STRESS-DEFORMED STATE OF A PACKING RING WITH ECCENTRIC HOLES

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Currently, various variants of physical and geometrical non-linear calculation of anisotropic bodies have been developed. In spite of the large and increasing number of studies on the theory of shells there are still many unsufficiently developed problems important both in scientific and applied fields, for example, development of practically convenient methods for calculating of anisotropic sealing composite materials weakened by eccentric holes under the influence of local loadings. Stress-deformed state of a packing ring with eccentric holes of sealing materials was studied. In composite materials, the Hooke's equation was used for this purpose. Also, using Lurie's symbolic method, the concrete solution of the sealing problem with eccentric holes was obtained.

Key words: *eccentricity, packing ring, stress, deformation, method of initial function, stress tensor components, Lurie's method, dimensionless geometric parameter.*

INTRODUCTION

Packing rings of composite polymer materials are widely used in the sealing unit on oilfield equipment. Different methods have been used to study such a stress deformation state of the packing: variation of parameters, the calculation of the operation, Muscalevshik's complex variables method and so on. However, mathematical calculation of these methods for such packing either is impossible to be calculated by the methods abovementioned or selected by "constructive selection" approaches of engineering designers. Such approaches have a negative impact on the capacity of packing parts and their efficiency is unproductive. In order to study stress deformation state of the packing, the methods developed according to Hook's law of equations system made for the stresses of elasticity theory are less suitable than the method of startup functions used for the border conditions. The solution of the problem not only meets the terms taking into account the geometric shape and working conditions of the packing, but also it considerably simplifies the number and form of mathematical conversions.

The stress-deformed state of complex geometric forms of packer with eccentric holes from composite polymer materials was determined by the method of initial functions in the elasticity theory for complex packers.

Derivative constants entering unit general were determined from boundary conditions at the edges of limned sealing in the direction ρ_1 . Various methods satisfying boundary conditions on the basis of variation equations of the mixed principle of Reisner stationary can be used for this. Point satisfaction of boundary conditions is orthogonalisation of the row related to any complete system of functions. Thus, solution of initial functions by the obtained method is precise. The aim of the paper was to provide a complete solution of the system of equations obtained by the method of initial functions and the boundary conditions at the edge.

METHODS

For the considered packer, differential equilibrium equations and the equation of Hook's law were written. Sizeless parameters of geometric and statistic values were accepted and symbols of differentiation on variables (r and φ) were introduced.

The symbolic operating method of system integrating at initial conditions on the plane was applied and operation of the initial values was determined. Composite polymer materials are widely used in the packing unit of the packing rings of oil-field equipment. Their application efficiency

largely depends on the stress state of the seal (Avrushenko, 1978; Naushirovanov, 1991; Razina *et al.*, 1991). Influence of excentricity of stress deformation on the condition of the packing has not been studied sufficiently. Study of the deformation processes on structurally homogeneous composite packing is the subject of the formation of a new model of real composite packing, where parameters determining properties of elasticity are the method of initial coordinates. All these make possible to carry out all round analyses of this method:

1. Considering components of the stress tensor and transmission vector, corresponding to coordinate directions the model based on differential equations of equilibrium was developed built and the Hook's law was written for the coefficients of elasticity for the plane strained condition and in the case of plane deformation;

Using the method of initial functions for solving the problem:

2. A method was proposed for calculating the boundary value problem of a static, geometric or mixed type of homogeneous and nonhomogeneous boundary conditions.

Thus, the solution of a boundary problem with nonhomogeneous conditions (static, geometric or mixed type) at the edges $\varphi = 0$ and $\varphi = \varphi_a$ can be obtained by imposing a particular solution satisfying all the boundary conditions set at these edges and a general solution of the corresponding homogeneous boundary problem where, according to the method of initial functions, solutions of some nonhomogeneous boundary-value problems for anisotropic bodies of sealing composite materials are constructed (Vlasov, 1960; Kutz, 2011; Babanli *et al.*, 2016; Mammadov, 2016).

Let us consider a packing O-ring having cylindrical coordinates (r, φ, z) with eccentric holes (Fig. 1).

