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BANKING AND MONEY IN AN EXTENDED SOLOW-UZAWA'S NEOCLASSICAL GROWTH MODEL

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Abstract:

This study deals with interactions between economic growth and structural change with banking. The study is influenced by the growth model of money and banking by Chang et al. (2007). It deviates from Chang et al.' model with regard to the monetary authority behavior, economic structure, and modeling behavior of household. The model deals with dynamic interactions between money, banking, economic structural change and growth in a perfectly competitive economy. The economic system consists of one capital goods sector, one consumer goods sector, and one banking sector. The two goods sectors are based on the Solow-Uzawa growth model. The motion is described by a set of differential equations. For illustration, we simulate the motion of the economic system. We identify the existence of a stable equilibrium point. We carry out comparative dynamic analysis. The comparative analyses provide some insights into the complexity of economic growth with banking.

Key words: *money; banking; neoclassical growth theory; propensity to save; reserve requirement*

1. Introduction

The purpose of this study is to introduce banking into a growth economic theory with economic structure. It is concerned with effects of money on economic growth. Many of the most intriguing and important questions in dynamic economic analysis involve money. It is generally agreed that modern analysis of dynamic interaction of inflation and capital formation begins with Tobin's seminal contribution in 1965. Tobin (1965) deals with an isolated economy in which "outside money" competes with real capital in the portfolios of agents within the framework of the Solow model. A monetary economy is characterized by that prices are expressed in money, transactions require money, and financial wealth can be held in the form of money or financial instruments competing with money. In Tobin's approach, a monetary economy has a real sector exactly like that in the Solow growth model, so that the monetary nature of the model depends on how money is introduced into the model. Since Tobin's pioneering analytical work, many approaches have been proposed within the literature of monetary growth theory (Patinkin, 1965, Sidrauski, 1967, Friedman, 1969, Wang and Yip, 1992, Gomme, 1993, van der Ploeg and Alogoskoufis, 1994, Jones and Manuelli, 1995, Chang and Lai, 2000, Dotsey and Starte, 2000, Jha et al., 2002; Chu and Cozzi, 2014; Gennaioli, et al., 2018; Oikaw and Ueda, 2018; and Chu et al.,

2019). It is well known that Schumpeter (1939) emphasizes the role of “credit creation” on economic development. Robinson (1969) also points out that financial services may affect investment decisions by enabling firms to get financing sources. In most of the literature of monetary economic growth the role of banking is not explicitly analyzed.

There are some studies which incorporate a banking sector in modeling economic dynamics (Greenwood and Jovanovic, 1990, Bencivenga and Smith, 1991, and Levine, 2002; Ennis, 2018; Magill, et al., 2019). These studies show how financial intermediaries may affect economic growth and development. Nevertheless, as observed by Chang et al. (2007), the literature of the growth of monetary policy had little concerns over “(i) exploring the role of an active banking sector played in evaluating the long-run growth effects of monetary policy and (ii) providing a more complete characterization of local and global dynamics so as to help assess how money supply and banking policies affect macroeconomic performance in a short-run transition.” We deal with similar issues like Chang et al. to provide insights into the issues not well examined in the literature. The purpose of this study is to examine dynamics of money and wealth growth with a banking sector. As far as banking is concerned, this paper is much inspired by a growth model of money and banking by Chang et al. (2007). In their model money is assumed to provide liquidity services to facilitate transactions and banks convert non-reserve deposits into productive capital. The model is concerned with long- and short-run effects of changes in the exogenous money growth rate and the reserve requirement ratio. Following Chang et al., we consider that money provides liquidity services to facilitate transactions and banks convert non-reserve deposits into productive input. This study deviates from the approach by Chang et al. in a few aspects. We assume that the speed is proportional to the economic growth rate. In Wang et al., it is assumed that the government issues money in an exogenously fixed speed. Instead of the Ramsey approach in modeling household behavior, this study accepts the approach proposed by Zhang (1993). We introduce banks into the two-sector growth model (e.g., Zhang, 2012, 2013) rather than in the one-sector framework. The paper is organized as follows. Section 2 develops the growth model with a banking sector. Section 3 examines dynamic properties of the model and simulates the model. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study.

