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Analysis of the effect of external heating in the human tissue: A finite element approach

Mridul SANNYAL^{a,*}, Abul Mukid Mohammad MUKADDES^a, Md. Matiar RAHMAN^b, M. A. H. MITHU^a

^aDepartment of Industrial and Production Engineering, Shahjalal University of Science and Technology, Sylhet, Bangladesh

^bDepartment of Mathematics, Shahjalal University of Science and Technology, Sylhet, Bangladesh

*E-mail address: sannymridul@gmail.com

Abstract

Thermal therapy which involves either raising or lowering tissue temperature to treat malignant cells needs precise acknowledgment of thermal history inside the biological system to ensure effective treatment. For this purpose, this study presents a two-dimensional unsteady finite element model (FEM) of the bioheat transfer problem based on Pennes bio-heat equation to analyze the thermal response of tissue subject to external heating. Crank-Nikolson scheme was used for the unsteady solution. A finite element code was developed using C language to calculate results. The obtained numerical result was compared with the analytical and other numerical results available in the literature. A good agreement was found from the comparison. Temperature distribution inside the human body due to constant and sinusoidal spatial and surface heating were analyzed. Response to point heating was also investigated. Moreover, a sensitivity analysis was carried out to know the effect of various parameters, i.e. blood temperature, thermal conductivity, and blood perfusion rate on tissue temperature. The outcome of this study will be helpful for the researchers and physicians involved in the thermal treatment of human tissue.

Key words: bio-heat transfer; finite element method; Pennes's equation; thermal therapy; external heating.

1. Introduction

Transferring thermal energy through external heating inside the biological tissue has remarkable success in many therapeutic treatments. Based on the amount of energy absorbed by the tissue and the nature of the heating, this process can serve as various biomedical problems i.e. malignant cell destruction, thermal comfort, burn analysis. This process is done by taking advantage of the lability of biological tissue at elevated temperatures. The design of such a modality strongly depends upon the knowledge of tissue response to thermal energy. Thus, acknowledgment of the thermal behavior of human tissue subjected to external power is utmost necessary. Undoubtedly experimental studies can provide such information closer to real scenarios. However, that requires suitable equipment and a proper experimental setup, which is costly as well as hazardous. Hence arises the necessity of mathematical modeling. With the advancement in computational methods and technology, its application in modeling biological systems has been a topic of interest by scientists and researchers. In modern days' mathematical models are widely used to ascertain safety issues in treating malignant cells, cryosurgery, laser eye surgery, etc. Moreover, due to its non-invasive nature and simplicity, it is welcomed by researchers and engineers.

Heat transfer in the biological tissue is governed by the bio-heat equation. Several forms of the bio-heat equation can be found in the literature. Among them, the most popular and most widely used one is the Pennes Bio-heat equation.¹ Uniform thermal conductivity, blood perfusion, and metabolism give it simplicity; that is why it is used in various biological research works.

Initially the Pennes bio-heat equation was developed to predict heat transfer in human forearm, but later on, it was used in various biological research works like human eye research^{2,3} and skin bio-heat transfer.^{4,5} This equation was used in modeling and optimization of temperature distribution around cancerous tissues during magnetic fluid hyperthermia,^{6,7} ultrasound thermal therapy.^{8,9} It was then used in determining tissue temperature by induced microwave,¹⁰ cryosurgery,¹¹ and radio-frequency heating^{12,13} and laser heating^{14,15}.

Various analytical and numerical solutions of the bio-heat equation can be found in the literature. In complex situations where analytical solutions are not available or very complicated, numerical methods are preferable, where an analytical solution works as a benchmark of numerical solutions. In many analytical solutions, a time-dependent green function is used in^{16,17}, Finite Difference solution is obtained in^{8,10,18}, Boundary Element Method in³, and the Finite Element Method in^{4,5}. The analytical and numerical results under different spatial and surface heating can be found in the

literature. In some cases, oscillatory surface heating¹⁸⁻²¹, step heating^{19,20}, or constant surface heating^{19,20} is considered in the bioheat transfer analysis. Sometimes constant spatial heating, even sinusoidal spatial heating^{17,22-24} is used as external heating. Deng et al. and Sharma et al. used point heating as an external heat source.^{17,24} The alternating heating and cooling strategy was introduced by Yuan et al.²⁵ Two-dimensional bioheat transfer problem with a rectangular domain with and without tumor was analyzed by Leilei et al.²⁶ Constant and sinusoidal heat flux boundary condition was used for estimation of blood perfusion using phase shift in temperature response in²⁷⁻²⁹.