Let us write differential (Grin *et al.*, 1965; Hasanov, 2016; Aslanov *et al.*, 2017) equations of equilibrium (in the absence of mass forces) and the Hooke's equation for the plane problem of the theory of elasticity in the form:

$$\begin{aligned} \frac{\partial(r\sigma_r)}{\partial r} - \sigma_\varphi + \frac{\partial\tau_{r\varphi}}{\partial\varphi} &= 0; \quad \frac{\partial(r\tau_{r\varphi})}{\partial r} + \tau_{r\varphi} + \frac{\partial\sigma_\varphi}{\partial\varphi} = 0; \\ \frac{\partial u}{\partial r} &= b_{11}\sigma_r + b_{12}\sigma_\varphi; \quad \frac{1}{r} \cdot \frac{\partial u}{\partial\varphi} + \frac{u}{r} = b_{21}\sigma_r + b_{22}\sigma_\varphi; \\ \frac{1}{r} \cdot \frac{\partial u}{\partial\varphi} + \frac{\partial\vartheta}{\partial r} - \frac{\vartheta}{r} &= \frac{1}{G_{12}} \cdot \tau_{r\varphi} \end{aligned} \quad (1)$$

Fig. 1. The design scheme of the stress-deformed state of an eccentric composite packing ring.

where $\sigma_r(r, \varphi)$; $\sigma_\varphi(r, \varphi)$ and $\tau_{r\varphi}(r, \varphi)$ are the stress tensor components; and u and v are the displacement vector components corresponding to the coordinate directions r and φ . For the coefficients of elasticity $b_{ij}(i, j = 1, 2)$ we have:

$$b_{11} = \frac{1}{E_1}; \quad b_{22} = \frac{1}{E_2}; \quad b_{12} = b_{21} = \frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}.$$

In the case of plane deformation

$$b_{11} = \frac{(1 - \nu_{13}\nu_{31})}{E_1}; \quad b_{22} = \frac{(1 - \nu_{23}\nu_{32})}{E_2}; \quad b_{12} = b_{32} = \frac{\nu_{12} + \nu_{13}\nu_{32}}{E_1}.$$

Here through $E_i(i = 1, 2)$, G_{12} and $\nu_{ij}(i, j = 1, 2, 3; i \neq j)$ we denote the tensile and compression modulus, shear modulus and Poisson's ratio; indices 1, 2, 3 at technical elastic constants correspond to main elastic directions r, φ, z (Kupradze *et al.*, 1976; Natroshvili *et al.*, 1978).

Let us write a new variable τ connected with variable r by formula:

$$\tau = \ln \rho; \quad \rho = \frac{r}{R} e^\tau \quad (2)$$

where ρ is a dimensionless radius (current radius related to the external radius).

For simplification let us write

$$\xi = \frac{1}{\sqrt{b_{11}b_{22}}}; \quad \theta = \sqrt{b_{22}/b_{11}}; \quad \delta = 1/\theta; \quad \nu = \xi\sqrt{b_{12}b_{21}};$$

$$\chi = (\xi/G_{12} - 2\nu)/2. \quad (3)$$

and instead of $u, \sigma_r, \sigma_\varphi, \tau_{r\varphi}$ we will further consider dimensionless geometric and statistic parameters

$$\begin{aligned} \phi_1 &= u/R; \quad \phi_2 = \vartheta/R; \quad \phi_3 = b_{11}\rho\sigma_\varphi; \quad \phi_4 = b_{11}\rho\tau_{r\varphi}; \\ \phi_5 &= b_{11}\rho\sigma_r \end{aligned} \quad (4)$$

Moreover, let us write symbols of differentiation on variables τ and φ :

$$\alpha^n = \partial^n / \partial \tau^n; \quad \beta^n = \partial^n / \partial \varphi^n. \quad (5)$$