2. The Basic Model

We consider an economic system consisting of capital goods, consumer goods, and banking sectors. The capital goods and consumer goods sectors are the same as in the Uzawa two sector model (Uzawa, 1961; Burmeister and Dobell 1970; Barro and Sala-i-Martin, 1995; and Zhang, 2009). Services are classified as consumer goods. Capital goods are used as inputs in the three sectors. Capital depreciates at a constant exponential rate δ_k ($0 < \delta_k < 1$), which is independent of the manner of use. Households own assets of the economy. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. The population $\bar{N}$ is homogenous. Let prices be measured in terms of capital

goods and the price of the commodity be unity. We denote the wage rates by $w(t)$. Let $p(t)$ denote price of consumer goods. The total capital stock $K(t)$ is allocated between the three sectors. We use subscript index, i, s , and b to stand for capital goods, consumer goods, and banking sectors, respectively. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector m . We consider $K_m(t)$ to be rented from households. The consumer and capital goods sectors convert bank loans, $X_i(t)$ and $X_s(t)$, into productive inputs. We assume that one unit of bank loan can be converted into τ_m unit of productive factor for sector m , $m = i, s$. We use $r(t)$ to stand for real rental rate which firms pay households and $R(t)$ for nominal loan rate which firms pay commercial banks.

The capital goods sector

Let $F_m(t)$ stand for the production function of sector m , $m = i, s, b$. The production function of capital goods sector is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t) (\tau_i X_i(t))^{\gamma_i}, \alpha_i, \beta_i, \gamma_i > 0, \alpha_i + \beta_i + \gamma_i = 1, (1)$$

where A_i , α_i , β_i and γ_i are positive parameters. It should be noted that in Wang et al. (2007), the production function of a single production sector is specified as

$$F_i(t) = A_i (K_i(t) + \tau_i X_i(t)).$$

As the financial sector employs no labor and there is a single production sector, the labor (which is normalized to be unity) disappears in the model by Wang et al. The capital consists of those rented from household and those converted from bank loans. In our approach, rather than use bank loans as substitutes of capital rented from households, we consider bank loans as different input factors. The marginal conditions for the capital goods sector are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, R(t) = \frac{\gamma_i F_i(t)}{X_i(t)}. (2)$$

Consumer goods sector

The production functions of the two sectors are

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t) (\tau_s X_s(t))^{\gamma_s}, \alpha_s + \beta_s + \gamma_s = 1, \alpha_s, \beta_s, \gamma_s > 0, (3)$$

where A_s , α_s , β_s and γ_s are the parameters of the sector. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_s p(t) F_s(t)}{K_s(t)}, w(t) = \frac{\beta_s p(t) F_s(t)}{N_s(t)}, R(t) = \frac{\gamma_s p(t) F_s(t)}{X_s(t)}. \quad (4)$$

Banking sector

Banks are operated in perfectly competitive markets. For simplicity we assume banks use physical capitals $K_b(t)$ to provide banking services. Banks convert household deposits $D(t)$ into loans. Nominal deposit rate $r_b(t)$ is determined in markets. Banks' profits are

$$\pi_b(t) = R(t) X(t) - r_b(t) D(t) - (r(t) + \delta_k) K_b(t), \quad (5)$$

where $X(t) = X_i(t) + X_s(t)$.

Let $\gamma(t)$ stand for banks' reserve ratio. We have the following balance sheet relationship

$$(1 - \gamma(t))D(t) = X(t), 0 < \gamma(t) < 1. \quad (6)$$

The equation implies loans equaling non-reserve deposits. There are some studies on effects of reserve requirement policy on economic growth (e.g., Tobin, 1969; Bhattacharya, 1997; Haslag and Young, 1998; Espinosa-Vega and Yip, 1999; Galati and Moessner, 2013; Chowdhury and Schabert, 2008; Chari and Phelan, 2014; and Chang et al., 2019). Different from the traditional approaches, this study introduces heterogeneity in $X(t)$ in the two-sector neoclassical growth model. We use $\bar{M}(t)$ to stand for real cashable deposits. We have:

$$\bar{M}(t) = \gamma(t)D(t). \quad (7)$$

Consider that the monetary authority requires the reserve requirement ratio γ_R to be maintained by banks. The reserve-deposit ratio consists of the reserve requirement ration and the excess reserve ration $\gamma_E(t)$. That is

$$\gamma(t) = \gamma_R + \gamma_E(t), 0 < \gamma_R, \gamma_E(t), \gamma(t) < 1, \quad (8)$$

The bank's efficiency has impact on excess reserve ratio. It is assumed

$$\gamma_E(t) = \varphi \left(\frac{K_b(t)}{D(t)} \right), \varphi' < 0, \varphi'' > 0. \quad (9)$$

This implies that the bank efficiency is dependent on resource per unit of deposits at a diminishing rate of return.