To the best of the authors' knowledge, most of the researchers concentrate on the one-dimensional case only. However, for a better understanding of the response of biological tissue to elevated temperature, it is beneficial to use a multidimensional heat transfer model. Three-dimensional cases of conventional heat transfer are available in the literatures³⁰ but cases of skin burn were few and with the use of commercial softwares³¹. It is quite difficult to have the temperature profile of a particular point or a line within the biological tissue with different time intervals using that software. Moreover, they operate on limited boundary conditions. The development of a free two-dimensional finite element code of bio-heat equations can meet the purpose. The objective of this research is to develop a two-dimensional finite element code for the solution of both steady and transient bio-heat equation. The popular Crank-Nicolson method was used in the time discretization of the transient analysis. The developed finite element code was used to simulate the thermal response of tissue during cancer hyperthermia, laser surgery, tissue heating with a hot disk, and point heating. Moreover, time-dependent spatial and surface heating were incorporated. The effect of surface heating, step heating, sinusoidal heating, and point heating was thoroughly investigated.

2. Finite element discretization of bio heat equation

2.1 Bio Heat Equation

To evaluate the temperature profile inside blood perfused tissue the most widely used Pennes Bio heat equation is used in this research. For the two-dimensional case, the equation can be expressed as:

$$k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial y^2} + \omega_b \rho_b c_b (T_a - T) + Q_m + Q_r = \rho c \frac{\partial T}{\partial t} \quad \text{Eq. 1}$$

where, k is the thermal conductivity of the tissue, ρ and c are the density and specific heat of the tissue respectively. And ρ_b , c_b denote density and specific heat of blood; ω_b is the blood perfusion; T_a and T is the known arterial temperature and unknown tissue temperature respectively; Q_m is the metabolic heat generation, and Q_r is the heat source due to spatial heating with respect to time t .

2.2 Finite element discretization

The simplified form of **Equation 1** can be simplified as:

$$k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial y^2} + CT_a + f = \rho c \frac{\partial T}{\partial t} \quad \text{Eq. 2}$$

Using the weighted residual method the weak form of the governing differential **Equation 2** is derived as:

$$0 = \int_{\Omega_e} [W \rho c \frac{\partial T}{\partial t} + k \frac{\partial T}{\partial x} \frac{\partial w}{\partial x} + k \frac{\partial T}{\partial y} \frac{\partial w}{\partial y} + CWT - fW] dA - \oint_{\Gamma_e} W \left(k \frac{\partial T}{\partial x} \eta_x + k \frac{\partial T}{\partial y} \eta_y \right) ds \quad \text{Eq. 3}$$

For a mixed boundary condition combining convective and heat flux boundary as $k \frac{\partial T}{\partial x} \eta_x + k \frac{\partial T}{\partial y} \eta_y = -h(T - T_0) + q$,

Equation 3 was modified as follows to account for the convective heat transfer term. Here, h is the convective heat transfer coefficient, T_0 is the ambient temperature, and q is the heat flux. And Ω_e is the element domain and Γ_e is the boundary of the element.

$$0 = \int_{\Omega_e} [w \rho c \frac{\partial T}{\partial t} + k \frac{\partial w}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial w}{\partial y} \frac{\partial T}{\partial y} + CwT - wf] dxdy - w \oint_{\Gamma_e} [q_n - h(T - T_0)] ds \quad \text{Eq. 4}$$

Taking a finite element approximation as $T = \sum_{j=1}^N T_j^e \psi_j^e$ **Equation 4** can be derived as:

$$0 = \int_{\Omega_e} \rho c \psi_i^e \psi_j^e \frac{\partial T_j^e}{\partial t} dxdy + \int_{\Omega_e} \left[k \frac{\partial \psi_i^e}{\partial x} \frac{\partial \psi_j^e}{\partial x} + k \frac{\partial \psi_i^e}{\partial y} \frac{\partial \psi_j^e}{\partial y} + C \psi_i^e \psi_j^e \right] dxdy + \oint_{\Gamma_e} h \psi_i^e \psi_j^e ds \left[T_j^e - \int_{\Omega_e} f \psi_i^e dxdy - \oint_{\Gamma_e} \psi_i^e h T_0 ds - \oint_{\Gamma_e} \psi_i^e q ds \right] \quad \text{Eq. 5}$$