Considering relations (2), (5) we will rewrite system of equations (1) in the form:

$$\begin{aligned} \alpha\phi_5 - \phi_3 + \beta\phi_4 &= 0; \quad (\alpha + 1)\phi_4 + \beta\phi_3 = 0; \\ \alpha\phi_1 &= \phi_5 - \nu\theta\phi_3 = 0; \\ \beta\phi_2 + \phi_1 &= \theta(\theta\phi_3 - \nu\phi_5); \\ \beta\phi_1 + (\alpha - 1)\phi_2 &= 2\theta(\chi + \nu)\phi_4. \end{aligned} \quad (6)$$

FINDINGS

General solution of the boundary task of inhomogeneous conditions (static, geometric or mixed type) on the edges of packer $\varphi = 0$ and $\varphi = \varphi_a$ can be obtained by finding the solution satisfying all set up boundary conditions at the edges and the general solution corresponding to initial boundary task. The solution can be determined by the method shown in the system of equations (1) where solu-

tions of some nonhomogeneous boundary tasks for an isotopic object (packer) have been set up due to the method of initial functions.

TASK SOLUTION

Applying the symbolic (operational) Lurie's method (Lurye, 1970; Golubev *et al.*, 1986; Gurbanov, 1991) by integrating the system (6) under the initial conditions on the plane $\varphi = 0$

$$\phi_j(\tau, 0) = \phi_j^0, (j = 1, 2, 3, 4) \quad (7)$$

we will get basic dependences of the method of initial functions in the form of the following transformation of the initial functions ϕ_j^0 into the required ones:

$$\phi_i(\tau, \varphi) = \sum_{j=1}^4 L_{ij}(\alpha, \varphi) \phi_j^0 \quad (i = 1, 2, 3, 4). \quad (8)$$

We determine the operators $L_{ij}(\alpha, \varphi)$ by formulae:

$$\begin{aligned} L_{11} &= \frac{C_1 P_1 - C_2 P_2}{K}; L_{12} = \frac{(\alpha - 1)(S_2 P_2 / K_2 - S_1 P_1 / K_1)}{K}; \\ L_{13} &= \frac{P_1 P_2 \cdot (C_1 - C_2)}{[K \alpha^2 (\alpha + 1)]}; L_{14} = \frac{(S_1 P_1 Q_1 / K_1 - S_2 P_2 Q_2 / K_2)}{(K \alpha^2)}; \\ L_{21} &= \frac{(S_2 Q_1 K_2 - S_1 Q_2 K_1)}{K}; L_{22} = \frac{(C_2 Q_1 - C_1 Q_2)}{K}; \\ L_{23} &= \frac{(S_2 P_1 Q_1 K_2 - S_1 P_2 Q_2 K_1)}{K}; L_{24} = \frac{Q_1 Q_2 (C_1 - C_2)}{[K \alpha^2 (\alpha - 1)]}; \\ L_{32} &= \frac{\alpha^2 (\alpha^2 - 1)(S_1 / K_1 - S_2 / K_2)}{K}; L_{33} = \frac{(C_2 P_1 - C_1 P_2)}{K}; \\ L_{34} &= \frac{(\alpha + 1)(S_2 Q_2 / K_2 - S_1 Q_1 / K_1)}{K}; \\ L_{41} &= \alpha^2 (S_2 K_2 - S_1 K_1) K; L_{42} = \frac{\alpha^2 (\alpha - 1)(C_2 - C_1)}{K}; \\ L_{51} &= \frac{\alpha(C_2 T_1 - C_2 T_2)}{K}; L_{52} = \frac{\alpha(\alpha - 1)(S_2 T_2 K_2 - S_1 T_1 K_1)}{K}; \\ L_{53} &= \frac{(C_1 P_2 T_1 - C_2 P_1 T_2)}{[K \alpha (\alpha + 1)]}; \\ L_{54} &= \frac{(S_1 Q_1 T_1 / K_1 - S_2 Q_2 T_2 / K_2)}{K \alpha}. \end{aligned} \quad (9)$$

where it is indicated

$$\begin{aligned} K_i &= \left[1 + \chi(\alpha^2 + (-1)^i \alpha) \sqrt{1 + 2\chi\theta + \theta^2 + (\chi^2 - 1)\theta^2 \alpha^2} \right]^{\frac{1}{2}}; \\ Q_i &= K_i^2 + (\alpha - 1)(1 + \nu\theta\alpha); P_i = K_i^2 - (\alpha - 1)(1 - \nu\theta\alpha); \\ T_i &= K_i^2 - (\alpha - 1); S_i = \sin K_i \cdot \varphi; (i = 1, 2); K = K_1^2 - K_2^2. \end{aligned}$$