From (6) and (7) we have (5) as follows

$$\pi_b(t) = (R(t) - R(t)\gamma_R - r_b(t) - R(t)\gamma_E(t))D(t) - (r(t) + \delta_k)K_b(t).$$

Banks maximize profits with capital as the sole decision variable. The marginal conditions are

$$\varphi' \left(\frac{K_b(t)}{D(t)} \right) = - \left(\frac{r(t) + \delta_k}{R(t)} \right). \quad (10)$$

Equation (10) enables us to determine $K_b(t)/D(t)$ as a function of $r(t)$ and $R(t)$. For simplicity of analysis we specify

$$\gamma_E(t) = \varphi_0 \left(\frac{K_b(t)}{D(t)} \right)^{-1/2}, \quad \varphi_0 > 0,$$

where φ_0 is a positive parameter. We thus solve (10):

$$K_b(t) = D(t) \left(\frac{\varphi_0 R(t)}{2(r(t) + \delta_k)} \right)^{2/3}. \quad (11)$$

Perfect competition results in zero profit. From (10) we determine the equilibrium deposit rate as

$$r_b(t) = R(t) - R(t)\gamma_R - R(t)\gamma_E(t) - (r(t) + \delta_k) \frac{K_b(t)}{D(t)}. \quad (12)$$

Current income and disposable income

In this study, we use the approach to modeling behavior of households by Zhang (1993). To simplify asset holding structure and make the analysis easily tractable, it is assumed that all households withdraw cashable deposits and instantaneously use the entire amount for purchasing consumer goods. Nevertheless, in order to analyze the role of money in facilitating transactions, it is assumed that there is a real resource cost of cash withdrawals. As specified in Chang et al. (2007: 1689), "In principle, one may consider each household's asset portfolio to have three components: physical capital, (real) deposits, and (real) money. It is, however, difficult (if not impossible) to characterize such a dynamical system. To reduce the dimensionality of the dynamical system, we therefore assume that all households withdraw cashable deposits and instantaneously use the entire amount for consumption purchases. That is, there are no household money hoardings at any instant in time...". We follow Chang et al. in simplifying portfolio structure. Let $\bar{k}(t)$ stand for per capita wealth j . We have $\bar{k}(t) = K(t)/\bar{N}$, where $K(t)$ is the total (capital) wealth held by the population. In order to model the role of money in transactions, Wang et

al. view cashable deposits as providing liquidity services to individual transactions. It is considered that the real resource cost of cash withdrawals is positively related to the ratio of consumption purchases to cashable deposits. This cost is further assumed to be proportional to the household's rental income $r(t)\bar{k}(t)$ with the unit resource cost $\eta(t)$ specified as follows:

$$\eta(t) = 1 - \frac{ap(t)c(t)}{\bar{m}(t)}, 0 < a < 1, (13)$$

where $\bar{m}(t) \equiv \bar{M}(t)/\bar{N}$ and a is a parameter. It should be noted that (13) is a simple form of the following general functional form specified by Wang et al.

$$\eta(t) = 1 - a \left(\frac{c(t)}{\bar{m}(t)} \right)^{-b}, 0 < b \leq 1.$$

For convenience of analysis, this study applies (13). The household current income $y(t)$ consists of the interest payment, returns from deposits, cost of liquidity services, cost of deposits due to inflation, real lump-sum transfers from the government $\tau(t)$, and the wage income

$$y(t) = r(t)\bar{k}(t) - \eta(t)r(t)\bar{k}(t) + r_b(t)d(t) - \pi(t)d(t) + w(t) + \tau(t),$$

where $d(t) \equiv D(t)/\bar{N}$. The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}(t) = y(t) + \bar{a}(t), (14)$$

where $\bar{a}(t) \equiv \bar{k}(t) + d(t)$ is the value of the total wealth consisting capital wealth and deposit.

Budget constraint

The disposable income is used for saving and consumption. The representative household would distribute the total available budget between savings $s(t)$ and consumer goods $c(t)$. The budget constraints are

$$p(t)c(t) + s(t) = \hat{y}_j(t). (15)$$

Insert (14) and (13) in (15)

$$\bar{p}(t)c(t) + s(t) = \bar{y}(t), (16)$$

where

$$\begin{aligned}\bar{p}(t) &\equiv \left(1 - \frac{a r(t) \bar{k}(t)}{\bar{m}(t)}\right) p(t), \bar{y}(t) \\ &= \bar{a}(t) + r_b(t) d(t) - \pi(t) d(t) + w(t) + \tau(t).\end{aligned}$$

Utility function

We assume that utility level $U(t)$ is dependent on $c(t)$ and $s(t)$ as follows

$$U(t) = c^{\xi_0}(t) s^{\lambda_0}(t), \xi_0, \lambda_0 > 0 \geq 0,$$

where ξ_0 is the propensity to consume consumer goods and λ_0 the propensity to save.

