On the Fractal Characteristics of Some Meteorological Extreme Series Recorded at Constanta, Romania

Alina Bărbulescu and Dumitriu Cristian Ștefan

Abstract – The study of hydro-meteorological series is important for planning agricultural activities, managing water resources, and taking measures to prevent the effects of natural hazards. Therefore, here we analyze the fractal characteristics of the temperature series recorded at Constanta, Romania. The studied series are the annual mean maximum annual and minimum annual temperatures recorded during 1961-2018.

Keywords – dimension, extreme, fractal temperature, trend

1. INTRODUCTION

The occurrence of extreme phenomena became more frequent in the last period than before due to climate change and the impact of anthropic activities on the environment. Romania is not an exception, with long periods of drought followed by high precipitations felt in very short time.

The IPCC Report [1] pointed out the global temperature increase with about 1.5°C by rapport to the level from the pre-industrial period. The evolution of the daily minimum temperature had the same trend, accompanied by the decrement in the number of frost days during 1985-2014 [3,4]. The augmentation of the minimum temperature was two times higher than that of the maximum temperature over the globe during 1950- 2005 [5,6].

Minimum temperature increased about twice as fast as maximum temperature over global land areas since 1950, resulting in a broad decline in the diurnal temperature range.

Constanta is situated in southeastern part of Romania, in Dobrogea, at 44.18°N and 28.63°E. It is the main city in the region and the biggest harbor at the Romanian Black Sea Shore (Figure 1). Studies on the climate change in Dobrogea showed an increase by 0.8°C of the annual mean temperature during the decade 1997- 2007 [7]. The same behavior is noticed for all the seasonal maximum and minimum annual series, but the autumn minimum series during 1961-2008 [4]. The trend slope for the autumn minim series is significant, while no trend has been detected for the corresponding minimum series.

Minimum temperature increased about twice as fast as maximum temperature over global land areas since 1950, resulting in a broad decline in the diurnal temperature range.

Different studies presented the research results of the hydro-meteorological series in this part of Romania [7-11], using statistical and artificial intelligence methods; still, the fractality of the extreme series was not addressed. This article fills a gap in this direction

and presents the fractal characteristics of the annual mean maximum and minimum temperature series recorded in the interval 1961-2018.

Fig. 1. The Black Sea region (<https://www.worldatlas.com/seas/black-sea.html>)

2. METHODOLOGY

Daily data from Constanta meteorological station have been downloaded from the European Climate Assessment & Dataset project website [12] and processed to obtain the study series, presented in Figure 2.

Fig. 2. The studied series

First, the KPSS test was applied, at a significance level of 5%, to test the hypothesis of the series stationarity in trend and level. Then, the assumption of the change points' existence was tested by the Lee and Heghinian test.

Mandelbrot [13] was the first to introduce the notion of fractal by comparing two types of dimensions. Leibovitch [14] defines the fractals as those objects with shapes similarities of the smaller and largest features.

The scale invariance properties can be addressed by using a parameter called the fractal dimension. Mandelbrot [13] introduced the box-counting method for this aim, while other scientists [15-17] proposed versions of this method or developed different approaches [18]. To determine the fractal dimension, the scaling exponent is computed as a linear

function of the fractal dimension (d). If Q is the analysed characteristic, the fractal dimension is calculated by fitting a linear regression of $\log Q(d)$ against $\log(d)$.

The box-count dimension (D_{BC}) of $\Gamma \subset \mathbb{R}^2$ is:

$$D_{BC} = \lim_{\varepsilon \rightarrow 0} N(\varepsilon) / \log(1/\varepsilon) \quad (1)$$

where $N(\varepsilon)$ ($\varepsilon > 0$) is the smallest number of squares with the side ε that can cover Γ [17].

The Hall – Wood dimension (D_{HW}) of $\Gamma \subset \mathbb{R}^2$ is:

$$D_{HW} = 2 - \lim_{\varepsilon \rightarrow 0} A(\varepsilon) / \log(\varepsilon), \quad (2)$$

where $N(\varepsilon)$ is the sum of the areas ($A(\varepsilon)$) of all boxes with ε - sides intersecting Γ [18].

The dimension estimator, D_G , is computed by plotting $\log(\hat{V}(d))$ vs. $\log(d)$, where

$$\hat{V}(d) = 4.924405 \{U_i(d) - U_j(d) : i < j\}_{(k)}^2, U_i(d) = X_{i/n} - X_{(i-d)/n}, \quad (3)$$

$k = C_{[(n-d)/2]+1}^2$ and d is the lag [20].

The variation estimator [21] is computed using

$$\gamma_p(t) = 1/2 E |X_u - X_{u+t}|^p \quad (4)$$

The variogram estimator is a particular case ($p=2$) of the variation estimator [19].

The periodogram and type-II discrete cosine transform (DCT-II) complete the set of estimators used in this article. For the description of these methods, the reader may see [21-23].

3. RESULTS AND DISCUSSION

The KPSS test rejected the hypothesis of series stationarity in trend and level. The Pettitt test detected the change point for both series in 1997.

The estimators of the fractal dimensions for the maximum series (T_{max}) are presented in Figs. 3 and 4. The Hall-Wood, variogram, and variation estimators have values around 2. Given that the periodogram and DCT-II computation is based on a different approach than the other estimators, their values are approximately equal but higher than for the previously mentioned estimators. The lowest value is that of the Box-Count dimension. A possible explanation is that the number of points used in the computation is small. Another one is the computation methodology that is the less elaborated among all the competitors.

