Relative role of sediment entrainments on log-law parameters of longitudinal velocity distributions in mobile bed flows

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Abstract: In alluvial channel, the non-cohesive bed particles are frequently accelerated by the flows and there has been an inconclusive debate on the deviations of logarithmic law parameters that demonstrate the velocity distributions in flows. Present study aims to elucidate the current knowledge of overwhelming theoretical and experimental evidences in this regard within the scope of near-bed turbulent flow characteristics. The study was conducted in two folds collecting instantaneous velocity of flow over a rigid sand bed under clear water flow conditions and compared to those over mobile sand beds under equilibrium bed-load. Results corroborated additional support to confirm the upward shifting of zero-velocity level in mobile bed flows. Most importantly, the conventional value of von Kármán coefficient significantly deviates in mobile bed flows compared to those in rigid sand bed. Also, the frictional velocity obtained from the bed slope consistently differs to those obtained from the Reynolds shear stress (RSS) distributions owing to transfer of stress aliquot to the bed particles. The mechanism is well demonstrated with the energy-momentum transfer within the framework of energy budget concept which shows near-bed negative pressure energy diffusion rates with increasing turbulence production in mobile bed flows.

Keywords: Alluvial channel; Energy budget; Hydrodynamics; Mobile bed; Zero-velocity level.

INTRODUCTION

Flow-stream-bed interaction in alluvial channel is a classical problem as they produce many morphodynamical changes. In mobile bed stream, individual bed-particles remain in contact with the bed and set into motion as a part of bed-load as shown in Fig. 1. It is worth mentioning that the sediment entrainment is directly governed by the flow characteristics and impetus new light on the near-bed turbulent flow characteristics (Balachandar et al., 2001; Guo, 1998; Kironoto and Graf, 1994; Yang, 2010). In recent years, several publications have appeared on mobile bed hydraulics and the interaction of overlying flows with the bed load transport imposes challenge when it comes to quantify the relative role of sediment mobility on the near-bed flow fields (Best et al., 1997; Dey and Raikar, 2007; Gaudio et al., 2011; Gust and Southard, 1983; Song et al., 1994). Many researchers reported that during sediment entrainment and collisions bed-particles receive kinetic energy from the overlying flow resulting enhanced flow resistance (Calomino et al., 2004;

Owen, 1964; Smith and McLean, 1977; Song et al., 1998; Wang and Larsen, 1994). In contrast, other researchers showed that sediment entrainment and bed-load transport cause reduction of flow resistance with an increase of streamwise velocity (Carboneau and Bergeron, 2000; Nikora and Goring, 2000). Present study is concerned with the impact of sediment mobility on the parameters of the formula that represent the mean velocity profiles in mobile bed flows. Traditionally, the streamwise velocity profile in the inner-layer of the flow ($z \leq 0.2h$, where z is the vertical coordinate and h is the flow depth in the channel) can be fitted to the universal logarithmic law of wall (log-law) as:

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{z'}{z_0} \right) \quad (1)$$

where, u^+ = non-dimensional time averaged streamwise velocity, κ = von Kármán coefficient, z' = function of displacement height (Δz) above the zero velocity level (z_0). Characterization

Fig. 1. Definition sketch of mobile bed flows: z_c = crest level, z_0 = zero velocity level, z_v = virtual bed level, z = vertical distance, h = flow depth.

