APPLICATION OF 3D ROUGHNESS PARAMETERS FOR WEAR INTENSITY CALCULATIONS

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In this paper, calculations of 3D parameter V_m (material volume) of surfaces with irregular roughness and comparison with experimental data were performed, with further application of this parameter in calculations of wear intensity. First, using Mountains Map software for profilometric measurements, 3D roughness processing and determination of material volume V_m at specific relative levels γ were performed. The next step was an additional analysis of the distribution of surface ordinates using a theoretical and experimental Laplace function. The given check confirmed that for mostly surfaces with irregular roughness the ordinate distribution corresponds to the normal Gaussian distribution law, but in cases when the asymmetry of the ordinate distribution function goes outside the permissible limits ($|\Delta Ssk| > 10\%$), errors $> 10\%$ occur. On this basis, the mathematical formula of the material volume V_m was derived, and the obtained calculations were compared with the measured values. The results showed that the calculated values of the parameter V_m were very close to the experimental data ($|\Delta V_m| < 10\%$), while at the relative level $\gamma = +3$, errors occurred that was related to the deviation from the normal distribution law. It was concluded that the given parameter could be used in the calculations of linear wear intensity, knowing the relative level γ .

Keywords: 3D roughness parameters, friction, material volume, wear.

1. INTRODUCTION

Nowadays, the issue of surface wear does not lose its relevance because all the time designers and technologists are trying to find effective solutions to increase the wear resistance of surfaces. As it is known, the wear intensity is influenced by several factors / parameters: the material and its properties, load, lubrication conditions, and roughness. The surface quality is very important in this regard because the type of surface treatment and the roughness parameters largely determine the service life of the friction pair.

Surfaces for the friction pairs of the machine apparatus mainly are treated with abrasive materials, which ensure the required surface smoothness and accuracy. Surfaces, after such treatment, are characterised by irregular arrangement of micro-irregularities, and roughness parameters for such surfaces are calculated using normal random field theory [1].

At this time, Standard ISO 25178-2, which defines 3D surface roughness param-

eters, does not give a complete picture of the physical meaning of these parameters. No relationships are shown between the surface roughness parameters, which would help determine the factors that affect a particular parameter and the way they can be changed / improved.

In the calculations of linear wear intensity, one of the important roughness parameters is the volume of the deformed material. To determine it, the following tasks have been performed in the given study:

1. Compliance check of the ordinate distribution function of surfaces with irregular roughness to the normal distribution law using an additional method – Laplace function analysis.
2. Determination of roughness parameter V_m at different relative levels γ in the Mountains Map software.
3. Output of parameter V_m , comparing the calculated and experimental values.
4. Calculations of linear wear intensity using the roughness parameter V_m .

2. SURFACE ORDINATE DISTRIBUTION

In the given study, six different surfaces were studied: flat ground, polished, lapped, shot peened, sandblasted and after electro-erosion.

As it was mentioned above, surface roughness after treatment with abrasive materials was irregular. For this type of surface, it is important to check compliance with the normal distribution law in order to make further calculations. In scientific works [2], the compliance of surface ordinate distribution function to the normal distribution law was substantiated using such criteria as:

1. asymmetry Ssk of the surface ordinate distribution function;
2. the excess Sku of the surface ordinate distribution function;
3. Pearson criterion;
4. correlation function.

An additional criterion that can be used to solve this issue is the Laplace function, which represents the area of a material at a given level c (in percent).

In the general case, the Laplace function is expressed by Eq. (1):

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}} dz. \quad (1)$$

The parameter given in the standard ISO 25178-2 is defined as a ratio of the area

of the material at a height c to the evaluation area (Fig. 1).

Fig. 1. Areal material ratio:

X – areal material ratio $Smr(c)$, in percent; Y – height; a – level c ; b – reference plane [3].

The graph of the Laplace function can be constructed for any of the studied sur-

faces at the levels c , using Eq. (2):

$$E\{Smr(c)\} = \frac{1}{A} \iint_{\Omega} \xi\{r(z, c)\} = 1 - \Phi\left(\frac{c}{Sq}\right), \quad (2)$$

where A – nominal surface area, mm^2 ; c – level, μm ; Sq – root mean square height, μm .

$$c = \gamma \cdot Sq, \quad (3)$$

where γ – relative level. $\gamma_{\text{standard}} = 3, 2, 1, 0, -1, -2, -3$.

