THE SHARE DISTRIBUTION AND PRECISION EVALUATION IN UNDERGROUND TOPOGRAPHIC ROUTES AND NETWORKS

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DOI: 10.2478/minrv-2021-0020

Abstract: *The underground topographic works of elevating and drawing are necessary for the execution of underground works (mining, hydrotechnics, communications, etc.) Fulfilling these works efficiently and safely requires an advance study for the precisions in topographic support routes and networks. This paper aims to establish the share distribution correlated with the underground works distribution.*

Keywords: *evaluation, underground route, underground network, topography, precision*

1. Introduction

The topographic works of elevating and drawing for the underground work execution use a lot of stable points which form planimetric and leveling routes and networks.

The underground objectives which have to be designed, executed and tracked for their stability in time, are very important from the technical, economic and social points of view. As a result, the topographic base used has to correspond to the quality and safety requirements.

The underground works are dynamic, as they are executed in stages. Consequently, the topographic base is also executed in stages, but with results corresponding to the entire results. It is necessary, based on the geometric configuration of the design underground objectives, to be established the distribution of shares for topographic routes and networks. The topographic leveling works will be analyzed.

2. Paper content

2.1. Geometric leveling route [1]

We consider the points A and B between which there were executed precise measurements of geometric leveling (fig. 1). If the values H_A H_B of the points A and B are known, the measured leveling differences h_1, h_2, h_3 are used to determine the values H_1 and H_2 for the points P_1^0 and P_2^0 situated on the leveling line AB .

Fig. 1.

For the determination of the values for the points P_1 and P_2 we will note:

$$\begin{aligned} H_1^0 &= H_A + h_1 \\ H_2^0 &= H_B + h_3 \end{aligned} \quad (1)$$

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and

$$\begin{aligned} H_1 &= H_1^0 + x_1 \\ H_2 &= H_2^0 + x_2 \end{aligned} \quad (2)$$

where:

H_1^0, H_2^0 – provisional values
 x_1, x_2 – probable corrections [2]

For the measured values, we will note the equations:

$$\begin{aligned} H_1^0 + x_1 - (H_A + h_1) &= v_1 \\ H_2^0 + x_2 - (H_1^0 + x_1 + h_2) &= v_2 \\ H_2^0 + x_2 - (H_B + h_3) &= v_3 \end{aligned}$$

or:

$$\begin{aligned} x_1 + \ell_1^0 &= v_1 & \ell_1^0 &= H_1^0 - H_A - h_1 \\ -x_1 + x_2 + \ell_2^0 &= v_2 & \ell_2^0 &= H_2^0 - H_1^0 - h_2 \\ x_2 + \ell_3^0 &= v_3 & \ell_3^0 &= H_2^0 - H_B - h_3 \end{aligned} \quad (3)$$

For the system (3) we will attach the condition:

$$[pvv] = \text{minimum} \quad (4)$$

where „ p ” are the shares of the measured values, given by the equations:

$$p_i = \frac{1}{s_i} ; \quad i=1,2,3 \quad (5)$$

where:

s_i – the length of the leveling lines

We will have the system:

$$\begin{aligned} (p_1 + p_2)x_1 - p_2x_2 + \ell_1 &= 0 \\ -p_2x_1 + (p_2 + p_3)x_2 + \ell_2 &= 0 \end{aligned} \quad (6)$$

where:

$$\begin{aligned} \ell_1 &= p_1\ell_1^0 - p_2\ell_2^0 \\ \ell_2 &= p_2\ell_2^0 + p_3\ell_3^0 \end{aligned} \quad (7)$$

From the system (6) we will remove the unknown value x_1 and will obtain:

$$\left(p_3 + \frac{1}{\frac{1}{p_1} + \frac{1}{p_2}} \right) x_2 + L_2 = 0 \quad (8)$$

or:

$$\begin{aligned} \left(p_3 + \frac{1}{\frac{1}{p_1} + \frac{1}{p_2}} \right) x_2 + L_2 &= 0 \\ \frac{1}{P_2} &= \frac{1}{p_1} + \frac{1}{p_2} ; \quad P_2 \text{ is the share of the point } P_2^0 \end{aligned} \quad (9)$$

In conclusion, the leveling lines with serial arrangement and shares p_1 and p_2 determine a share P_2 among which we have the relation (9). Therefore, the equation (8) is:

$$(P_2 + p_3)x_2 + L_2 = 0 \quad L_2 = \ell_2 + \frac{p_2\ell_1}{p_1 + p_2} \quad (10)$$

We notice that the equation which gives the correction x_2 is placed right after the design of the leveling route (fig.1).

After calculating the correction x_2 , we calculate the correction x_1 from the system (6).

For the precision evaluation we determine:

- m_0 mean square deviation of the level difference on the length unit
- m_1, m_2 mean square deviations we use to obtain the probable amounts of the values H_1 and H_1 .