Here i and j donate the element number which ranges from 1 to N , where N is the total no of elements. Finally, the finite element model of the governing differential equation is derived as follows:

$$0 = \sum_{j=1}^N \left\{ \int_{\Omega_e} \rho c \psi_i^e \psi_j^e \frac{\partial T_j^e}{\partial t} dxdy + \int_{\Omega_e} \left[k \frac{\partial \psi_i^e}{\partial x} \frac{\partial \psi_j^e}{\partial x} + k \frac{\partial \psi_i^e}{\partial y} \frac{\partial \psi_j^e}{\partial y} + C \psi_i^e \psi_j^e \right] dxdy + \oint_{\Gamma_e} h \psi_i^e \psi_j^e ds \right\} T_j^e - \int_{\Omega_e} f \psi_i^e dxdy - \oint_{\Gamma_e} \psi_i^e h T_0 ds - \oint_{\Gamma_e} \psi_i^e q ds \quad \text{Eq. 6}$$

A simplified form of **Equation 6** can be written as:

$$0 = \sum_{j=1}^N \{ C_{ij}^e T_j^e + [K_{ij}^e + H_{ij}^e] T_j^e - q_i^e - Q_i^e - P_i^e \} \quad \text{Eq. 7}$$

Here,

$$C_{ij}^e = \int_{\Omega_e} \rho c \psi_i^e \psi_j^e$$

$$K_{ij}^e = \int_{\Omega_e} k \frac{\partial \psi_i^e}{\partial x} \frac{\partial \psi_j^e}{\partial x} + k \frac{\partial \psi_i^e}{\partial y} \frac{\partial \psi_j^e}{\partial y} + C \psi_i^e \psi_j^e$$

$$H_{ij}^e = \oint_{\Gamma_e} h \psi_i^e \psi_j^e$$

$$q_i^e = \int_{\Omega_e} f \psi_i^e dA$$

$$P_i^e = \oint_{\Gamma_e} q_f \psi_i^e ds$$

$$Q_i^e = \oint_{\Gamma_e} h_0 T_0 \psi_i^e ds$$

In matrix notation, the finite element model can be expressed as:

$$[C]\{\dot{T}\} + [K]\{T\} = \{Q\} + \{Q_f\} + \{P\} \quad \text{Eq. 8}$$

where, $[C]$ is the capacitance matrix and $[K]$ is the conductivity matrix and $\dot{T} = \frac{T^{n+1} - T^n}{\Delta t}$.

2.3 Time discretization

A simple time integration scheme for **Equation 8** is derived by assuming that C and K are constant. In such a case, the matrix differential equation can be discretized with the response to time as:

$$C \frac{T^{n+1} - T^n}{\Delta t} + \alpha K T^{n+1} + (1 - \alpha) K T^n = Q + q_f + P \quad \text{Eq. 9}$$

where T^{n+1} and T^n are the vectors of unknown nodal temperature values at times $n\Delta t$ and $(n+1)\Delta t$, respectively and α is the weighting factor, which must be chosen in the interval between 0 and 1. When the value of α is considered 0.5, the process is called the popular Crank-Nicolson method. The discretized **Equation 9** can be written as:

$$\left(C \frac{1}{\Delta t} + \alpha K\right) T^{n+1} = \left[C \frac{1}{\Delta t} - (1 - \alpha) K\right] T^n + q_f + Q + P \quad \text{Eq. 10}$$

Equation 10 was solved using an iterative procedure. The initial temperature is known and then the temperature of the next step can be calculated from the solution of Eq. (10) through the Gauss elimination technique.

3. Computational model

3.1 Tissue model

Our two-dimensional computational model of the human tissue is shown in **Figure 1**. Here $X=0$ is the skin surface, and $X=L_1$ is the body core. L_1 is the problem length along the X-axis and L_2 is the length along Y-axis.