ANALYSIS OF RESULTS

Introducing the conditions of correspondence between the operators $\alpha - 1$ and $\alpha + 1$, P_i , and Q_i , it can be noted that the

formulas for the operators L_{ij} located symmetrically with respect to the secondary diagonal of the transformation matrix (8), go one into the other:

$$L_{31} \leftrightarrow L_{42}; L_{21} \leftrightarrow L_{43}; L_{11} \leftrightarrow L_{44}; L_{221} \leftrightarrow L_{33}; L_{12} \leftrightarrow L_{34};$$

$$L_{13} \leftrightarrow L_{24}.$$

This property of operators S_i and C can be presented into series of:

$$\begin{aligned} S_i &= \sum_{n=0}^{\infty} (-1)^n \frac{(K_i \varphi)^{2n+1}}{(2n+1)!} \cdot \beta^{n+1}; \\ C_i &= 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(K_i \varphi)^{2n}}{(2n)!} \cdot \beta^{2n} \end{aligned} \quad (10)$$

Using dependencies (8) and (10) for the desired functions, we can obtain expressions in the form of power series in the angular coordinate φ , the coefficients of which are the values of various combinations of operators $\alpha, \alpha \pm 1, K, K_i, P_i, Q_i$ and T_i over the initial functions. In turn, the operators K, K_i, P_i, Q_i and T_i can be formally decomposed into rows as ordinary irrational functions from α .

SOLUTION FOR ANOTHER CONDITION

Let us suppose that on $\varphi = 0$ and $\varphi = \varphi_a$ a set of homogeneous boundary conditions (geometric, static or mixed) is given as

$$\phi_{\partial} = \phi_n = 0 \text{ in } \varphi = 0; \quad (11)$$

$$\phi_p(r, \varphi) = \phi_q(\tau, \varphi_a) = 0 \text{ in } \varphi = \varphi_a. \quad (12)$$

Let us indicate two unknown initial function ϕ_s^0 and ϕ_i^0 without concretising values of the indices g, h, p and q .

Considering condition (11) instead of dependence (8) we have

$$\phi_i = L_{is}(\alpha, \varphi) \phi_s^0 + L_{it}(\alpha, \varphi) \phi_t^0 \quad (i = 1, 2, 3, 4, 5). \quad (13)$$

From the dependence (13) considering condition (12) we get the system of two ordinary differentials transcendent equations relatively to ϕ_s^0 and ϕ_i^0 .

$$\begin{aligned} L_{ps}(\alpha, \varphi_a) \phi_s^0 + L_{pt}(\alpha, \varphi_a) \phi_t^0 &= 0; \\ L_{qs}(\alpha, \varphi_a) \phi_s^0 + L_{qt}(\alpha, \varphi_a) \phi_t^0 &= 0. \end{aligned} \quad (14)$$

Putting the decision function $F(\tau)$ by the formulas

$$\phi_s^0 = L_{ps}(\alpha, \varphi_a) F; \phi_t^0 = -L_{ps}(\alpha, \varphi_a) F \quad (15)$$

we will bring the system (14) to a resolving equation

$$L_{qs}(\alpha, \varphi_a) L_{pt}(\alpha, \varphi_a) - L_{qt}(\alpha, \varphi_a) \cdot L_{ps}(\alpha, \varphi_a) F = 0. \quad (16)$$

Considering formula (15) instead of the dependence (13), we get

$$\phi_i = [L_{is}(\alpha, \varphi) L_{pt}(\alpha, \varphi_a) - L_{it}(\alpha, \varphi) L_{ps}(\alpha, \varphi_a)] F. \quad (17)$$

The solution of equation (16) we find in the form $F = e^{\lambda\tau}$. To determine λ we have the characteristic transcendental equation

$$L_{qs}(\lambda, \varphi_a)L_{pt}(\alpha, \varphi_a) - L_{qt}(\lambda, \varphi_a) = 0. \quad (18)$$