Equilibrium in household's asset portfolio choice

The two assets should yield the equal return. That is

$$r(t) = r_b(t) - \pi(t). \quad (18)$$

Optimal household behavior

Maximizing the utility function subject to (9) yields:

$$\bar{p}(t)c(t) = \xi \bar{y}(t), \quad s(t) = \lambda \bar{y}(t), \quad (19)$$

where

$$\xi \equiv \rho \xi_0, \quad \lambda_j \equiv \rho \lambda_0, \quad \rho_j \equiv \frac{1}{\xi_0 + \lambda_0}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{a}}(t) = s(t) - \bar{a}(t). \quad (20)$$

This equation simply states that the change in wealth is equal to saving minus dissaving.

The monetary authority

In this model the government is the monetary authority. The monetary authority fully controls the monetary supply change rate and the reserve requirement ratio.

Money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money, $M(t) > 0$. The scheme according to which

the money stock evolves over time is deterministic and known to all agents. With $\mu(t)$ being the net growth rate of the money stock, $M_j(t)$ evolves over time according

$$\dot{M}(t) = \mu(t)M(t).$$

At t the government brings $\mu(t) M(t)$ additional units of money per capita into circulation in order to finance all government expenditures via seigniorage. It should be noted that in most of models in the theoretical literature of monetary growth theory $\mu(t)$ is assumed to be constant. It is exogenous and fixed over time (Zhang, 2009). The model by Wang et al. (2007) also uses this assumption. It is unrealistic to assume that money is infinite as the above equation implies: $M(t) = M_0 \exp(\mu t)$. In this type of models per capita consumption level is also infinite even in the saddle equilibrium point. As shown in many examples in Zhang (2009), the models with this type of government monetary policy tend to be unstable. Most of theoretical economic models in the literature of economic growth (based on the Ramsey approach) are characterized of saddle equilibrium points. This implies that when simulating the model with many variables, it is rare to follow the path of economic evolution with finite values of the variables as they tend to be infinite over time if the initial state is not located on a special path. Saddle-point models will not disturb traditional theoretical economists as they are only concerned with general analytical properties or equilibrium points (and corresponding static comparative analysis) of the models without putting data to simulate the models or conduct dynamic comparative analysis.

Let $m(t)$ stand for the real value of money per capita measured in units of the output good, that is, $m(t) = M(t)/P(t)$. The government expenditure in real terms per capita, $\tau(t)$, is given by

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu(t) M(t)}{P(t)} = \mu(t) m(t). \quad (21)$$

The representative household receives $\mu(t)m(t)$ units of paper money from the government. It should be noted that when $\mu(t)$, is negative, we interpret the term $\mu(t)m(t)$ as the government's lump-sum tax. By the definition of the monetary base and from (7), we have

$$m(t) = \gamma(t) d(t) = \bar{m}(t). \quad (22)$$

From (21) and the definition of inflation rate, we have

$$\frac{\dot{m}(t)}{m(t)} = \frac{\dot{M}(t)}{M(t)} - \frac{\dot{P}(t)}{P(t)} = \mu(t) - \pi(t).$$

We introduce the total payment to all the production factors by:

$$Y(t) = w(t) h \bar{N} + (r(t) + \delta_k)K(t) + R(t) X(t).$$

We assume that the real money growth rate is proportional to the change rate of $Y(t)$ as follows:

$$\mu(t) - \pi(t) = \mu_0 \frac{\dot{Y}(t)}{Y(t)}, \mu_0 > 0. (23)$$

If μ was fixed in our model, the system will also has a saddle point. Because of (23), we will demonstrate that the system has a locally stable equilibrium point. It is straightforward to generalize (23), for instance, by the following formation

$$\mu(t) - \pi(t) = f\left(\frac{\dot{Y}(t)}{Y(t)}\right).$$

As far as we can solve the equation function in the following form

$$\frac{\dot{Y}(t)}{Y(t)} = f^{-1}(\mu(t) - \pi(t)).$$

The model is still technically tractable. It should be noted that (23) is closely related to the quantity theory of money. A simple formation of the theory is summarized by the following (Fisher) equation:

$$M(t)V(t) = P(t)T(t),$$

where $M(t)$ is the money supply, $V(t)$ is velocity of circulation, $P(t)$ is the average price level, and $T(t)$ is the volume of transactions of goods and services. Take derivatives of the Fisher equation with regard to time

$$\frac{\dot{M}(t)}{M(t)} + \frac{\dot{V}(t)}{V(t)} = \frac{\dot{P}(t)}{P(t)} + \frac{\dot{T}(t)}{T(t)}.$$

Assuming a constant velocity of circulation implies (24). It should be noted that it is important to examine how the velocity is affected by other variables in the economic system. According to the Keynesian economics the velocity should be an endogenous variable.