3.2 Boundary Conditions

The skin surface is in contact with the environment, hence heat exchange between the environment and skin surface is a convection process. So the boundary condition at the skin surface is governed by

$$k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = -h_0 (T_0 - T) \text{ on } \Gamma_1 \quad \text{Eq. 11.1}$$

Here h_0 is the ambient heat transfer coefficient, and T_0 is the ambient temperature.

In some cases, water is circulated over the skin surface to reduce the skin surface temperature. The forced convection boundary condition in such cases can be expressed as

$$k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = -h_f (T_f - T) \text{ on } \Gamma_1 \quad \text{Eq. 11.2}$$

Here h_f is the forced convection coefficient between the environment and tissue, and T_f is the cold water temperature.

In the case of surface heating, the skin surface is exposed to the applied heat flux. So the boundary condition in such a case can be expressed as:

$$k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = -q_f \text{ on } \Gamma_1 \quad \text{Eq. 11.3}$$

Here q_f is the applied heat flux, which can be both constant and time-dependent.

Both sides of the Y-Axis $Y=0$ and $Y=30$ mm is considered as thermally insulated. No heat transfer occurs through those boundaries, therefore:

$$k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = 0 \text{ on } \Gamma_2 \quad \text{Eq. 11.4}$$

Tissue temperature within a narrow range tends to keep constant the same as the body core temperature. Thus, considering T_c as the body core temperature, the boundary condition at the body core is expressed as:

$$T = T_c \text{ on } \Gamma_3 \quad \text{Eq. 11.5}$$

In all the numerical investigations, the boundary condition **Equation 11.5** is the temperature boundary condition used at the body core (Γ_3). **Equation 11.4** used both sides of the Y axis (Γ_2 and Γ_4). However, based upon the heating and cooling conditions the natural convection boundary condition (**Equation 11.1**), forced convection boundary condition (**Equation 11.2**) and heat flux boundary condition (**Equation 11.3**) are applied at the skin surface (Γ_1) as the boundary condition.

Figure 1. Human Tissue Model

3.3 Mesh and materials properties

The tissue properties and values of other control parameters used in this study are summarized in **Table 1** as given in³². As tissue temperature tends to keep constant within a narrow range, such as for 20 mm to 30 mm. Our problem domain length is considered as 30 mm for both axes, so $L_1=30$ mm and $L_2=30$ mm. The linear triangular element was used for space discretization. Moreover, in this study, the body core temperature is considered as 37°C. Besides, the initial temperature (at $t=0$) of all nodes is considered 37°C. For the transient solution, a time step of 0.25 s is considered in this study.

3.4 Code Validation

Before using the computer program for further investigation, it is necessary to verify it with the analytic solution. For this purpose, we develop a test problem considering the following boundary conditions of the problem described in **Figure 1**. The analytical result for the steady-state problem is calculated from **Equation 12**. The typical tissue properties used in this section are summarized in **Table 1**. In this case, heat generation due to metabolism is ignored; hence Q_m is considered as 0.

Boundary conditions for the test problem are as below:

$$T = 37^\circ\text{C} \text{ on } \Gamma_3; T = 45^\circ\text{C} \text{ on } \Gamma_1; k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = 0 \text{ on } \Gamma_2 \text{ and } k \frac{\partial T}{\partial x} \eta_x + \frac{\partial T}{\partial y} \eta_y = 0 \text{ on } \Gamma_4$$

For the steady-state case, the analytical result is derived from the following expression³³:

$$T(x) = T_a + \frac{(T_s - T_a) \sinh[\mu(L_1 - x)] + (T_c - T_a) \sinh(\mu x)}{\sinh(\mu L_1)} \quad \text{Eq. 12}$$

where, $\mu = \sqrt{(\omega_b \rho_b C_b)}$; T_s = skin surface temperature; L_1 = problem length along X-axis.