We will obtain the equation of specific boundary value problems solutions (18) by assigning the corresponding values to the indices g, h, p, q, s, t. These equations are given in Table 1, where

$$K_{1,2} = \left[1 + \chi\theta\lambda^2 = \lambda\sqrt{(1 + 2\chi(\theta + \theta^2) + (\chi^2 - 1)\theta^2\lambda^2)^{1/2}} \right]$$

$$\omega_{1,2} = k_1 \pm k_2; \varepsilon_{1,2} = 2k_1k_2(\chi + \nu) \pm \theta(\lambda^2 - 1)(1 - \nu^2).$$

Applying in expression (17) the general integrals for F corresponding to the characteristic numbers of a particular boundary value problem, it is not difficult to obtain the corresponding general integrals for ϕ_i ($i = 1, 2, 3, 4, 5$).

However, in the case of a boundary value problem with homogeneous static conditions at the edges $\varphi = 0$ and $\varphi = \varphi_a$, the characteristic equation (18) has four multiple zero roots $\lambda(1, 2, 3, 4)$ and two roots $\lambda = \pm 1$, which, unlike other roots, do not depend on the angle φ_a . The general integral for F , determined by these roots is:

$$F = A_1 + A_2\tau + A_3\tau^2 + A_4\tau^3 + A_2\tau^4 + A_6\tau^5 \quad (19)$$

When substituting integral (19) into expression (17), the part of the solution corresponding to arbitrary constants A_1 and A_2 is lost as a result of differentiation.

Therefore, in determining common integrals ϕ_i corresponding to multiple zero characteristic numbers, we should proceed directly from dependence (13) and equation (14). Representing in system (14) transcendental operators in the form of series, restricted in these series only the first members, we obtain two independent differential equations for the unknown initial functions ϕ_1^0 and ϕ_2^0 :

$$\alpha^r \phi_1^0 = 0; \alpha^r \phi_2^0 = 0.$$

From these equations we find

$$\phi_1^0 = A_1 + A_2\tau; \phi_2^0 = A_3 + A_4\tau, \quad (20)$$

where A_i ($i = 1, 2, \dots$) are arbitrary constants. Substituting the expression (20) into the dependence (13) we get the integrals for ϕ_i , corresponding to the four multiple zero roots of equation (18). We find integrals for ϕ_i determined by the roots $\lambda_{5,6} = \pm 1$. As a result from the equation (17) for ϕ_i , meeting elementary typical numbers $\lambda_{1,2,\dots,6}$, we write

