.7984
	ELM	6.7347e+04	183.8473	0.5334	0.5669	0.6823	0.8310
Tucuruí							
	Model	MSE	MAE	MSEd	MAEd	NSE	CORR
M	PAR	8.2451e+06	1.6697e+03	0.3216	0.4162	0.8914	0.9449
	ELM	5.8830e+06	1.4679e+03	0.2580	0.3698	0.9055	0.9577
A	AR	7.5509e+06	1.6553e+03	0.2974	0.4087	0.9005	0.9504
	ELM	7.5304e+06	1.6285e+03	0.3053	0.4055	0.9108	0.9552

Table 6 presents the MSE achieved by the models for each series related to the hydroelectric plants and prediction approach to $P=3$, 6, and 12 step ahead. The acronyms are: monthly recursive (M-R), monthly direct (M-D), annual model for the entire series (A-R), and direct prediction with a single forecaster (A-D).

Table 6. MSE of predicting models by means of approaches proposed for $P = 3, 6$, and 12.

14 de Julho					Água Vermelha				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	5.5155e+04	5.7452e+04	5.7596e+04	M-R	PAR	7.1255e+05	7.0557e+05	8.5390e+05
	ELM	5.0878e+04	5.6104e+04	5.8155e+04		ELM	5.7068e+05	6.8496e+05	7.2656e+05
M-D	PAR	5.6052e+04	5.8434e+04	6.6208e+04	M-D	PAR	9.2621e+05	7.1423e+05	1.2135e+06
	ELM	4.4124e+04	5.4047e+04	6.0889e+04		ELM	5.5554e+05	5.6353e+05	6.5140e+05
A-R	AR	5.6373e+04	5.7646e+04	5.7709e+04	A-R	AR	5.6951e+05	6.7890e+05	6.8005e+05
	ELM	5.6594e+04	5.8150e+04	5.8011e+04		ELM	5.4907e+05	6.1027e+05	6.9728e+05
A-D	AR	5.5424e+04	5.7974e+04	6.0228e+04	A-D	AR	5.7437e+05	6.7592e+05	6.5666e+05
	ELM	5.6105e+04	5.4488e+04	6.0698e+04		ELM	5.5927e+05	6.4755e+05	6.5970e+05
Belo Monte					Boa Esperança				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	8.5568e+06	8.8506e+06	9.6666e+06	M-R	PAR	3.7241e+04	1.7825e+05	2.4198e+04
	ELM	6.6622e+06	7.2863e+06	6.7983e+06		ELM	1.5031e+04	1.7585e+04	1.4971e+04
M-D	PAR	5.6483e+06	7.2862e+06	5.9839e+06	M-D	PAR	2.2008e+04	1.9990e+04	1.6191e+04
	ELM	6.0493e+06	6.5406e+06	5.9614e+06		ELM	1.5786e+04	1.5379e+04	1.5428e+04
A-R	AR	5.9579e+06	6.3235e+06	6.0845e+06	A-R	AR	1.6680e+04	1.7342e+04	1.5459e+04
	ELM	6.1416e+06	6.5057e+06	6.0169e+06		ELM	1.6667e+04	1.6515e+04	1.5659e+04
A-D	AR	5.9297e+06	8.8506e+06	6.3216e+06	A-D	AR	1.7054e+04	1.9543e+04	1.4976e+04
	ELM	6.6217e+06	7.2863e+06	5.5853e+06		ELM	1.6670e+04	1.8223e+04	1.4994e+04
Cana brava					Chavantes				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	1.1733e+05	1.2758e+05	1.2420e+05	M-R	PAR	3.5018e+04	3.8890e+04	3.6171e+04
	ELM	1.1508e+05	1.1566e+05	1.1179e+05		ELM	3.3467e+04	4.0245e+04	4.0342e+04
M-D	PAR	1.1401e+05	1.3086e+05	1.0206e+05	M-D	PAR	3.2125e+04	4.1397e+04	3.7437e+04
	ELM	9.8560e+04	9.9610e+04	9.7562e+04		ELM	3.2635e+04	3.5095e+04	3.5292e+04
A-R	AR	1.1750e+05	1.1533e+05	1.0572e+05	A-R	AR	3.1053e+04	3.6047e+04	3.6278e+04