Table 1. Thermal Properties of biological tissue and values of control parameters

Parameters	Symbol	Value
Thermal conductivity	k	0.5 W/(m.°C)
Convection Coefficient	h_o	10 W/(m ² .°C)
Forced convection coefficient	h_f	100 W/(m ² .°C)
Environmental Temperature	T_0	25°C
Temperature of the Artery	T_a	37°C
Body core temperature	T_c	37°C
Metabolic heat generation	Q_m	33800 W/m ²
Density of blood	ρ_b	1000 kg/m ³
Density of tissue	P	1000 kg/m ³
Specific heat of blood	c_b	4200 J/(kg.°C)
Specific heat of the tissue	c	4200 J/(kg.°C)
Blood perfusion	ω_b	0.0005 m ³ /s/m ³
Laser Heating Power	P_0	2500 W/m ²
Point Heating Power	P_1	2500 W/m ²
Scattering Coefficient	η	200 s ⁻¹
Heat Flux	q_f	1000 W/m ²

The steady and transient temperature (after the 2000 s) in comparison with other analytical and numerical results available in the literature is shown in **Figure 2** and **Figure 3**, respectively. A good agreement with the analytical result is found in **Figure 2**. While a slight deviation of our transient solution from analytical results²⁶ is noticed in **Figure 3**. However, our result seems better than Trefftz-FEM and RBF-MFS results.

4. Computational results and discussion

4.1 Spatial heating

Laser therapy uses a strong beam of light to cut, burn, or destroy tissue. It is delivered from a safe source. That is why it poses no health risks to the patient or the medical team. The laser beam is tiny and precise, hence allows doctors to treat tissue without injuring the surrounding area. Laser therapy can be used to treat cervical, lung, and skin cancer. Along with laser, microwaves are also used in non-surgical, minimally invasive therapy.

Figure 2. Code verification (Steady-State Case)

Figure 3. Code verification (Transient Case)

In this section, we aim to know the response of biological tissue to laser or microwave heating. For this purpose, we consider convection boundary condition at the skin surface and temperature boundary condition at the body core, and laser or microwave power is used as external heating.

In the case of heating by laser and microwave, the heat absorption rate can be simplified by Beers Law, which can be expressed as $Q_r = \eta P_0(t) e^{-\eta x}$.³⁴ Where, $P_0(t)$ is the time-dependent heating power on the skin surface and η is the scattering coefficient. Since $P_0(t)$ and η vary from one apparatus to another, so it is important to know the influence of these parameters on tissue temperature. $P_0(t)$ can be both constant and time-dependent. In this study, tissue temperature is calculated considering both cases. In addition, the natural convection boundary condition (Equation 11.1) is used here as the skin surface boundary condition. Figure 4 depicts the transient temperature at various times along the x-axis when tissues subject to spatial heating. Figure 4a shows the case of constant spatial heating where heating power $P_0(t) = 250 \text{ W/m}^2$ and while Figure 4b is for sinusoidal spatial heating where $P_0(t) = 250 + 200\cos(0.02t) \text{ W/m}^2$. In both cases, at the early

stage, the tissue temperature increases along with the distance from the skin surface due to metabolism and spatial heating but later it decreases towards the body core. Moreover, an intercross is noticed between temperature curves at different times in Figure 4b, which indicates the oscillation of the temperature inside the tissue over time due to sinusoidal spatial heating. Here the maximum temperature lies approximately 7 mm below the skin surface. But in Figure 4a it lies below 11 mm. Thus, the laser heating shifts the point of maximum temperature near the skin surface.

Figure 5 shows the effect of the scattering coefficient. Where, Figure 5a shows the result of constant spatial heating and Figure 5b shows the sinusoidal case. Both figures show an increasing trend in temperature over time. However, the increasing rate decreases as time passes. In both cases, the larger coefficient results in a higher temperature. Moreover, in the case of sinusoidal spatial heating, the larger coefficient returns a higher amplitude. It is to be noted that the sinusoidal effect is clearly visible from Figure 5b. Besides, under the current set up, tissue temperature begins to stabilize approximately after 3000 s.

Figure 4. Temperature along with time (a) $P_0(t) = 250 \text{ W/m}^2$; (b) $250 + 200\cos(0.02t) \text{ W/m}^2$.

Figure 5. Effect of Scattering Coefficient (a) $P_0(t) = 250 \text{ W/m}^2$; (b) $250 + 200\cos(0.02t) \text{ W/m}^2$.

Figure 6. Sinusoidal surface heating at different heating frequency $(P_0(t) = 250 + 200\cos(\omega t)) \text{ W/m}^2$

4.1.1. Effect of heating frequency

Figure 6 shows the temperature distribution inside the human tissue subject to sinusoidal spatial heating for three different heating frequencies. The sinusoidal oscillation of tissue temperature is clearly visible in all three curves. Apparently, the larger heating frequency decreases temperature fluctuation. However, it does not have any significant effect on the tissue temperature instead it changes the wavelength and frequency. Such kind of information is significant in the case of parameter estimation.