Using the Mountains Map software [4], the area of the material at the relative levels $\gamma = +3, +2, +1, 0, -1, -2, -3$ was determined for the studied surfaces. In Table 1, the calculated and measured values of the

parameter $Smr(c)$ for a flat ground surface were summarised and the absolute error was calculated. The calculated values of this parameter are fully compliant with the Laplace function indicators.

Table 1. Comparison of Theoretical and Experimental Values of Roughness Parameter $Smr(c)$ for Flat Ground Surface

γ	$Smr(c)_{\text{calc}}, \%$	$Smr(c)_{\text{measured}}, \%$	$ \Delta Smr(c) , \%$
+3	0.13	0.12	8.30
+2	2.28	2.20	3.30
+1	15.87	17.30	9.04
0	50.00	47.90	4.20
-1	84.13	85.40	1.50
-2	97.72	97.80	0.08
-3	99.87	100.00	0.14

In Fig. 2, the graphical representation of the experimental values of the parameter $Smr(c)$ and the Laplace function are shown. It can be concluded that for the given sur-

face the distribution of ordinates very exactly corresponds to the normal distribution law and falls within the deviation range of $\pm 10\%$.

Fig. 2. Graphical representation of the calculated and experimental values of the parameter $Smr(c)$.

In turn, for the other studied surfaces there are deviations from the standard values of the Laplace function at the upper relative levels $\gamma = +2$ and $+3$ (Fig. 3). The value of the absolute error $\Delta Smr(c)$ is greater than 10 % in several cases. This result is influ-

enced by the asymmetry Ssk of the surface ordinate distribution function. The experimental values of $Smr(c)$ at $Ssk > 0.05$ and $Ssk < -0.05$ differ from the theoretical ones by up to 60 %.

Fig. 3. Influence of asymmetry Ssk of ordinate distribution function on absolute error $|\Delta Smr|$.

3. CALCULATION OF MATERIAL VOLUME V_m

The volume of the micro-irregularities of the rough surface above the level c is a very important feature in the calculations of the contact deformation. The volume of surface material at a given level expresses the amount

of deformed material in the friction process.

The average volume of material can be obtained analogously to the surface profile area by integrating the surface cross-sectional area $A(c)$ at levels c [5].

$$Vm = \int_c^\infty A(c)dc. \quad (4)$$

Due to the fact that

$$A(c) = Smr(c) \cdot Aa, \quad (5)$$

the mathematically expected value of the material volume, taking into account Eq. (2), will be equal to:

$$E\{Vm\} = Aa \int_c^\infty E\{Smr(c)\}dc, \quad (6)$$

$$E\{Vm\} = Aa \int_c^\infty \left[1 - \Phi\left(\frac{c}{Sq}\right)\right]dc, \quad (7)$$

$$E\{Vm\} = Sq \left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{c^2}{2Sq^2}\right) - \frac{c}{Sq} \left[1 - \Phi\left(\frac{c}{Sq}\right)\right] \right\}. \quad (8)$$

Expression (8) indicates that the material volume Vm varies depending on the relative level γ . As γ changes from $-\infty$ to $+\infty$, the material volume increases. At γ values

$-\infty < \gamma < -1$, it can be considered that the volume of deformed material practically does not change according to the linear law.

Fig. 4. Topography of a flat ground surface.

In Fig. 4, a flat ground surface is visible, the ordinate distribution of which corresponds to the normal Gaussian distribution law, which was justified in the previous paragraph. For a given surface Vm calculations were performed at different levels c subtracted from the mean plane. Theoretical values of material volume were calculated according to Eq. (8), while the experimental data were obtained by Mountains Map soft-

ware, using a function “slices” to generate a surface divided into two parts – material and voids. In Fig. 5, the material volume is marked in grey colour, and void volume – in black. By selecting the required level, it is possible to obtain the required Vm values. In addition, this function represents the area of the material at a given level, which corresponds to the values of the parameter $Smr(c)$.

Fig. 5. Representation of surface material and void volume at level c .

Table 2 shows the calculated and experimental values of parameter Vm and their comparison in the form of absolute error for the flat ground surface. At all relative levels, except the highest ($\gamma=+3$), the calculated values coincide very precisely with the

experimental data, and the absolute error falls within the range of $\pm 10\%$. The discrepancy at the relative level $\gamma=+3$ arises because the Ssk value of the ordinate distribution function has a large deviation ($> 10\%$) from the standard values of normal distribution law.