Table 1

Boundary condition	$\varphi = 0$	$\varphi = \varphi_a$	$L_{qs}(\lambda, \varphi_a)L_{pi}(\lambda, \varphi_a) - L_{qt}(\lambda, \varphi_a)L_{ps}(\lambda, \varphi_a) = 0$
1	$\mathcal{G} = \tau_{r\varphi} = 0$	$\mathcal{G} = \tau_{r\varphi} = 0$	$\theta^2\lambda^2 - 1/k_1k_2 \cdot \sin k_1\varphi_0 \sin k_2\varphi_a = 0$
2	$u = \sigma_\varphi = 0$	$u = \sigma_\varphi = 0$	$\frac{\lambda^2 - 1}{k_1k_2} \sin k_1\varphi_a \sin k_2\varphi_a = 0$
3	$u = \sigma_\varphi = 0$	$\mathcal{G} = \tau_{r\varphi} = 0$	$\cos k_1\varphi_a \cos k_2\varphi_a = 0$
4	$\mathcal{G} = \tau_{r\varphi} = 0$	$u = \sigma_\varphi = 0$	$\cos k_1\varphi_a \cos k_2\varphi_a = 0$
5	$\sigma_\varphi = \tau_{r\varphi} = 0$	$\tau_{r\varphi} = \sigma_\varphi = 0$	$\frac{\lambda^4(\lambda^2 - 1)}{k_1k_2} \cdot \sum_{i=1}^2 (-1)^i \left(\frac{1}{\omega_i} \sin \frac{\omega_2\varphi_a}{2} \right)^2 = 0$
6	$\mathcal{G} = \tau_{r\varphi} = 0$	$\sigma_\varphi = \tau_{r\varphi} = 0$	$\lambda^2 \sum_{i=1}^2 \frac{1}{\omega_i} \sin \omega_i \varphi_a = 0$
7	$\sigma_\varphi = \tau_{r\varphi} = 0$	$\mathcal{G} = \tau_{r\varphi} = 0$	$\lambda^2 \sum_{i=1}^2 \frac{1}{\omega_i} \sin \omega_i \varphi_a = 0$
8	$u = \sigma_\varphi = 0$	$\sigma_\varphi = \tau_{r\varphi} = 0$	$\frac{\lambda^2(\lambda^2 - 1)}{k_1k_2} \sum_{i=1}^2 (-1)^i \frac{1}{\omega_i} \sin \omega_i \varphi_a$
9	$\sigma_\varphi = \tau_{r\varphi} = 0$	$u = \sigma_\varphi = 0$	
10	$u = \mathcal{G} = 0$	$u = \sigma_\varphi = 0$	$\frac{1}{k_1k_2} \sum_{i=1}^2 \frac{\varepsilon_i}{\omega_i} \sin \omega_i \varphi_a = 0$
11	$u = \sigma_\varphi = 0$	$u = \mathcal{G} = 0$	
12	$u = \mathcal{G} = 0$	$\mathcal{G} = \tau_{r\varphi} = 0$	$\frac{1}{\lambda^2 - 1} \sum_{i=1}^2 (-1)^i \frac{\varepsilon_i}{\omega_i} \sin \omega_i \varphi_a = 0$
13	$\mathcal{G} = \tau_{r\varphi} = 0$	$u = \mathcal{G} = 0$	
14	$u = \mathcal{G} = 0$	$u = \mathcal{G} = 0$	$\frac{1}{k_1k_2(\lambda^2 - 1)} \sum_{i=1}^2 (-1)^i \left(\frac{\varepsilon_i}{\omega_i} \sin \frac{\omega\varphi_a}{2} \right)^2 = 0$
15	$u = \mathcal{G} = 0$	$\sigma_\varphi = \tau_{r\varphi} = 0$	$\frac{\lambda^2}{k_1k_2} \sum_{i=1}^2 \varepsilon_i \left(\frac{1}{\omega_i} \sin \frac{\omega_i \varphi_a}{2} \right)^2 - 1 = 0$
16	$\sigma_\varphi = \tau_{r\varphi} = 0$	$u = \mathcal{G} = 0$	

$$\begin{aligned}
\phi_1 &= \frac{A_1 \cos \varphi + A_2 (\theta_1 \varphi \sin \varphi + \ln \rho \cos \varphi) + A_3 \sin \varphi -}{\rho} \\
&\quad \frac{-A_4 [(\theta_2 - \ln \rho) \sin \varphi + \theta_1 \varphi \cos \varphi] + A_6 \theta_3 [\cos \theta_3 \varphi -}{\rho} \\
&\quad \frac{-\cos \theta_3 (\varphi - \varphi_a)]}{\rho}, \\
\phi_2 &= \frac{-A_1 \sin \varphi + A_2 [\theta_1 \varphi \cos \varphi - (\theta_2 + \ln \rho) \sin \varphi] +}{\rho} \\
&\quad \frac{+A_3 \cos \varphi + A_2 [(\theta_1 \varphi \cos \varphi - (\theta_2 + \ln \rho) \sin \varphi) + A_3 \cos \varphi +}{\rho} \\
&\quad \frac{+A_4 (\theta_1 \sin \varphi + \ln \rho \cos \varphi + A_5 \rho + 2A_6 [\theta_1 [\sin \theta_3 (\varphi - \varphi_a) -}{\rho} \\
&\quad \frac{-\sin \theta_3 \varphi] + (\theta_1 - \theta_3^2 / 4) \sin \theta_3 \varphi_a \}}{\rho}, \\
\phi_3 = 0; \phi_4 &= \frac{A_6 [\sin \theta_3 (\varphi - \varphi_a) - \sin \theta_3 \varphi + \sin \theta_3 \varphi_a]}{\rho}, \\
\phi_5 &= \frac{A_2 \cos \varphi + A_4 \sin \varphi + A_6 \theta_3 \cdot [\cos \theta_2 (\varphi - \varphi_a) -}{\rho} \\
&\quad \frac{-\cos \theta_3 \varphi]}{\rho},
\end{aligned} \tag{21}$$