of u^+ with z_0 and Δz are very essential in order to fit the velocity profiles with logarithmic law of the wall. Since many decades the process of delineating z_0 has been largely studied in accordance with the assumptions charted in the definition of zero boundaries (Schlichting and Gersten, 1979). In general, the lower boundary of the inner flow region located Δz above or below the definition boundary zero is considered as z_0 whereas, the vertical distance between z_0 and zero boundary indicate Δz . Some researchers argued that the zero plane indicate the lower limit of flow penetration depth below the crest level and there is no significant exchange of momentum occurs below the zero boundary line (Ferreira et al., 2012; Nikora et al., 2002). Secondly, another important parameter is the von Kármán coefficient (κ) in the log-law of velocity profile and many researchers adopted the value as 0.41 irrespective of channel wall conditions (Kirkgoz and Ardiclioglu, 1997; Miguthanna et al., 2020). Importantly, some recent experimental studies reported significant deviations of κ in mobile bed open channel flows and found to be in the range of 0.28 to 0.34 (Dey and Raikar, 2007; Gaudio et al., 2010; Nikora and Goring, 2000). A short review in this regard is presented in Fig. 2. The results provide confirmatory evidence on the influence of sediment mobility on κ . It is argued that the enhancement in velocity gradients and near bed turbulence intensity owing to sediment mobility leads to the reduction in κ . Further, the assessment of frictional velocity (u^*) to plot the log-law profile is crucial as it correlates the sediment mobility to the flow characteristics. In mobile bed stream, it is worth noting that the transports of grains are governed by the momentum extracted from the flow. Hence, a significant portion of the flow energy transfers to turbulence through turbulent energy generation resulting large vortices during bed-load transport and associated to the RSS aliquot transferred to the bed particles (Grass, 1970; Yeganeh-Bakhtiary et al., 2009). The momentum extracted from the flow and energy dissipation can be well presented through the local balance of turbulent kinetic energy (TKE) budget components. The TKE-budget analyses are already demonstrated by many authors in the domain of atmospheric boundary layers and hydraulically smooth, transitionally rough and flows over vegetated canopies (Balachandar and Bhuiyan, 2007; Dey et al., 2012; Mignot et al. 2009; Raupach and Shaw, 1982). Recent

growth in the volume of research offers significant impetus on TKE-budget concept for assessing especially the pressure energy diffusion rates over the mobile bed which cannot be measured directly. Therefore, the present laboratory work is devoted to clarify how sediment mobility during bed-load transport influences the parameters of the time-averaged velocity distributions and energy-moment exchange over a mobile sand bed. The paper is organized as follows: the experimental procedure and bed configuration is followed by a brief introduction. The results and discussions are described next. Finally, the summary and conclusions are given.

EXPERIMENTAL PROCEDURE

A rectangular open-channel of 15.0 m length was used to perform the laboratory experiments as shown in Fig. 3 (a). The width and height of the channel was 0.91 m and 0.80 m, respectively. Both sides of the channel were made of glass walls for visual observations. Uniform grains of sand ($d_{50} = 0.65$ mm) was glued on the flume bottom with cement slurry for preparation of rigid sand bed having longitudinal bed-slopes (S) of 0.1 and 0.075% (Fig. 3 b). All the tests were performed under uniform flow conditions. Three Vernier point gauges were used to check the uniform flow depth at different points in the flow. In mobile bed experimental runs, sediment feeding was imposed into the flow at the upstream end maintaining dynamic equilibrium conditions (Fig. 3c). In each bed, three tests were conducted with the flow rates of 0.1, 0.07 and 0.055 m^3/s . The mean velocity (U), flow Reynolds number (Re), flow Froude number (F) and bed-load transport rate (g_b) varied in the following ranges: $0.48 \text{ m/s} \leq U \leq 0.67 \text{ m/s}$, $1.8 \times 10^5 \leq Re \leq 3.4 \times 10^5$, $0.44 \leq F \leq 0.53$ and $1.03 \times 10^{-2} \text{ kg/(ms)} \leq g_b \leq 3.28 \times 10^{-2} \text{ kg/(ms)}$. The experimental details can be found in Malakar et al. (2020). In order to achieve dynamic equilibrium conditions, the sediment transport rate was initially quantified by collecting the transported grains in a sediment collector installed at downstream. During the tests either deposition or erosion was observed. After finalising the desirable flow conditions that enables sediments rolling over the bed, the velocity measurements were undertaken along the vertical line at the mid cross-section of the flume. An acoustic Doppler velocimeter (ADV) of

Fig. 2. A review of von Kármán coefficient, κ .

Fig. 3. (a) Experimental setup (b) rigid sand bed and (c) mobile sand bed.

10 MHz frequency was used to measure the velocity with 100 Hz sampling rate. The sampling volume was cylindrical that had 6 mm diameter and 1 – 4 mm adjustable height. The acoustic sensor comprises of one transmitting transducer and four receiving transducers. Addition of a fourth receiver improves turbulence measurements and provides redundancy. As the measuring location was 5 cm below the probe, there was no significant influence of the instrument on the measured data.