Table 2. Comparison of Theoretical and Experimental Values of Roughness Parameter Vm for Flat Ground Surface

γ	$Vm_{calc}, mm^3/mm^2$	$Vm_{measured}, mm^3/mm^2$	$ \Delta Vm , \%$
+3	1.50E-08	1.08E-08	28.03
+2	3.33E-07	3.24E-07	2.74
+1	3.27E-06	3.46E-06	5.90
0	1.56E-05	1.57E-05	0.38
-1	4.25E-05	4.23E-05	0.39
-2	7.87E-05	7.87E-05	0.04
-3	1.18E-04	1.18E-04	0.33

Graphically, the experimental and theoretical values of the parameter Vm are shown in Fig. 6. In the graph, we can see

only those levels, at which deformations during friction can actually occur depending on the applied load [6].

Fig. 6. Graphical representation of the calculated and experimental values of the parameter Vm .

Calculations of material volume V_m and comparison with experimental data were performed for all six studied surfaces to understand the extent to which the asymmetry of the ordinate distribution function affected the results. From the data in Table

3, it can be concluded that very large errors between theoretical and experimental values occur at the relative levels $\gamma = +2$ and $+3$; it is also related to the deviations of the asymmetry Ssk .

Table 3. Influence of Asymmetry Ssk of Ordinate Distribution Function on Absolute Error $|\Delta V_m|$

No. Nr.	Type of surface treatment	Ssk	$ \Delta V_m , \%$		
			$\gamma=+3$	$\gamma=+2$	$\gamma=+1$
1	Flat grinding	0.051	28.03	2.74	5.9
2	Lapping	0.039	14.50	0.28	1.16
3	Sand blasting	-0.196	2.53	23.55	8.95
4	Electro erosion	-0.176	53.79	40.79	9.26
5	Shot peening	-0.026	17.42	1.31	1.31
6	Polishing	-0.247	98.41	86.76	3.83

4. CALCULATION OF WEAR INTENSITY J_H

In the given study, the analysis of the surface contacting process was performed using the equivalent surface. The application of such a method for solving engineering tasks has been justified in several studies [5]–[7].

In general, there is a direct deformation of the material volume V_m in the contact zones, but it will not localize in a specific volume, but will spread deeper in the body of the micro-irregularity. In turn, it is substantiated that the deformation spread factor at the deeper layers of the material can be disregarded.

According to [8], the linear wear intensity J_h is determined by Eq. (9):

$$E\{J_h\} = \frac{V_m}{n \cdot d \cdot Aa}, \quad (9)$$

where V_m – material volume, mm^3/mm^2 ;

n – a number of cycles to failure;

d – contact length, mm ;

Aa – nominal contact area, mm^2 .

In Eq. (10), the roughness parameter V_m , which expresses the amount of deformed material, is figured out. Knowing the relative level γ , this parameter can be

easily determined for any surface. In turn, at the beginning of solving an engineering task, it is important to determine how the micro-irregularities of the studied surface will be deformed.

The contact area to a greater extent is characterised by the ability of surface micro-irregularities to be elastically deformed, what can be described by the contact type criterion CC [6]. The calculation formula of CC includes roughness parameters – Sa , RSm and material mechanical properties – micro-hardness and elastic constant:

$$CC = \frac{RSm \cdot \theta \cdot H_\mu}{Sa}, \quad (10)$$

where RSm – an average step of micro-irregularities, mm ;

θ – elastic constant of material, m^2/MN ;

H_μ – surface microhardness, N/mm^2 ;

Sa – arithmetic mean height, μm ;

$$\theta = \frac{1 - \mu^2}{E_2}, \quad (11)$$

where μ – Poisson's ratio;

E – elastic modulus, MN/m^2 .

If the value of the contact type criterion is >1.7 , then the condition of elastic deformation is fulfilled, but if $CC < 0.7$, the contact is plastic.

Accordingly, depending on the type of contact, the level of deformation can be expressed by the following equations:

$$F_1\{\gamma\} = \left(\frac{q \cdot \theta_{sum}}{k_{elast}}\right) \cdot \left(\frac{RSm}{Sa}\right), \quad (12)$$

$$F_2\{\gamma\} = \frac{q}{H \cdot k_{plast}}, \quad (13)$$

where $F_{1,2}(\gamma)$ – tabulated functions [8];
 q – load, MN/m²;
 $k_{elast, plast}$ – a coefficient that depends on the anisotropy of the surface (the parameter Str);
 θ_{sum} – total elastic constant of material, m²/MN.