	ELM	1.1507e+05	1.1245e+05	1.0737e+05		ELM	3.2298e+04	3.7934e+04	3.8549e+04
A-D	AR	1.1843e+05	1.1672e+05	1.0341e+05	A-D	AR	3.1270e+04	3.7917e+04	3.7372e+04
	ELM	1.1365e+05	1.1152e+05	1.0391e+05		ELM	3.3175e+04	3.7923e+04	3.7694e+04
Furnas					Governador Parigot de Souza				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	1.6151e+05	1.7519e+05	1.7839e+05	M-R	PAR	8.3288e+01	8.4083e+01	8.4377e+01
	ELM	1.6274e+05	1.7729e+05	1.7782e+05		ELM	7.8756e+01	7.4673e+01	8.8624e+01
M-D	PAR	1.5852e+05	1.7017e+05	1.7121e+05	M-D	PAR	8.1980e+01	8.0682e+01	8.1503e+01
	ELM	1.5296e+05	1.5803e+05	1.5756e+05		ELM	7.5597e+01	7.1562e+01	7.7426e+01
A-R	AR	1.5503e+05	1.7126e+05	1.6935e+05	A-R	AR	7.8338e+01	8.1725e+01	8.4139e+01
	ELM	1.5127e+05	1.6122e+05	1.7538e+05		ELM	7.5425e+01	7.7421e+01	8.5190e+01
A-D	AR	1.5588e+05	1.7076e+05	1.6448e+05	A-D	AR	7.7971e+01	8.0020e+01	8.2946e+01
	ELM	1.5422e+05	1.6943e+05	1.6213e+05		ELM	7.4722e+01	7.8388e+01	8.3213e+01
Itaipu					Itapebi				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	1.3303e+07	1.4966e+07	1.2602e+07	M-R	PAR	4.7323e+04	5.0712e+04	4.9257e+04
	ELM	1.1896e+07	1.5512e+07	1.3377e+07		ELM	4.4688e+04	4.7938e+04	4.8161e+04
M-D	PAR	1.2439e+07	1.6884e+07	1.3589e+07	M-D	PAR	5.4774e+04	5.0956e+04	5.0380e+04
	ELM	1.1876e+07	1.5351e+07	1.0533e+07		ELM	4.7174e+04	5.0208e+04	5.1491e+04
A-R	AR	1.3469e+07	1.6393e+07	1.2338e+07	A-R	AR	5.1919e+04	4.8657e+04	4.9543e+04
	ELM	1.2184e+07	1.4932e+07	1.3471e+07		ELM	4.9451e+04	4.9815e+04	4.7755e+04
A-D	AR	1.3754e+07	1.6435e+07	1.2266e+07	A-D	AR	5.1935e+04	4.7198e+04	5.3440e+04
	ELM	1.2141e+07	1.6402e+07	1.2373e+07		ELM	5.1878e+04	4.7219e+04	5.2138e+04
Jirau					Machadinho				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	1.4308e+07	1.8272e+07	1.8385e+07	M-R	PAR	3.3944e+05	3.3969e+05	3.5043e+05
	ELM	9.8725e+07	1.7252e+08	3.6801e+07		ELM	3.0149e+05	3.3351e+05	3.6146e+05
M-D	PAR	1.5261e+07	2.4278e+07	2.1842e+07	M-D	PAR	3.3845e+05	3.3631e+05	3.7309e+05
	ELM	4.8930e+07	1.6533e+08	1.9482e+07		ELM	2.9785e+05	3.2552e+05	3.4374e+05
A-R	AR	2.2972e+07	4.0591e+07	2.9428e+07	A-R	AR	3.4501e+05	3.4961e+05	3.5118e+05
	ELM	1.6813e+07	1.8305e+08	2.9883e+08		ELM	3.3986e+05	3.5018e+05	3.5395e+05
A-D	AR	2.2714e+07	4.0983e+07	3.0916e+07	A-D	AR	3.4580e+05	3.4235e+05	3.5542e+05
	ELM	6.8510e+07	1.4721e+08	5.2202e+07		ELM	3.4406e+05	3.4168e+05	3.5011e+05
Paulo Afonso					Santo Antônio				
Model		$P=3$	$P=6$	$P=12$	Model		$P=3$	$P=6$	$P=12$
M-R	PAR	1.0399e+06	1.5074e+06	5.1011e+07	M-R	PAR	1.4955e+07	1.8892e+07	1.9015e+07
	ELM	9.3479e+05	1.1754e+06	1.1386e+06		ELM	2.4670e+07	1.1783e+8	2.1892e+07