where

$$\theta_1 = (1 - \nu\theta) / 2; \theta_2 = (1 + \nu\theta) / 2; \theta_3 = \sqrt{2(1 + \chi\theta)}.$$

For the remaining homogeneous boundary value problems whose characteristic equations have real roots independent

of φ_a , the corresponding elementary solutions are given in Table 2.

These solutions, including those determined by multiple zero characteristic numbers, are in a common way. In the case of boundary conditions 7 or 9 (see Table 1) in the corresponding formulas of Table 2, φ should be replaced by $\varphi - \varphi_a$. It is not difficult to find out the mechanical meaning of separate parts of the obtained elementary solutions. For example, in the general solution (21), the part of this solution defined by the constants A_1, A_3 and A_5 corresponds to rigid displacements and rotations in the plane (ρ, φ) , the other part of the solution with constants A_2 and A_4 corresponds to the solution for the sealing ring with an eccentric hole, loaded by a lateral concentrated force, and finally, part of the solution with constant A_6 corresponds to the solution for a packing ring with an excentric hole loaded by a concentrated bending moment. The angle-dependent characteristic numbers φ_a of boundary problems with homogeneous mixed conditions at the edges $\varphi = 0$ and $\varphi - \varphi_a$ are determined from ratios

$$k_{1,2} = d_n \quad (n = 1, 2, \dots, \infty),$$

where in the case of boundary conditions 1 or 2 (see Table 1) $d_n = \pi n / \varphi_a$; in the case of boundary conditions 3 and 4 (see Table 1) $d_n = \pi(n - \frac{1}{2}) / \varphi_a$. Hence, we find the following values of characteristic numbers:

$$\lambda_{1,2,3,4} = \pm(\delta_n \pm \sqrt{\delta_{1n} \delta_{2n}})^{\frac{1}{2}} / \sqrt{2\theta} \tag{22}$$

where it is indicated

$$\delta_n = 2\chi d_n^2 + \theta + \delta;$$

$$\delta_{1n} = \delta_n - 2(d_n^2 - 1) = (\sqrt{\theta} - \sqrt{\delta})^2 + 2d_n^2(1 + \chi);$$