The total elastic constant is calculated according to the following equation:

$$\theta_{sum} = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}. \quad (14)$$

The number of cycles to failure also depends on the type of deformation – elastic or plastic, and n_{el} and n_{plast} can be expressed by the equations:

$$E\{n_{el}\} = \left(\frac{3\pi\theta\sigma_0}{4kf} \cdot \sqrt{\frac{r}{Sz \cdot \varepsilon}}\right)^{t_{el}}, \quad (15)$$

$$E\{n_{plast}\} = \left(\frac{e_0}{\sqrt{2}} \sqrt{\frac{\sigma_T - 2fH_\mu}{\sigma_T + 2fH_\mu} \cdot \frac{r}{Sz \cdot \varepsilon}}\right)^{t_{plast}}, \quad (16)$$

where e_0 , σ_0 – extrapolated values of the fatigue curve, MN/m²;
 k – constant depending on friction-fatigue characteristics, $k=3(1+\mu)$;
 f – coefficient of friction;
 $t_{el, plast}$ – fatigue curve parameters;
 σ_T – material yield strength, N/mm²;
 r – mean summit curvature [1], mm⁻¹;
 Sz – max height of surface, μ m;
 ε – relative convergence, mm⁻².

$$r = \frac{1}{2}\pi^2 \cdot Sq \cdot \left(\frac{4}{RSm^2}\right) (1 + Str^2)\gamma, \quad (17)$$

$$\varepsilon = 1,54 \cdot \left(\frac{r}{Sz}\right)^{\frac{1}{5}} (q \cdot \theta)^{\frac{2}{5}}. \quad (18)$$

Determination of the wear intensity according to Eq. (9) requires the study of the contact size, which is expressed as the length of the actual contact area in the direction of friction. According to [8], the mathematically expected value of the contact length $E\{d\}$ is determined by the equation:

$$E\{d\} = RSm[1 - \Phi(\gamma)]e^{\frac{\gamma^2}{2}}. \quad (19)$$

Linear wear is expressed by the following formula:

$$h = J_h \cdot L_{tr}, \quad (20)$$

where L_{tr} – a friction path, μ m;

$$L_{tr} = V \cdot t, \quad (21)$$

where V – traverse speed, m/min;
 t – working time, st.

Thus, knowing the output calculation data: roughness parameters, physical-mechanical, friction-fatigue and constructive characteristics of friction pair materials, it is possible to calculate the linear wear intensity according to Eq. (9). In this study, wear calculations for a lathe friction pair guide-table (Fig. 7) were made.

Fig. 7. Friction pair guide – table.

Table 4 summarises the data on the material characteristics of the friction pair,

as well as tabulated values of different coefficients. In this example, the wear intensity

was calculated for the sliding guide – the part that absorbs the load during contact.

Table 4. Output Data of Friction-Pair Guide-Table for Wear Intensity Determination

Parameters		Guide	Table	Units
Cast iron 300 ISO 185		Steel C45 EN 10083-2		
Roughness parameters	Sa	0.24	0.24	μm
	Sq	0.3	0.3	μm
	RSm_1	0.12	0.12	mm
	RSm_2	0.6	0.6	mm
	Str	0.22	0.24	-
	V_m	$1.19 \cdot 10^{-6}$	-	mm^3/mm^2
	Sz	2.25	-	μm
Physical-mechanical characteristics	E	$1.3 \cdot 10^5$	$2.1 \cdot 10^5$	MN/m^2
	μ	0.25	0.3	-
	H_μ	1800	1960	N/mm^2
Friction-fatigue characteristics	σ_0	1600	-	MN/m^2
	t_{er}	6.45	-	-
	f	0.1		-
	k	3.75		-
Constructive characteristics	q	0.5		MN/m^2
	V	0.5		m/min
	t	5000		st.

1. First, the elastic constant of the material was determined according to Eq. (11):

$$\theta = \frac{1-0.25^2}{1.3 \cdot 10^5} = 7.21 \cdot 10^{-6} (\text{m}^2/\text{MN}).$$

2. The contact type criterion CC according to Eq. (10):

$$CC = \frac{0.12 \cdot 7.21 \cdot 10^{-6} \cdot 1800}{0.24 \cdot 10^{-3}} = 6.49.$$

It can be assumed that in this case the elastic contact is provided ($CC > 1.7$).