(a)

(c)

4.2 Surface Heating

The practical application of this type of heating can be found in the case of industrial accidents when human tissue is exposed to hot fluid or flash flame or in contact with a hot object. Moreover, in hyperthermia treatment, this can be used for increasing blood perfusion rate and metabolism. Surface heating is also used in thermal burn injuries or thermal comfort analysis. To investigate the temperature distribution during surface heating, we used a heating plate that delivers heat flux continuously to the skin tissue at a constant rate. Heat flux boundary condition is used here at the skin surface as a boundary condition. **Figure 7a** shows our tissue model with a heating plate. Tissue temperature after 1000 s due to heating with a hot plate is shown in **Figure 7b** ($q_f = 1000 \text{ W/m}^2$). Where tissue temperature along time for three different heat fluxes is shown in **Figure 7c**. Moreover, tissue temperature along with the distance from the skin surface at different times is shown in **Figure 7d**. It is clearly visible in **Figures 7b** and **7d** that tissue layers adjacent to the heat source are affected more significantly than others, which results in a hot temperature region at the skin surface. Moreover, temperature declined from the skin surface to the body core. Here, from **Figure 7c** the maximum recorded temperature after 2500 s for $q_f = 1000 \text{ W/m}^2$ is almost 83°C . For $q_f = 500 \text{ W/m}^2$ the maximum temperature decreases to 65°C and for $q_f = 200 \text{ W/m}^2$ it is 55°C .

(b)

(d)

Figure 7. Temperature profile of human tissue during surface heating; (a) Model (b) Tissue temperature at $q_f = 1000 \text{ W/m}^2$ (c) Tissue temperature along time at three different heat flux (d) Tissue temperature along distance at $q_f = 1000 \text{ W/m}^2$

So, depending upon the required temperature a combination between heat flux and exposure duration can be made using this result. For high heat flux, there is a great possibility of skin burn due to high temperature. So careful heating is necessary in the case of thermal therapy.

4.2.1 Step heating

In some cases, the tissue is heated in two stages one is the heating stage and another is the cool-down stage. In the cool-down stage heat source becomes idle and allows tissues to cool. This type of scenario is considered in this section. **Figure 8** shows the effect of step heating at three different locations of the tissue $x = 0$ mm, 15 mm, and 30 mm while $y=0$ mm. Here **Figure 8a** shows the result when tissue is heated in one step, where **Figure 8b** represents the tissue temperature when tissue is heated in two steps. After heating up to 500 s heat source is removed, the heat source again becomes active after 1000 s and heated up to 1500 s. The external heat source for both cases is expressed as below

For single-step heating $q_f = \begin{cases} 1000 \text{ Wm}^{-2} & \text{if } t < 1200 \text{ s} \\ 0 & \text{if } t > 1200 \text{ s} \end{cases}$

For double step heating $q_f = \begin{cases} 1000 \text{ Wm}^{-2} & \text{if } t < 500 \text{ s or } 1000 \text{ s} < t < 1500 \text{ s} \\ 0 & \text{Otherwise} \end{cases}$

Figure 8a shows that the temperature increases along time up to 1200s and decreases afterward as the heat source is idle at that time. However, both figures draw a similar conclusion, and that is tissue temperature increases as time increases and distance from the skin surface decreases.

4.2.2 Sinusoidal surface heating

Figure 9 shows the temperature distribution subject to sinusoidal surface heating where $q_f = 1000 + 500 \cos(\omega t)$, here ω is the heating frequency (0.02s^{-1}). **Figure 9a** shows the result along time. The three curves show the result for three different locations of $X=0$ mm, 15 mm, and 30 mm (Here $Y=0$ mm). Meanwhile, **Figure 9b** shows the temperature inside human tissue at three different blood perfusion rates. From **Figure 9a**, the frequency of the oscillating temperature decreases when we move from the skin surface to towards the body core (From $X=0$ mm to $X=30$ mm). **Figure 9b** shows that higher blood perfusion results in a decrease of tissue temperature as much heat is carried away by blood vessels.