Full employment of labor force

The total qualified labor supply N is $N = h \bar{N}$, in which h is the human capital. The assumption of labor force being fully employed implies

$$N_i(t) + N_s(t) = N. \quad (24)$$

Demand and supply of the capital and consumer goods sectors

The demand and supply equilibrium for the consumer goods sector is

$$c(t)\bar{N} = F_s(t). \quad (25)$$

As output of the capital goods sector is equal to the depreciation of capital stock and the investment, we have

$$\dot{K}(t) = F_i(t) - \delta_k K(t). \quad (26)$$

Capital being fully utilized

Total capital stock $K(t)$ is allocated to the three sectors

$$K_i(t) + K_s(t) + K_b(t) = K(t). \quad (27)$$

We completed the model. The model is obviously built on some strict assumptions. Nevertheless, it is structurally general in the sense that some well-known models in economics can be considered as its special cases. For instance, if there is no banking industry, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). The new aspect is the introduction of banking sector and the quantity theory of money into the neoclassical growth theory.

3. The dynamics and its properties

As the dynamic system is nonlinear and highly dimensional, it is difficult to get explicitly analytical properties. The following lemma shows that the dimension of the dynamical system is three. We also provide a computational procedure for calculating all the variables at any point in time. First, we introduce two new variables $z(t)$ and

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}, Z(t) \equiv \frac{r(t) + \delta_k}{R(t)}.$$

Lemma

The motion of the economic system is determined by 3 differential equations with $z(t)$, $Z(t)$, and $\bar{k}(t)$ as the variables

$$\begin{aligned}\dot{z}(t) &= \Psi_z(z(t), Z(t), \bar{k}(t)), \\ \dot{Z}(t) &= \Psi_Z(z(t), Z(t), \bar{k}(t)), \\ \dot{\bar{k}}(t) &= \Psi_{\bar{k}}(z(t), Z(t), \bar{k}(t)),\end{aligned}\quad (28)$$

in which $\Lambda_j(t)$ are unique functions of $z(t)$, $Z(t)$, and $\bar{k}(t)$ defined in the appendix. At any point in time the other variables are unique functions of $z(t)$, $Z(t)$, and $\bar{k}(t)$ determined by the following procedure: $r(t)$, $w(t)$, and $R(t)$ by (A2) $\rightarrow p(t)$ by (A3) $\rightarrow K(t) = \bar{k}(t)\bar{N}$ by (A3) $\rightarrow d_0(t)$ by (A5) $\rightarrow \gamma_E(t) = \varphi_0\Lambda^{-1/2}(t) \rightarrow \gamma(t)$ by (A7) $\rightarrow r_b(t)$ by (A6) $\rightarrow \pi(t)$ by (A8) $\rightarrow K_b(t)$ by (A14) $\rightarrow D(t)$ by (A15) $\rightarrow d(t) = D(t)/\bar{N} \rightarrow K_i(t)$ by (A13) $\rightarrow K_s(t)$ by (A12) $\rightarrow N_i(t)$ and $N_s(t)$ by (A1) $\rightarrow N_i(t)$ and $N_s(t)$ by (A2) $\rightarrow X(t) = X_i(t) + X_s(t)$ by (A2) $\rightarrow m(t)$ by (A17) $\rightarrow \bar{m}(t) = m(t) \rightarrow \mu(t)$ by (A19) $\rightarrow \tau(t) = \mu(t)m(t) \rightarrow \bar{a}(t) = \bar{k}(t) + d(t) \rightarrow \bar{p}(t)$ and $\bar{y}(t)$ by the definitions $\rightarrow c(t)$ and $s(t)$ by (A19) $\rightarrow F_j(t)$ by the definitions $\rightarrow Y(t)$ by the definition.