The signal-noise ratio (SNR) was maintained equal to or above 15. The ADV was fixed 8 m away from the flume inlet and at the mid cross-section of flume. Instantaneous velocity for 180s was captured along the vertical direction of flow depth at 1 mm interval. The longitudinal direction of flow is considered as x -axis, whereas transverse and vertical directions are considered as y and z -axis, respectively. Therefore, the measuring location has coordinates $(0, 0, z)$. In all the test runs, the flume width to flow depth ratio was more than 5 in order to avoid any secondary current effects induced by the side-glass walls. The measured data were filtered with a spike removal algorithm (Goring and Nikora, 2002).

RESULTS AND DISCUSSION

Zero velocity level (z_0) and law of the wall:

The vertical distributions of dimensionless time-averaged streamwise velocity, u^+ are shown in Figure 4. In the present study, the time-averaged streamwise velocity ($\bar{u}$) is normal-

ized by u^* whereas, the vertical coordinates (z) is normalized by d_{50} which enable us to fit data with log-law of velocity distributions as (Dey et al., 2012):

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln \left(\frac{z/d_{50} + \Delta z/d_{50}}{z_0/d_{50}} \right) \quad (2)$$

where, $\bar{u}/u_* = u^+$, $z/d_{50} = z^+$, $\Delta z/d_{50} = \Delta z^+$ and $z_0/d_{50} = z_0^+$. A comparative study of log-law fittings of the velocity profiles over both the test beds are illustrated in Fig. 4 (a, b, c) and Fig. 4 (d, e, f) for longitudinal bed slopes (S) = 0.1 and 0.075%, respectively. The u^+ profiles obtained from both the slopes exhibit a similar trend and it is worth noting that the use of parameters from Table 1 for an individual run produces a very good matching between the experimental data and the log-law confirming the legitimacy of the findings. The assessment of z_0^+ and Δz^+ is a challenging task. z_0 is the height above virtual bed for which the estimated velocity is zero (Eq.1) whereas, it is very common to consider the virtual bed level at $0.25d_{50}$ below the crest level of bed (Van Rijn, 1984). z_0 is directly governed by the size of sand grains that compose the bed and to our knowledge extensive works have been carried out by many authors pertaining to indirect assessment of this parameter as a function of geometric roughness scale, k_s and given as (Schlichting and Gersten, 1979)

Fig. 4. Vertical distributions of dimensionless time-averaged streamwise velocity (u^+) (a, b, c) $S = 0.1\%$ and (d, e, f) $S = 0.075\%$.

Table 1. Measured Log-law parameters.

Group	Tests	Test category	u_{*s}	u_{*r}	κ	Δz^+	z_0^+
$S = 0.1\%$	Test 1	Rigid bed	0.0343	0.0420	0.41	0.384	0.033
		Mobile bed	0.0343	0.0401	0.38	0.207	0.039
	Test 2	Rigid bed	0.0316	0.0379	0.41	0.169	0.029
		Mobile bed	0.0316	0.0360	0.38	0.207	0.036
	Test 3	Rigid bed	0.0295	0.0353	0.40	0.384	0.033
		Mobile bed	0.0295	0.0334	0.35	0.205	0.039
$S = 0.075\%$	Test 1	Rigid bed	0.0306	0.0350	0.41	0.384	0.0215
		Mobile bed	0.0306	0.0316	0.34	0.184	0.040
	Test 2	Rigid bed	0.0282	0.0304	0.41	0.385	0.0215
		Mobile bed	0.0282	0.0281	0.37	0.210	0.032
	Test 3	Rigid bed	0.0268	0.0274	0.41	0.384	0.0218
		Mobile bed	0.0268	0.0264	0.38	0.215	0.027

S = streamwise bed slope, u_{*s} = frictional velocity obtained from slope, u_{*r} = frictional velocity obtained from RSS, κ = von Kármán coefficient, Δz^+ = nondimensional depth of virtual bed level, z_0^+ = nondimensional depth of zero-velocity level

$$\left. \begin{aligned} z_0 &= 0.11 \frac{V}{u_*} \quad \text{for smooth flow } R_* \leq 5 \\ z_0 &= 0.11 \frac{V}{u_*} + 0.033k_s \quad \text{for transition flow } 5 < R_* < 70 \\ z_0 &= 0.033k_s \quad \text{for rough flow } R_* \geq 70 \end{aligned} \right\} \quad (3)$$