Figure 8. Temperature distribution of human tissue during step heating at three different locations of x , 0 mm, 15 mm, and 30 mm; (a) $q_f=1000\text{W/m}^2$ (When $t < 1200$ s) else $q_f=0$; (b) $q_f=1000\text{W/m}^2$ (When $t < 500$ s and $1000 \text{ s} < t < 1500$ s) else $q_f=0$;

Figure 9. Sinusoidal surface heating; (a) Tissue temperature along distance and time (b) Tissue temperature at three different blood perfusion

4.3 Effect of irregular heating frequency

The thermal response of tissue subject to simultaneously surface and spatial heating is shown in **Figure 10**. Here the expression of surface and special heating are used as $q_f = 1000 + 500 \cos(0.02t)$ and $P_0(t) = 250 + 200 \cos(0.01t)$ respectively. From **Figure 10**, curve A shows the result of sinusoidal spatial heating ($\omega = 0.01 \text{ s}^{-1}$), curve B of sinusoidal surface heating ($\omega = 0.02 \text{ s}^{-1}$). While in curve C both surface and spatial heating are applied simultaneously. From this figure, curve A and curve B bears the frequency of the external heating. However, irregular frequency is noticed from Curve C, which bears both frequencies. Such information is useful for selecting the appropriate heating pattern. Moreover, those results can be used in parameter estimation also.

Figure 10. Effect of irregular heating frequency

4.4 Point Heating

In treating deep tumors located at the kidney, lung, or rectum, it is challenging to adopt surgical treatment. In such a case, point heating is an alternative to surgical operations. In this heating style, the total heating power deposits at the tumor site. There is an inverse relationship between elevated temperature and exposure duration. For the same amount of tissue necrosis, a high temperature needs low exposure duration. On the contrary, a low temperature needs high exposure duration. So, for effective thermal treatment, it is necessary to evaluate the required temperature and exposure duration precisely. Moreover, in some cases, to protect the skin surface cells from excess heat, cooling water is circulated over the skin surface during treatment.

To deposit total heating power at the desired site, the expression of the external heating was used like as ¹⁷.

$$Q_r(x, t) = P_1(t)\delta(x - x_0)\delta(y - y_0)$$

where $P_1(t)$ is the point heating power, and $\delta(x - x_0)$ is the Dirac delta function. It has a value of 1 at our desired location (X_0 , Y_0) and 0 elsewhere. That's why it allows depositing total heating power at the tumor site. In this investigation, the force convection boundary condition is applied at the skin surface. Where $P_1(t) = 2500 \text{ W/m}^2$, force convection coefficient $h_f = 100 \text{ W/(m}^2 \cdot ^\circ\text{C)}$ and $T_f = 15^\circ\text{C}$.

Figure 11a shows the temperature subjected to single heating source heating the point at (15 mm, 15 mm), where **Figure 11b** shows the result for two heating sources at (10.5 mm, 10.5 mm) and (20.5 mm, 20.5 mm). Thermal response of tissue subject to three heating sources at (10.5 mm, 10.5 mm), (10.5 mm, 20.5 mm), and (20.5 mm, 15 mm) is depicted in **Figure 11c**. Here the maximum recorded temperature is 70°C , 75°C , and 80°C , respectively in **Figure 11a**, **11b**, and **11c**. Note that each figure showing the skin surface temperature below 30°C . This scenario happens as cooling water is circulated over the skin surface.

Figure 11. Thermal response of tissue during point heating

However, applying three heat sources increase the temperature more rapidly and heated more region than using a single and double heat source. Multiple heat sources generate more heat through the tissue. So when the target region is quite larger it is beneficial to use multiple heat sources.

4.5 Effect of Blood Temperature

The arterial temperature, which is considered as constant, may be interrupted for many situations, i.e. for hypothermia or fever case. Thus, it is necessary to know its effect on the thermoregulation of biological tissues. **Figure 12** shows the effect of arterial temperature for four sets of blood temperatures 34°C, 37°C, 39°C, 41°C. The result shows that tissue temperature increases as the arterial temperature increases. This information can be beneficial for thermal diagnosis analysis. After getting the tissue temperature, it is possible to detect abnormality in the human body. For example, in this study, when the tissue temperature is 45.5°C it ensures that blood temperature is 41°C, so it is the hyperthermia case. Moreover, in the case of hypothermia, the blood temperature is about 34°C, which reduces the tissue temperature to almost 41.6°C. Here, the natural convection boundary condition (**Equation 11.1**) is used as a skin surface boundary condition.