Using the matrices calculation, we note:

$$m_0 = \sqrt{\frac{v'pv}{n-k}} \quad (11)$$

n – number of equations

k – number of unknown values

But:

$$v'pv = \ell'(p - pAQ_{xx}A'p)\ell$$

where:

$$Q_{xx} = (A'pA)^{-1}$$

and:

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{pmatrix} \quad \ell = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \quad p = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}$$

We easily notice that:

$$Q_{xx} = \frac{1}{D} \begin{pmatrix} p_2 + p_3 & p_2 \\ p_2 & p_1 + p_2 \end{pmatrix}$$

where:

$$D = p_1(p_2 + p_3) + p_2p_3 = p_3(p_1 + p_2) + p_1p_2$$

Using:

$$Q_{xx} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

where

$$Q_{11} = \frac{1}{p_1 + p_2} \quad \frac{1}{p_2} = \frac{1}{p_1} + \frac{1}{p_3}$$

$$Q_{22} = \frac{1}{p_3 + p_1} \quad \frac{1}{p_1} = \frac{1}{p_3} + \frac{1}{p_2}$$

Finally, we can note:

$$m_1 = m_0 \sqrt{Q_{11}} \quad ; \quad m_2 = m_0 \sqrt{Q_{22}} \quad (12)$$

The share coefficients Q_{11} , Q_{22} are easily obtained from the design of the leveling network.

2.2. Simple leveling network [3]

We consider a leveling network formed by closed and open routes (fig. 2).

Fig. 2.

$$\begin{aligned} (p_1 + p_2 + p_3)x_1 - (p_2 + p_3)x_2 + \ell_1^0 &= 0 \\ -(p_2 + p_3)x_1 + (p_2 + p_3 + p_4)x_2 + \ell_2^0 &= 0 \end{aligned}$$
$$(P_2 + p_4)x_2 + \ell_2 = 0$$
$$\ell_2 = \ell_2^0 + \frac{(p_2 + p_3)}{p_1 + p_2 + p_3} \ell_1^0 \quad (13)$$
$$\frac{1}{P_2} = \frac{1}{p_1} + \frac{1}{p_2 + p_3} \quad (14)$$

2.3. Complex leveling network [3]

$$\begin{aligned}(P_2 + p_4)x_2 - p_4x_3 - 0x_4 + \ell_2 &= 0 \\ (P_3 + p_5 + p_6)x_3 - 0x_4 + \ell_3 &= 0 \\ (P_4 + p_7)x_4 + \ell_4 &= 0\end{aligned}\tag{15}$$
$$\begin{aligned}\frac{1}{P_2} &= \frac{1}{p_1} + \frac{1}{p_2 + p_3}; \\ \frac{1}{P_3} &= \frac{1}{P_2} + \frac{1}{p_4}; \\ \frac{1}{P_4} &= \frac{1}{P_3} + \frac{1}{p_5 + p_6}\end{aligned}\tag{16}$$

3. Case Study

The diagram shows a network flow problem with nodes A, C, D, E, and F. The edges and their associated costs are: S_1 (A to D) with cost h_1 , S_2 (D to C) with cost h_2 , S_3 (D to F) with cost h_3 , and S_4 (F to E) with cost h_4 .

The following are known:

- absolute dimensions H_A, H_B, H_C of points A, B, C, given quantities, considered without errors;
- level differences $h_1, h_2, \dots, h_4$
- quantities measured, with errors $m_1, m_2, \dots, m_4$
- lengths $S_1, S_2, \dots, S_6$

Determine the weight in point E.

For the calculation of the elevation of point E starting from points A and C (the most disadvantageous case as precision) the level differences are used: $h_1, h_2, \dots, h_3$. E.g:

$$H_E = H_D + h_3$$

where:

$$H_D = H_A + h_1 = H_C + h_2$$

The weighting in point E is determined taking into account that:

- the paths AD and CD with lengths S_1 and S_2 are parallel and consequently the weight in point D, P_1 is:

$$P_1 = p_1 + p_2$$

- next, the route DE with length S_3 is added in series, and the weight in point E is P_2 which is obtained by writing:

$$\frac{1}{P_2} = \frac{1}{P_1} + \frac{1}{p_3}$$

The weights are obtained with the general relation:

$$p = \frac{1}{S}$$

so we admit:

$$S_1 = 0,5 \text{ km}$$

$$S_2 = 0,5 \text{ km}$$

$$S_3 = 1,0 \text{ km}$$

With these you get:

$$P_1 = \frac{1}{S_1} + \frac{1}{S_2} = 2 + 2 = 4$$

and

$$\frac{1}{P_2} = \frac{1}{4} + \frac{1}{1} = 1,25$$

so

$$P_2 = \frac{1}{1,25}$$

and weighting coefficient:

$$Q_E = \frac{1}{P_2} = 1,25$$

As a result the precalculated error in point E is:

$$m_E = m_0 \sqrt{Q_E} = m_0 \sqrt{1,25}$$

If the error m_0 is $\pm 10 \text{ cm / km}$, the result will be :

$$m = 10 \sqrt{1,25} \cong 11,2 \text{ cm}$$

which can be admitted.

If not allowed, m_0 is reduced by appropriately chosen instruments and measurement methods.

4. Conclusions

The underground topographic based is formed by points for which we determine their spatial positions and which on the whole form leveling routes and networks and similarly polygonal networks different from the leveling ones.

The measured values for the flat coordinates calculation may be processed by generalizing the applied methods in the leveling routes and networks, noting that the shares are given by the opposite of the measured angles number or route length, for the orientation and coordinates.

It is important to establish the distribution of the measured values' shares, corresponding to the resolving stages. Consequently, there exists the possibility for the advance evaluation of the determined values' precisions in the main points of the topographic networks.

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