where, k_s is the Nikuradse equivalent sand roughness and R_* is the particle shear Reynolds number. As the primary focus of this study is to illustrate the effects of sediment mobility on different parameters of the formula that express the velocity distributions, estimation of z_0^+ and Δz^+ obtained from the measured velocity are preferred in comparison to aforementioned correlations. Therefore, we started with a trial value of $\Delta z^+ = 0$ for regression analysis of Eq. (2) and subsequent estimation of κ and z_0 . In the next step, Δz^+ is increased at regular interval by a small magnitude and the values of κ , Δz^+ are re-estimated. The values of Δz^+ , κ and Δz^+ correspond to the maximum regression coefficient (RC) are considered for log-law fittings and the procedure is well defined with a flow chart in Fig. 5. The average values of $\Delta z^+ = 0.39$, $\kappa = 0.413$ and $z_0^+ = 0.034$ over the rigid sand beds under clear water conditions are compatible to those for the traditional log-law over any rough beds. In mobile-bed runs, the average values of $\Delta z^+ = 0.21$, $\kappa = 0.37$ and $z_0^+ = 0.04$ are evaluated with some scatter around the average log-law curve as shown in Fig. 6a, b. A close observation of z_0^+ reveal upward shifting of the virtual bed level in

mobile bed experimental runs indicating enhanced zero-velocity levels owing to the fact of sediment entrainment into the flows. Secondly, majority of prior research has adopted κ as 0.41 and it is contradictory to consider the traditional value of κ for unrest conditions of bed particles as reported by many authors (Kironoto and Graf, 1994; Song et al., 1994). Nikora and Goring (2000) reported decreasing trend of κ in mobile bed runs. In the present study, the value of κ is assessed from the mixing-length, l based on the similarity hypothesis followed by smooth curve fitting to the measured data. The values of κ obtained for all the experimental runs are plotted in Figs. 6 (c, d). The slope of the linear portion demonstrate $\kappa = l/z$. In the inner-layer, all the data points over the rigid sand bed and mobile bed tests are found well clustered on a single band as shown in Fig. 6c, d. The experimental data plots are compared with the theoretical curves, $l/h = \kappa(z/h)(1 - z/h)^{0.5}$ (Dey et al., 2012) and noted well in agreement. A closer insight to the data plots reveal deviations in gradient for mobile sand bed with respect to the rigid sand bed under clear water conditions. In mobile-bed conditions the values of κ were evaluated smaller than those in clear-water flows.

Frictional velocity (u_*):

A proper assessment of friction or shear velocity (u_*) is very much essential in order to fit the measured velocity profile to log-law of the wall. Estimation of u_* from the bed slope is very common and in many cases, the results obtained does not enable to insight the relative role of sediment entrainment on

Fig. 5. Steps involved for estimation of displacement height.

Fig. 6. Estimation of log-law parameters for rigid sand bed and mobile sand beds: (a, b) zero velocity level, z_0 , (c, d) von Kármán constant, κ and (e, f) frictional velocity, u_* .

the flow characteristics in the inner-layer of flows. Therefore, $u_{*\tau}$ obtained from the RSS ($-u'w'$) distributions and extrapolating them to the bed as given below is preferred than those obtained from the bed slope (u_{*s}).

$$\tau_t = \left[1 - \left(\frac{z}{h} \right) \right] \tau_0 \quad (4)$$

where, τ_t = total shear stress and τ_0 = bed shear stress. In particular, the τ_t in mobile bed flows is balanced by the sum of bed

shear stress associated to fluid (τ_f) and that of bed particles (τ_b) to overcome the frictional resistance. Therefore, the total bed shear stress balance yields

$$\tau_t = \tau_f + \tau_b \quad (5)$$

Table 1 presents the shear velocity obtained from the u_{*s} and $u_{*\tau}$ in both rigid and mobile bed runs. Importantly, it is noted that the shear velocity obtained from u_{*s} are consistently deviated from $u_{*\tau}$ in both the experimental runs. The vertical distribu-

tions of nondimensional time-averaged RSS (τ_{uw}^+) are shown in Figs. 6 (e, f). In the outer flow region, the measured data points are well matched with the so-called gravity line for both the cases. An excessive damping in the near-bed flow region over mobile sand bed is noticed compared to rigid sand bed. In order to address the deviations above, we interpret that during bed load transport individual grains are moved by the flows and the resistance at the contacts of bed particles decelerates them and the resulting wakes induces lower turbulence level in comparison to rigid sand beds. The results are quite convincing and the analysis of RSS distributions in mobile-bed streams shows that: 1) the decreasing turbulence level in mobile bed runs with respect to clear-water conditions produce a near-bed damping in RSS distributions; 2) the sand grains in motion in mobile bed tests extract momentum from the flow to overcome the bed resistance inducing a reduced level of RSS distributions in the near bed flow region.