The lemma gives a computational procedure for following the motion of the economic system. As it is difficult to get explicit analytical solution, we simulate the model. We specify parameter values as follows:

$$\begin{aligned}\bar{N} = 10, h = 1.2, \lambda_0 = 0.7, \xi_0 = 0.15, A_i = 1.1, A_s = 1, \alpha_i = 0.33, \gamma_i = 0.04, \alpha_s \\ = 0.3, \\ \gamma_s = 0.03, a = 0.1, \gamma_R = 0.1, \varphi_0 = 0.1, \tau_i = 0.6, \tau_s = 0.7, \mu_0 = 0.4, \delta_k \\ = 0.06.\end{aligned}\quad (29)$$

The population is fixed 10 and human capital at 1.2. The capital goods and consumer goods sector's total productivities are respectively 1.1 and 1. We specify the values of the parameters, α_j , in the Cobb-Douglas productions for the capital goods and consumer goods sectors approximately equal to 0.3 (for instance, Miles and Scott, 2005; Abel et al., 2007). The depreciation rate of physical capital is specified at 0.05. We specify the initial conditions as follows

$$z(0) = 0.11, Z(0) = 0.41, \bar{k}(0) = 4.65.$$

Due to the fixed positions of the initial state, the national output and wealth/capital rise over time. The output levels of the capital goods and consumer goods sector rise. The two sectors' capital and labor inputs are slightly changed. The capital input of the banking sector is slightly changed. The national deposit $D(t)$ and total loans $X(t)$ are increased.

The two sectors use more loans. The real rental rate of capital $r(t)$ falls and the wage rate rises. The price of consumer goods is changed slightly. The bank reserve ratio $\gamma(t)$ rises and then falls. The inflation policy $\mu(t)$ and inflation rate $\pi(t)$ initially rise and then fall. The nominal loan rate $R(t)$ and the nominal deposit rate $r_b(t)$ fall. The real money $m(t)$ rises initially and then changes slightly. The total wealth $\bar{a}(t)$, physical wealth $\bar{k}(t)$, and deposit $d(t)$ are all increased over time. The government expenditure on the household $\tau(t)$ is positive. The consumption level $c(t)$ initially falls and then rises.

Figure 1. The Motion of the Economic System

It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms that the system has an equilibrium. We list the equilibrium values in (30)

$$Y = 16.9, K = 47.17, D = 2.98, X = 2.07, M = 0.91, F_i = 2.83, F_s = 14.38, N_i = 1.71,$$

$$N_s = 10.3, K_i = 7.54, K_s = 38.92, K_b = 0.71, X_i = 0.39, X_s = 1.68, r = 0.064, w = 1.05,$$

$$p = 1.12, R = 0.29, \gamma = 0.31, \gamma_E = 0.21, \mu = 0.11, \pi_0 = 0.11, m = 0.09, \tau = 0.01,$$

$$d = 0.3, c = 1.44. (30)$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-141179.4, -2.85, -0.11\}.$$

The eigenvalues are real and negative. The equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis. It should be remarked that the system has at least another equilibrium point with the following value:

$$z = 0.93, Z = 15, \bar{k} = 0.66.$$

The equilibrium point is unstable. We simulated the model near this equilibrium point. The system moves to the stable equilibrium point in (30).

4. Comparative Dynamic Analysis

We simulated the motion of the national economy under (29). It is significant to examine how the economic system reacts to exogenous changes. As the lemma provides the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $\bar{\Delta}x_j(t)$ which stands for the change rate of the variable $x_j(t)$ in percentage due to changes in the parameter value.

A rise in the reserve requirement ratio

We now examine the effects of the following change in the reserve requirement ratio: $\gamma_R: 0.1 \Rightarrow 0.11$. The simulation result is plotted in Figure 2. The bank reserve ratio is enhanced. The nominal loan rate and the nominal deposit rate are increased. The inflation policy and inflation rate are enhanced. The national deposit and total loans are reduced. The national output falls. The wealth/capital rises initially and remains almost unchanged in the long term. The capital goods and consumer goods sectors employ less loans. The real rental rate of capital, price of consumer goods and wage rate fall. The real money rises initially and falls in the long term. The household's total wealth rises initially and falls in the long term. The physical wealth rises initially and changes slightly in the long term. The deposit falls and the government expenditure rises. The consumption level falls.

Figure 2. A Rise in the Reserve Requirement Ratio

A fall in the bank efficiency

We now study what happens to the economic system when the bank efficiency is falling in the following way: $\varphi_0: 0.1 \Rightarrow 0.15$. The simulation result is plotted in Figure 3. The bank reserve ratio is enhanced. The nominal loan rate and the nominal deposit rate are increased. The inflation policy and inflation rate are enhanced. The national deposit and total loans are reduced. The national output falls. The wealth/capital rises initially and falls in the long term. The capital goods and consumer goods sectors employ less loans. The real rental rate of capital, price of consumer goods and wage rate fall. The real money rises initially and falls in the long term. The household's total wealth and physical wealth rise initially and fall in the long term. The deposit rises initially and falls in the long term. The government expenditure rises. The consumption level falls. We see that the effects in the rise in the reserve requirement ratio in the previous case are qualitatively similar to the effects in the falling in the bank efficiency.