3. The relative level γ according to Eq. (12) in the case of elastic contact is determined as follows:

$$F_1\{\gamma\} = \left(\frac{0.5 \cdot 1.15 \cdot 10^{-5}}{0.4562} \right) \cdot \left(\frac{0.12 \cdot 10^3}{0.24} \right) = 0.06 \cdot 10^{-2},$$

where $k_{elast} = 0.4562$ – a tabulated value obtained at $Str = 0.12 / 0.6 = 0.2$ [8].

In turn, the total elastic constant of the contact is equal to:

$$\theta_{sum} = \frac{1-0.3^2}{2.1 \cdot 10^5} + \frac{1-0.25^2}{1.3 \cdot 10^5} = 1.15 \cdot 10^{-5} (\text{m}^2/\text{MN}).$$

4. In this case, the calculated relative level $\gamma = 2.3$ [8].

5. The mean summit curvature is equal to:

$$r = \frac{1}{2} \pi^2 \cdot 0.3 \cdot \left(\frac{4}{0.12^2} \right) (1 + 0.2^2) \cdot 2.3 = 0.99 (\text{mm}^{-1}).$$

6. The relative convergence was determined by Eq. (18):

$$\varepsilon = 1.54 \cdot \left(\frac{0.99}{2.25 \cdot 10^{-3}} \right)^{\frac{1}{5}} (0.5 \cdot 7.21 \cdot 10^{-6})^{\frac{2}{5}} = 2.18 \cdot 10^{-3} (\text{mm}^{-2}).$$

7. The number of cycles to failure was determined by Eq. (15):

$$E\{n_{el}\} = \left(\frac{3\pi \cdot 7.21 \cdot 10^{-6} \cdot 1600}{4 \cdot 3.75 \cdot 0.1} \cdot \sqrt{\frac{0.99}{2.25 \cdot 10^{-3} \cdot 2.18 \cdot 10^{-3}}} \right)^{6.45} = 5693844682.$$

8. The value of the contact length is equal to:

$$d = 0.12 \cdot [1 - \Phi(2.3)] e^{\frac{2.3^2}{2}} = 9.13 \cdot 10^{-5} (\text{mm}).$$

9. The volume of the deformed material according to theory will be equal to:

$$Vm = 0.001 \cdot 0.3 \left\{ \frac{1}{\sqrt{2\pi}} e^{\frac{2.3^2}{2}} - 2.3[1 - \Phi(2.3)] \right\} = 1.1 \cdot 10^{-6} (\text{mm}^3/\text{mm}^2).$$

10. The linear wear intensity was determined by Eq. (9):

$$J_h = \frac{1.1 \cdot 10^{-6}}{5693844682 \cdot 9.13 \cdot 10^{-5}} = 2.12 \cdot 10^{-12}.$$

11. The friction path was determined by the traverse speed and working time:

$$L_{tr} = 0.5 \cdot 5000 \cdot 60 = 15 \cdot 10^{10} (\mu\text{m}).$$

12. The linear wear was determined as follows:

$$h = 2.12 \cdot 10^{-12} \cdot 15 \cdot 10^{10} = 0.32 (\mu\text{m}).$$

In addition, calculations were performed using the measured values of roughness parameters Vm and Str (Table 4). The

linear wear intensity $J_h = 2.3 \cdot 10^{-12}$ and linear wear $h = 0.35 \mu\text{m}$. The absolute error in both cases is less than 10 %.

5. CONCLUSIONS

The study investigated the dependence of linear wear intensity J_h of the surface on the volume of the material V_m , the surface texture index Str and the arithmetic mean height Sa . In addition, the obtained equation of the material volume V_m contains such parameters as Sq – the root mean square height and the level c . To check the theoretical relationships of the parameter V_m , roughness measurements were performed using a 3D measuring equipment Taylor Hobson Talysurf Intra 50. The Mountains Map software was used to process the obtained data. A comparison of the

theoretical values and experimental data shows that the absolute error of the surface material volume $|\Delta V_m|$ at the relative level $\gamma=+3$ in some cases reaches 100 %, at the level $\gamma=+2$ - $|\Delta V_m| < 45$ %, at lower levels - $|\Delta V_m| < 10$ %, what can be explained by the asymmetry of the surface ordinate distribution function.

Calculations of linear wear intensity J_h show that the offered roughness parameters V_m , Str and Sa simplify the calculation of wear because using measured values, it is not necessary to calculate each parameter separately.

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