Performance tests of log-law parameters:

The results obtained in the previous sections indicate several further questions on compatibility of the estimated parameters for the log-law fitting of velocity profiles. In order to address the questions properly, we present here the performance tests of the computed parameters by grouping into different categories as shown in Table 2. The deviations of z_0^+ with sediment mobility factor (θ) are importantly relevant in our study to interpret the results for both mobile and rigid sand bed tests. To demonstrate the correlation among the parameters, θ is considered as dependent variables while the other characteristic parameters as independent variables. Table 2 summarizes the Pearson correlation coefficient and significant p -value for each of the characteristic parameter given in Table 2. The criteria used to select the best regression model are based on: (a) coefficient of determination P^2 (b) mean square error and (c) Adjusted P^2 (Fig. 7a, b and c). Only characteristic parameters

with weak correlation (between +0.3 and +0.7 or −0.3 and −0.7) and above were selected for regression analysis and given as

$$\theta = \left[\begin{array}{l} 0.362 \left(\frac{R}{d_{50}} \right)^{0.254} \left(\frac{z_0}{d_{50}} \right)^{0.815} \\ 0.00652 \left(\frac{R}{d_{50}} \right)^{0.644} \kappa^{0.7026} \\ 1.305 \left(\frac{R}{d_{50}} \right)^{0.658} \left(\frac{\Delta z}{d_{50}} \right)^{0.02639} \end{array} \right] \quad (6)$$

Here, it is pertinent to imply that the deviation of z_0^+ in the log-law profile and the virtual bed level with θ is a simple consequence of increasing bed thickness in mobile sand bed tests. It is apparent from the data plots of Fig. 8a that z_0^+ of the log-law profile is comparatively located deeper in rigid sand beds. The data plots of mobile and rigid sand bed experimental runs demonstrate the influence of the sediment mobility that associated to the upward shifting of the zero velocity level relative to the rigid sand bed. A closer insight of the data plots in Fig. 8a, b reveal z_0^+ and κ as 0.03 and 0.41, respectively in rigid sand bed under clear water condition and well in conformity with the customary values for the rough immobile bed given as $z_0^+ = 0.033$ and $\kappa = 0.41$ (Van Rijn 1984). In case of mobile bed tests, the average values are observed as $z_0^+ = 0.04$ and $\kappa = 0.37$ as shown in Fig. 8a, b. A short review of the results in mobile-bed flows indicate some degree of deviations and scatter with the average log-law profile. The slight increase of z_0^+ in mobile bed occurs owing to the fact of bed load transport whereas, the flow over immobile sand bed produce higher bed shear stress introducing larger form drag due to particles rigidity on the bed. The probable justification can be interpreted as gluing of grains to develop rigid sand surface otherwise they would be movable on the bed. In mobile bed tests z_0^+ are in the range of 0.04–0.045 which configures an

Table 2. Regression analysis.

Characteristic parameter	Sediment mobility, $\theta = \frac{u_*^2}{\Delta g d_{50}}$
Particle diameter, $D_* = d_{50} (\Delta g / \nu^2)^{\frac{1}{3}}$	0 (p -value = 1)
Particle shear Reynolds number, $R_* = \frac{u_* d_{50}}{\nu}$	0.999 (p -value = 0)
Zero velocity level, $\frac{z_0}{d_{50}}$	0.752 (p -value = 0.084)
Displacement height, $\frac{\Delta z}{d_{50}}$	−0.272 (p -value = 0.6)
von Kármán constant, κ	0.122 (p -value = 0.81)
Degree of submergence, $\frac{R}{d_{50}}$	0.485 (p -value = 0.32)
Bed slope, S	0.698 (p -value = 0.122)
Aspect ratio, $\frac{h}{B}$	0.484 (p -value = 0.33)

d_{50} = median size of bed materials, $\Delta = s - 1$; s = relative density of sediment particles, g = acceleration due to gravity, ν = kinematic viscosity of fluid, R = hydraulic mean radius, h = flow depth and B = width of the channel

Fig. 7. Comparison of experimental data plots with Regression model: (a) variations of θ with z_0 , (b) variations of θ with κ , (c) variations of θ with Δz .