M-D	PAR	9.7506e+05	1.2841e+06	9.7919e+05	M-D	PAR	1.5968e+07	2.4984e+07	2.2556e+07
	ELM	9.3237e+05	9.0422e+05	8.3284e+05		ELM	3.2154e+07	1.9860e+08	1.1205e+07
A-R	AR	9.8348e+05	9.7881e+05	1.0099e+06	A-R	AR	2.3802e+07	4.1802e+07	3.0263e+07
	ELM	1.0122e+06	9.7298e+05	9.7087e+05		ELM	2.7794e+07	1.1422e+08	2.7921e+07
A-D	AR	9.9471e+05	9.8596e+05	1.0394e+06	A-D	AR	2.3532e+07	4.2289e+07	3.1870e+07
	ELM	9.9192e+05	9.6880e+05	1.0011e+06		ELM	6.8314e+07	1.4721e+08	5.3713e+07

Serra da mesa					Sinop						
Model		P=3	P=6	P=12	Model		P=3	P=6	P=12		
M-	R	PAR	1.4683e+05	9.4877e+4	1.8215e+05	M-	R	PAR	1.0741e+05	1.5520e+05	6.9934e+04
	ELM	9.6654e+04	9.6615e+04	9.3975e+04	M-		R	ELM	5.0167e+04	5.4662e+04	5.4178e+04
M-	D	PAR	1.0407e+05	1.6288e+05		2.3069e+05	M-	D	PAR	6.1374e+04	1.0576e+05
	ELM	8.7269e+04	8.6777e+04	8.3729e+04	M-	D		ELM	5.4431e+04	5.4706e+04	5.5066e+04
A-	R	AR	9.8673e+04	9.6364e+04		8.8187e+04	A-	R	AR	5.0601e+04	5.6389e+04
	ELM	9.5990e+04	9.5182e+04	9.0861e+04	A-	R		ELM	5.0094e+04	5.6472e+04	5.4937e+04
A-	D	AR	9.9589e+04	9.7744e+04		8.6756e+04	A-	D	AR	5.1355e+04	5.9207e+04
	ELM	9.7705e+04	9.2552e+04	7.5969e+04	A-	D		ELM	4.9851e+04	5.7424e+04	5.8099e+04
Sobradinho						Teles Pires					
Model		P=3	P=6	P=12	Model		P=3	P=6	P=12		
M-	R	PAR	4.5133e+05	1.1998e+06	5.0590e+05	M-	R	PAR	4.0565e+05	4.5270e+05	4.2352e+05
	ELM	3.5880e+05	4.1236e+05	4.0946e+05	M-		R	ELM	3.5657e+05	3.4639e+05	3.2055e+05
M-	D	PAR	4.0617e+05	1.0699e+06		4.3672e+05	M-	D	PAR	3.8729e+05	4.3994e+05
	ELM	3.1979e+05	2.7024e+05	3.3465e+05	M-	D		ELM	3.5514e+05	2.7438e+05	3.4297e+05
A-	R	AR	3.3513e+05	3.4327e+05		3.5499e+05	A-	R	AR	3.4091e+05	3.6936e+05
	ELM	3.7163e+05	3.4543e+05	3.7176e+05	A-	R		ELM	3.2955e+05	3.4759e+05	3.6146e+05
A-	D	AR	3.2963e+05	3.3238e+05		3.6706e+05	A-	D	AR	3.4209e+05	3.8763e+05
	ELM	3.5676e+05	3.2513e+05	3.8213e+05	A-	D		ELM	3.4676e+05	3.7845e+05	4.0286e+05
Três Irmãos						Tucuruí					
Model		P=3	P=6	P=12	Model		P=3	P=6	P=12		
M-	R	PAR	1.1288e+05	1.1585e+05	1.1722e+05	M-	R	PAR	1.6745e+07	2.0727e+07	1.4444e+07
	ELM	1.0025e+05	1.1321e+05	1.1290e+05	M-		R	ELM	1.2114e+07	1.3634e+07	1.1967e+07
M-	D	PAR	1.0347e+05	1.1920e+05		1.2109e+05	M-	D	PAR	1.3470e+07	1.3196e+07
	ELM	9.8422e+04	1.1988e+05	1.1686e+05	M-	D		ELM	1.0936e+07	1.2235e+07	1.0986e+07
A-	R	AR	9.6518e+04	1.1494e+05		1.1354e+05	A-	R	AR	1.3166e+07	1.4458e+07
	ELM	9.6874e+04	1.1308e+05	1.2271e+05	A-	R		ELM	1.2135e+07	1.2116e+07	1.1627e+07
A-	D	AR	9.6028e+04	1.1907e+05		1.1349e+05	A-	D	AR	1.3391e+07	1.4728e+07
	ELM	9.7129e+04	1.1316e+05	1.1531e+05	A-	D		ELM	1.2234e+07	1.5125e+07	1.1760e+07