Figure 3. A Fall in the Bank Efficiency

An improvement in the total factor productivity of the capital goods sector

We now examine changes in the path of economic growth when the total factor productivity of the capital goods sector is improved as follows: $A_i: 1.1 \Rightarrow 1.15$. The simulation result is plotted in Figure 4. The capital goods sector produces more output and employs more capital goods and has more loans from the banks. Some of the labor force is initially shifted to the capital goods sector from the consumer goods sector. In the long term there is almost no change in the labor distribution. Initially the consumer goods sector produces less output and employs less capital goods. In the long term the sector produces more output and employs more capital goods. The consumer goods sector has more loans from the banks. The national income, total wealth, total deposit and total loans are augmented. The banks employ more capitals and have more businesses. The real rental rate of capital, price of consumer goods and wage rate are increased. The bank reserve ratio is enhanced initially and is almost not affected in the long term. The nominal loan rate and the nominal deposit rate are increased initially and are almost not affected in the long term. The inflation policy and inflation rate are lowered initially and are almost not affected. The real money, total wealth and physical wealth rise. The deposit and the government expenditure rise. The consumption level falls initially and rises in the long term.

Figure 4. An Improvement in the Total Factor Productivity of the Capital Goods Sector

More efficient utilization of loans by the capital goods sector

We now study what happens to the economic system when the capital goods sector more effectively utilize the banks' loans in the following way: $\tau_i: 0.6 \Rightarrow 0.7$. The simulation result is plotted in Figure 5. It is straightforward to see that the effects in the rise in the more efficient utilization of loans by the capital goods sector should be qualitatively similar to the effects in the improvement in the sector's total factor productivity.

Figure 5. More Efficient Utilization of Loans by the Capital Goods Sector

A fall in the capital goods sector's output elasticity of financial input factor
We now examine changes in the path of economic growth when the capital goods sector's output elasticity of financial input factor is reduced as follows: $\gamma_i: 0.04 \Rightarrow 0.035$. The simulation result is plotted in Figure 6. The capital goods sector uses less loans. It produces more output and employs more capital goods. Some of the labor force is shifted to the capital goods sector from the consumer goods sector. Initially the consumer goods sector produces less output and employs less capital goods. In the long term the sector produces more output and employs more capital goods. The consumer goods sector has

more loans from the banks. The national income, total wealth, and total deposit are augmented. The total loan is increased slightly. The banking sector employ less capitals initially and more in the long term. The real rental rate of capital rises initially and falls in the long term. The price of consumer goods and wage rate are increased. The bank reserve ratio is enhanced. The nominal loan rate and the nominal deposit rate are reduced. The inflation policy and inflation rate are lowered. The real money is increased. The national income and national wealth are increased. The household's total wealth and physical wealth rise. The deposit rises and the government expenditure falls. The consumption level falls initially and rises in the long term.

Figure 6. A Fall in the Capital Goods Sector's Output Elasticity of Financial Input Factor

A rise in the household's propensity to save

We now examine changes in the path of economic growth when the household's propensity to save is enhanced as follows: $\lambda_0: 0.7 \Rightarrow 0.72$. The simulation result is plotted in Figure 7. The household accumulates more physical wealth and total wealth. The real money is reduced. The real rental rate of capital is reduced. The price of consumer goods and wage rate are increased. The consumption level falls initially and rises in the long term. The capital goods sector produces more output and employs more capital goods and has more loans from the banks. Some of the labor force is shifted to the capital goods sector from the consumer goods sector. Initially the consumer goods sector produces less output and employs less capital goods. In the long term the sector produces more output and employs more capital goods. The consumer goods sector has less loans from the banks. The national income and total wealth are increased. The total deposit falls initially and rises in the long term. The total loans fall initially and rise slightly in the long term. The banks employ more capitals. The bank reserve ratio is lowered. The nominal loan rate and the nominal deposit rate are increased.