Fig. 8. (a) variations of θ with z_0 (b) variations of θ with κ in mobile bed.

increase of z_0^+ compared to the immobile bed experiments. It can be concluded that this increase is related to larger bed thickness in mobile sand bed. This implies that z_0^+ is not a universal scale. Further to broaden our understanding, we interpret the obtained results in each case with three scenarios as presented in Fig. 9a, b and c. In the first case, z_0^+ is set from the regression analysis of the measured velocity profile, u^* values are considered from RSS distributions and κ is considered from mixing length theory (case1). In the second case, z_0^+ is set from the correlations with equivalent sand roughness (Eq. 3), u^* values are considered from RSS distributions, and κ is considered from mixing length theory (case 2) and finally, for the third case, we set z_0^+ from regression analysis of measured

velocity profile, u^* are considered from the RSS distributions and κ is considered as universal constant ≈ 0.41 (case 3). In all the cases, Δz is defined as the depth of virtual bed level from the bed surface for which the velocity profile is logarithmic. For each scenario, the streamwise velocity profiles are shown in Fig. 9a, b and c. Comparison of the data plots over mobile sand bed for all the three cases (Fig. 9) reveal that the case 1 for which z_0^+ is computed from the maximum regression coefficient, κ from the mixing length and u^* from the RSS distributions are well in conformity to the logarithmic law of walls. On the other hand, the frictional velocity obtained from the bed slope (u_{*s}) and κ as 0.41 are applicable to rigid sand bed under clear water conditions only.

Fig. 9. Fittings of longitudinal velocity profiles with log-law of the walls for rigid and mobile sand beds: (a) case 1, (b) case 2 and (c) case 3.

Fig. 10. (a, b) TKE dissipation rate, (c, d) TKE budget components and (e, f) TKE flux assessment for rigid sand bed and mobile-bed.

Relative role of sediment mobility on turbulent kinetic energy (TKE) balance:

The results presented in the earlier sections revealed near-bed damping of RSS aliquot resulting lower value of u_{*r} in the mobile-bed tests (Table 1). Further, we attempt to analyse the issue in the tune of TKE-budget concept. In particular, the sediment entrainment as bed load transport is associated to the momentum extracted from the flow and this section demonstrate the improved description of TKE-budget in the frameworks of higher order turbulent statistics. For uniform flow conditions, the TKE-budget is expressed as, $t_P = (t_D + p_D + \epsilon - \nu_D)$ (Nezu and Nakagawa, 1993) where, t_P = TKE-production rate, t_D = TKE-diffusion rate, p_D = pressure

energy diffusion rate, ϵ = TKE-dissipation rate, ν_D = viscous diffusion rate. The viscous diffusion rate ν_D is not considered because of large flow Reynolds numbers. Therefore, the pressure energy diffusion rate is obtained as $p_D = t_P - \epsilon - t_D$. In order to resolve ϵ satisfactorily, the inertial subrange of the velocity power spectrum, $S_{uu}(k_w)$ is fitted with the theoretical slope, i.e., the Kolmogorov, $-5/3$ curve and expressed as

$$S_{uu}(k_w) = \omega \epsilon^{2/3} k_w^{-5/3} \quad (7)$$

where, k_w is the radian wave number and ω is an empirical constant, which is set to 0.5 (Monin and Yaglom, 2007). Traditionally, the wave number spectrum $S_{uu}(k_w)$ is transformed to the frequency spectrum $S_{uu}(f)$ and ϵ is computed as

$$S_{uu}(f) = \left(\frac{\bar{u}_i}{2\pi} \right)^{\frac{2}{3}} \omega \varepsilon^{\frac{2}{3}} f^{-\frac{5}{3}} \quad (8)$$