4.6 Effect of Thermal Conductivity

In our study, we have considered constant thermal conductivity. In practice, the tissue thermal conductivity is temperature-dependent. Moreover, in some cases, it changes from person to person and for the tissue contaminant. So, it is convenient to know the effect of such variation on the tissue temperature. To investigate the effect of thermal conductivity data values 0.25 W/(m·°C), 0.50 W/(m·°C) and 0.75 W/(m·°C) is used. Here natural convection boundary condition (**Equation 11.1**) is used as a skin surface boundary condition. **Figure 13** shows the result. Three curves show the temperature distribution along with the x-axis for three different values. Clearly, as the thermal conductivity increases tissue temperature decreases. It is because as thermal conductivity increases, much heat is transferred to the nearby tissue. But due to temperature boundary conditions at the body core, it further starts to decrease to keep the core temperature constant.

4.7 Effect of Blood perfusion

Another important parameter that plays a significant role in heat transport in human tissue is blood perfusion. In the human body, heat is carried away by the blood vessel. So a high blood flow rate results in reducing tissue temperature, conversely a lower blood flow rate increases the temperature. In this study blood perfusion is considered as constant. However, other researchers do not agree with such an assumption.

Figure 12. Effect of blood temperature

Figure 13. Effect of thermal conductivity

Figure 14. Effect of blood perfusion

Expression of blood perfusion as a temperature-dependent function is made in ¹⁸ as $\omega = \omega_0 + \gamma T + \delta T^2$ where ω_0 is the base perfusion, γ and δ are the linear and quadratic coefficients of temperature dependency. Moreover, in ²³ it was expressed as $\omega = \omega_0(1 + \alpha T)$. Where ω_0 is the base perfusion, α is a linear coefficient of temperature dependency. Where in some cases blood perfusion is expressed as a function of temperature and damage rate ³⁵.

To know the effect of blood perfusion data values 0.00025 s⁻¹, 0.0005 s⁻¹, 0.01 s⁻¹ have been used for investigation. Besides, natural convection boundary condition (**Equation 11.1**) was used as a skin surface boundary condition. The result is shown in **Figure 14**.

It is clear from this figure that higher blood perfusion results in lower temperatures. Conversely, a low blood perfusion rate leads to higher tissue temperature. However, the effect of temperature boundary condition bound the body core temperature to be constant 37°C for different blood perfusion rate. Another vital piece of information that can be drawn from this figure is that a slight change in blood perfusion rate results in a massive change in tissue temperature. So, we can say that blood perfusion plays a crucial role in human skin thermoregulation. So the accuracy of thermal modeling of human tissue greatly depends on the accurate prediction of blood perfusion rate.

5. Conclusions

This research presents the two-dimensional Finite Element Model based on Pennes bio-heat equation. The finite element system has been developed to measure the temperature of tissue for different heating conditions. Results are verified with the analytical and other numerical solutions available in the literature. Temperature distributions inside the human tissue subject to spatial heating, surface heating, and point heating are analyzed and briefly discussed. Spatial heating shifts the position of maximum temperature towards the body core.

Moreover, there is an exponential decay form of the temperature curve from the skin surface to the body core. Both the effect of constant and sinusoidal spatial heating is almost the same except for the presence of sinusoidal oscillation in sinusoidal spatial heating. Surface heating shows the same results. Surface heating mainly increases the temperature of the skin surface more rapidly. Its effect decreases towards the body core. The effect of heating frequency is related to the oscillation of the tissue temperature, but it keeps the mean temperature almost the same. The effect of irregular frequency remarkably influences the tissue temperature. However, its effect on the mean temperature is not significant enough. Point heating increases the temperature of the target region more rapidly with less effect on the nearby tissue. In this particular heating style, the temperature of the malignant tissue can be selectively controlled keeping the temperature of the healthy tissues below the injury threshold. That's why it is the most accepted method for treating deep tumors. The result found in this research shows that there is an effect of thermal conductivity of tissue and blood perfusion rate on the tissue temperature. This solution can also be used to determine the required temperature to kill the malignant cells as well as optimizing the treatment procedure.

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