Fig. 3. Box-Count and Hall-Wood estimators for T_{max} series

Fig. 4. Variogram, Variation, Periodogram, DCT-II for $T_{\max}$ series

Figures 5 and 6 display the fractal dimensions estimators for the minimum series ($T_{\min}$). All but the Box-Count has values between 1.84 and 1.91, so close to each other. The smallest dimension is the Box-Count.

Fig. 5. Box-Count and Hall-Wood, Variogram, and Variation estimators for $T_{\min}$ series

Fig. 6. Periodogram and DCT-II for $T_{\min}$ series

Comparison of the values obtained for $T_{\max}$ and $T_{\min}$ indicate slightly lower values of the first four dimensions computed for $T_{\min}$. A high difference between the Periodogram and DCT-II estimators calculated for the $T_{\max}$ and $T_{\min}$ is also noticed.

4. CONCLUSION

This article estimated the fractal dimensions of the maximum and minimum average annual temperature series recorded at Constanta in 1961-2018. Both series present fractal characteristics, with the smallest fractal dimensions corresponding to the minimum series. In a future study, we intend to analyze the multifractality of these series.

5. REFERENCES

- [1] 2018 IPCC Report, <https://www.ipcc.ch/sr15/>
- [2]. D. R. Easterling et al. (1997), Maximum and minimum temperature trends for the globe, *Science*, 277(5324), pp. 364–367
- [3]. R. Heino et al. (1999), Progress in the study of climatic extremes in northern and central Europe, *Climatic Change*, 42, pp. 151–181
- [4]. C. A. Mocanu-Vargancsik, A. Barbulescu (2019), Study of the Temperature's Evolution Trend on the Black Sea Shore at Constanta, *IOP Conference Series: Journal of Physics Conference Series*, no. 1297, art. 012010. doi:10.1088/1742-6596/1297/1/012010
- [5]. C. K. Folland, et al. (2001), Observed climate variability and change, in *Climate Change 2001: The Scientific Basis*, Cambridge University Press, New York, pp. 108–109
- [6]. R. S. Vose, D. R. Easterling, B. Gleason (2005), Maximum and minimum temperature trends for the globe: An update through 2004, *Geographical Research Letters*, 32(23), L23822, doi:10.1029/2005GL024379
- [7]. C. Maftai, A. Barbulescu (2008), Statistical Analysis of Climate Evolution in Dobrudja Region, *Proceedings of the World Congress on Engineering 2008*, vol II, WCE 2008, July 2 - 4, 2008, London, U.K, pp.1082-1087
- [8]. A. Barbulescu (2016), Models for temperature evolution in Constanta area (Romania), *Romanian Journal of Physics*, 61(3-4), pp. 676 - 686
- [9]. A. Barbulescu (2016), Modeling temperature evolution. Case study, *Romanian Reports in Physics*, 68(2), pp. 788 - 798
- [10]. A. Barbulescu, J. Deguenon (2015), About the variations of precipitation and temperature evolution in the Romanian Black Sea Littoral, *Romanian Reports in Physics*, 67(2), pp. 625 – 637.

- [11]. A. Bărbulescu, C. Maftai (2015), Modeling the climate in the area of Techirghiol Lake (Romania), *Romanian Journal of Physics*, 60(7-8), pp.1163 – 1170
- [12]. European Climate Assessment & DataSet, <https://www.ecad.eu/>
- [13]. D. Kwiatkowski, P.C.B. Phillips, P. Schmidt, Y. Shin (1992), Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, 54(1–3), pp. 159–178
- [14]. A. N. Pettitt (1979), A non-parametric approach to the change point problem, *Journal of the Royal Statistical Society Series C, Applied Statistics*, 28, pp. 126-135.
- [15]. B. Mandelbrot (1977), *Fractals: form, chance and dimension*, W.H. Freeman and co., San Francisco
- [16]. L. S. Leibovitch (1998), *Fractal and Chaos Simplified for Life Sciences*, Oxford University Press, New York
- [17]. K. J. Falconer (1990), *Fractal Geometry: Mathematical Foundations and Applications*, Wiley, New York
- [18]. P. Hall, A. Wood (1993), On the performance of box-counting estimators of fractal dimension, *Biometrika*, 80, pp. 246–252
- [19]. S. Davies, P. Hall (1999), Fractal analysis of surface roughness by using spatial data, *Journal of the Royal Statistical Society, Series B*, vol. 61 (1999), pp. 3–37
- [20]. M. G. Genton (1998), Highly robust variogram estimation, *Mathematical Geology*, 30, (pp. 213–221
- [21]. T. Gneiting, H. Sevcikova, D. B. Percival (2012), Estimators of fractal dimension: Assessing the smoothness of time series and spatial data, *Statistical Science*, 27(2), pp. 247-277 (Version as technical report available at <https://stat.uw.edu/sites/default/files/files/reports/2010/tr577.pdf>)
- [22]. N. Ahmed, T. Natarajan, K. R. Rao (1994), Discrete cosine transform, *IEEE Transactions on Computers*, 23, 90–93.
- [23]. G. Strang (1999), The discrete cosine transform, *SIAM Review*, 41, pp. 135–147.

Note:

Alina Bărbulescu - Transilvania University of Braşov, Faculty of Civil Engineering, 5, Turnului Str, 900152, Braşov, Romania (e-mail: alina.barbulescu@unitbv.ro)

Cristian Ştefan Dumitriu – Transilvania University of Braşov, Faculty of Civil Engineering, 5, Turnului Str, 900152, Braşov, Romania (corresponding author e-mail: cristian.dumitriu@unitbv.ro)