Figure 10a, b presents the average value of ε computed from Eq. (8) at $z = 0.00125$ m for both immobile and mobile beds. Other nondimensional TKE-budget components are estimated over the mobile and rigid sand beds for $Q = 0.1 \text{ m}^3/\text{s}$ and shown in Fig. 10c, d. The nondimensional TKE-production rate (T_P) in both the cases depict near-bed enhancement. Data plots of nondimensional TKE-dissipation rate (E_D) shows similar trend of the T_P profile with distinct lag in the distributions. Both the T_P and E_D profiles are compared with the theoretical curves (Nezu and Nakagawa, 1993) expressed as:

$$T_P = \kappa^{-1} \left(\frac{1 - z/h}{z/h} \right) \quad (9)$$

$$E_D = \frac{9.8}{(z/h)^{0.5}} \exp(-3z/h) \quad (10)$$

Nevertheless, the T_P and E_D obtained from Eqs. (9) and (10) overestimate the results of present study. The influence of sediment mobility is apparent in the near bed distributions of T_P and E_D , where a markedly reduced lag is observed. In the near-bed flow zone, the T_P reduction in mobile sand bed is attributed to the damping in RSS distributions and the difference of T_P and E_D at each vertical distance are compensated by the combined effect of nondimensional TKE-diffusion rate (T_D) and pressure energy diffusion rate (P_D). The results has an implication to influence T_D in presence of bed load by augmenting it with respect to the immobile bed conditions, whereas, P_D is noticed almost equal to E_D . In mobile-bed runs, the near-bed distributions of P_D is estimated by the action of bed-load transport with a negative value ($P_D = -12$), which offers an evidence of increasing turbulence production which is well in conformity to the Detert et al. (2010). Further, the phenomenon can be well represented analyzing TKE flux components. The data plots in Fig. 10e, f present the vertical distributions of dimensionless longitudinal and vertical TKE-flux components. In case of immobile sand bed under clear water condition, the TKE-flux for longitudinal velocity component, F_{ku} ($= f_{ku}/u_*^3$) depicted negative value with a positive transport of vertical TKE-flux, F_{kw} ($= f_{kw}/u_*^3$) throughout the flow depth. This has an agreement on F_{ku} transports against the longitudinal direction of flow due to the higher frictional resistance induced by the inertia of flowing fluid layer in rigid sand bed. On the contrary, the influence of bed load transport under mobile bed conditions depicted longitudinal and downward transports of TKE-flux components indicating positive F_{ku} and the negative F_{kw} responsible to induce an accelerating effect in the near-bed flow zone.

CONCLUSIONS

The influence of sediment entrainment in flows as bed-load transport on the log-law parameters that express the velocity distributions has been studied experimentally and compared with those in rigid sand bed under clear-water conditions. The instantaneous velocities in flows were measured by an acoustic Doppler velocimeter. Experimental findings are discussed and summarised below:

1. Sediment entrainment in flows changes the parameters of the logarithmic law: the von Kármán coefficient (κ) decreases, whereas the virtual bed and the zero-velocity levels (z_0) in-

crease. The frictional velocity (u_*) decreases indicating a reduction of flow resistance.

2. The streamwise velocity profile with z_0 set from regression analysis and κ from mixing length theory offers best fittings to the log-law of the wall.

3. The universal value of κ allows a better fitting of the streamwise velocity profile over immobile bed only whereas; the log-law of the wall in presence of bed-load is characterized by a decrease in κ .

4. A reduction in RSS distributions in presence of bed-load is associated with the extraction of momentum from the main flow to maintain grain motion overcoming the bed resistance resulting an excessive near-bed damping and reduction in u_* .

5. The influence of sediment entrainment on the TKE-budget is noticeable inducing the pressure energy diffusion rate to negative magnitude in the near-bed flow zone. Moreover, in the near-bed flow zone, sediment entrainment as bed load is associated with a positive streamwise and a negative vertical TKE-flux, respectively.

The aforementioned results offer to solve number of issues on mobile bed hydraulics as a future research aim. The bed shear stress computed from the bed slope in presence of bed-load is disadvantageous. Hence, our results permit to elaborate carefully the reduction of RSS in presence of bed-load. Moreover, the log-law of wall in mobile-bed streams and its application with traditional κ is strongly doubtful owing to the reduced value in mobile bed. The TKE budget analysis reveals near-bed negative pressure energy diffusion rate indicating of low-pressure field which offer significant impetus on the future scope of research towards sediment dynamics in mobile bed flows. Nevertheless, the present study offers how best to consider the turbulent production and energy dissipation events into a theoretical model describing the sediment transport phenomenon as future scope of research.

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