Table 2

$\varphi = 0$	$\varphi = \varphi_a$	$\lambda_1 \phi_i (i = 1, 2, 3, 4, 5)$
$\mathcal{G} = 0$ $\tau_{r\varphi} = 0$	$\mathcal{G} = 0$ $\tau_{r\varphi} = 0$	$\lambda_{1,2} = \pm \delta; \phi_1 = A_1(1 - \nu)\rho^\delta + A_2(1 + \nu)/\rho^\delta$ $\phi_2 = \phi_4 = 0; \phi_3 = \delta^2(A_1\rho^\delta + A_2/\rho^\delta)$ $\phi = \delta(A_1\rho^\delta - A_2/\rho^\delta)$
$u = 0$ $\sigma_\varphi = 0$	$u = 0$ $\tau_{r\varphi} = 0$	$\lambda_{1,2} = \pm 1; \phi_1 = \phi_3 = \phi_5 = 0$ $\phi_2 = A_{1p} + \theta(\chi + \nu)A_2/\rho; \phi_4 = -A_2/\rho$
$\mathcal{G} = 0$ $\tau_{r\varphi} = 0$	$\sigma_\varphi = 0$ $\tau_{r\varphi} = 0$	$\lambda_{1,2} = 0; \phi_1 = (A_1 + A_2 \ln \rho) \cos \varphi + \theta_1 A_2 \varphi \sin \varphi$ $\phi_2 = -(A_1 + A_2 \ln \rho) \sin \varphi - (\theta_2 \sin \varphi - \theta_1 \varphi \cos \varphi) A_2$
$\sigma_\varphi = 0$ $\tau_{r\varphi} = 0$	$\mathcal{G} = 0$ $\tau_{r\varphi} = 0$	$\phi_3 = \phi_4 = 0; \phi_5 = A_2 \cos \varphi$
$u = 0$ $\sigma_\varphi = 0$	$\sigma_\varphi = 0$ $\tau_{r\varphi} = 0$	$\lambda_{1,2} = 0; \lambda_{3,4} = \pm 1;$ $\phi_1 = (A_1 + A_2 \ln \rho) \sin \varphi - (\theta_2 \sin \varphi + \theta_1 \varphi \cos \varphi) A_2 + (2A_4 \theta_3 \sin \theta_3 \varphi / \rho);$ $\phi_2 = (A_1 + A_2 \ln \rho) \cos \varphi + \theta_1 A_2 \varphi \sin \varphi + A_3 \rho + A_4 [4\theta_1 \cos \theta_3 \varphi + (\theta_3^2 - 4\theta_1) \cos \theta_3 \varphi_a] / \rho$
$\sigma_\varphi = 0$ $\tau_{r\varphi} = 0$		$\phi_3 = 0; \phi_4 = 2A_4 (\cos \theta_3 \varphi - \cos \theta_3 \varphi_a) / \rho;$ $\phi_5 = A_2 \sin \varphi - (2A_4 \theta_3 \sin \theta_3 \varphi) / \rho$

$\delta_{2n} = \delta_n - 2(\sqrt{\theta} + \sqrt{\delta}) / \sqrt{2(1-\chi)}$, we have complex characteristic numbers $\lambda_{n1,2,3,4} = \pm(b_n \pm ic_n)$,

where

$$n_n = \frac{1}{2} \sqrt{\delta_{1n} / \theta}; c_n = \frac{1}{2} \sqrt{-\delta_{2n} / \theta}; i = \sqrt{-1}.$$

General integrals for ϕ_i ($i = 1, 2, 3, 4, 5$) corresponding to these characteristic numbers are represented according to dependence (17) in the form

$$\begin{aligned} \phi_i = & \sum_{n=1}^{\infty} \sum_{j=1}^4 A_{ij} [L_{is}(\lambda_{nj}, \varphi) L_{pt}(\lambda_{nj}, \varphi_a) - \\ & - L_{it}(\lambda_{nj}, \varphi) L_{ps}(\lambda_{nj}, \varphi_a)] \cdot \rho^{\lambda_{nj}} + \\ & + \sum_m \sum_{k=1}^2 A_{mk} [L_{is}(\lambda_{mk}, \varphi) L_{pt}(\lambda_{mk}, \varphi_a) - \\ & - L_{it}(\lambda_{mk}, \varphi) L_{ps}(\lambda_{mk}, \varphi_a)] \cdot \rho^{\lambda_{mk}} \end{aligned} \quad (23)$$

where A_{nj}, A_{mk} are arbitrary constants.

CONCLUSIONS

1. The conclusion obtained by the method of initial functions solution satisfies exactly the system of equations (1) and boundary conditions at the edges $\varphi = 0$ and $\varphi = \varphi_a$.

2. If in the solution obtained, the functions from φ are orthogonal, then the boundary conditions at the edges $\rho_1 = a$ and $\rho_2 = b$ can be satisfied exactly, otherwise the boundary conditions at the edges bounding the body in the direction ρ are approximately satisfied.

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BLĪVĒJUMA GREDZENA SPRAIGI DEFORMĒTAIS STĀVOKLIS AR EKSCENTRISKAJIEM CAURUMIEM

Rakstā pētīti blīvējuma gredzena spraiģi deformētie stāvokļi ar blīvējuma materiālu ekscentriskajiem caurumiem. Kompozītmateriālos šim nolūkam izmanto Guka vienādojumu. Izmantojot Lurijas simbolisko metodi, ir iegūts konkrēts risinājums blīvējuma problēmai ar ekscentriskiem caurumiem.