Figure 7. A Rise in the Household's Propensity to Save

5. Concluding Remarks

This study deals with interactions between economic growth and structural change with banking. It was influenced by the growth model of money and banking constructed by Chang et al. (2007). Our model deviates from Chang et al.' model with regard to the monetary authority behavior, economic structure, and modeling behavior of household. Rather than assuming a fixed money growth rate, this study assumes that the money growth rate is proportional to the growth rate of the national product. Rather than a simplified version of the AK model, this study uses the neoclassical two-sector growth model to more properly deal with the impact of the banking sector on the economic structural change. This study uses Zhang's approach to behavior of the household. The model deals with dynamic interactions between money, banking, economic structural change and growth in a perfectly competitive economy. The economic system consists of one capital goods sector, one consumer goods sector, and one banking sector. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system. We identified the existence of two equilibrium points. We carry out comparative dynamic analysis with regard to the stable equilibrium point. The comparative analyses provide some insights into the complexity of economic growth with banking. For instance, we demonstrate how a rise in the reserve requirement ratio affects the transitory and long-term economic growth as follows: (i) the bank reserve ratio, nominal loan rate, nominal deposit rate, inflation policy and inflation rate being enhanced; (ii) the national deposit and total loans being reduced; (iii) the national output being reduced, the national wealth/capital being augmented initially and being almost unaffected in the long term; (iv) the capital goods and consumer goods sectors utilizing less loans; (v) the real rental rate of capital, price of consumer goods and wage rate being reduced; the real money being increased initially and reduced; (vi) the deposit being reduced and the government expenditure being enhanced; and (vii) the household's total wealth being enhanced initially and reduced in the long term, the physical wealth being increased initially and affected slightly in the long term, and The consumption level being lowered. We may relax the assumption about the liquidity service role of money. This can be done by either accepting

money in utility approach or cash-in-advance approach. We may also make the behavior of the monetary authority more realistic.

Appendix: Proving the lemma

By (2) and (4), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s}, \quad (A1)$$

$$Z \equiv \frac{r + \delta_k}{R} = \frac{X_i}{\bar{\gamma}_i K_i} = \frac{X_s}{\bar{\gamma}_s K_s}, \quad (A2)$$

where

$$\bar{\beta}_m \equiv \frac{\beta_m}{\alpha_m}, \bar{\gamma}_m \equiv \frac{\gamma_m}{\alpha_m}.$$

Insert (A1) and (A2) in (2)

$$r(z, Z) = \alpha_i \theta - \delta_k, w(z, Z) = \frac{\beta_i \theta}{\bar{\beta}_i z}, R(z, Z) = \frac{\gamma_i \theta}{\bar{\gamma}_i Z}, \quad (A2)$$

where we also use (1) and

$$\theta(z, Z) \equiv A_i \bar{\beta}_i^{\beta_i} \tau_i^{\gamma_i} \bar{\gamma}_i^{\gamma_i} z^{\beta_i} Z^{\gamma_i}.$$

From (3) and (4), we have

$$p(z, Z) = \frac{w(\bar{\beta}_s z)^{1-\beta_s}}{\beta_s A_s \tau_s^{\gamma_s} (\bar{\gamma}_s Z)^{\gamma_s}}. \quad (A3)$$

By (4) we get:

$$X_s = \frac{\gamma_s}{\alpha_s} \left(\frac{r + \delta_k}{R} \right) K_s. \quad (A4)$$

From (11) we have

$$\frac{K_b}{D} = d_0(z, Z) \equiv \left(\frac{\varphi_0 R}{2(r + \delta_k)} \right)^{\frac{2}{3}}. \quad (A5)$$

From (A5) and (12) we have

$$r_b(z, Z) = R - R \gamma_R - R \varphi_0 d_0^{-\frac{1}{2}} - (r + \delta_k) d_0. \quad (A6)$$

From (8) we have

$$\gamma(z, Z) = \gamma_R + \varphi_0 d_0^{-1/2}. \quad (A7)$$

By (18) we have

$$\pi(z, Z) = r_b - r. \quad (A8)$$

By (25) and (19)

$$\frac{\xi \bar{N} \bar{y}}{\bar{p}} = \frac{w N_s}{\beta_s p}, \quad (A9)$$

where we also use (4). Insert (A2) in $(1 - \gamma)D = X$

$$(1 - \gamma)D = (\bar{\gamma}_i K_i + \bar{\gamma}_s K_s)Z. \quad (A10)$$

Insert (A5) in (A10)

$$K_b = d_i K_i + d_s K_s, \quad (A11)$$

where

$$d_i(z, Z) \equiv \frac{\bar{\gamma}_i d_0 Z}{1 - \gamma}, d_s(z, Z) \equiv \frac{\bar{\gamma}_s d_0 Z}{1 - \gamma}.$$

Insert (A1) in (24)

$$K_s = \frac{N}{\bar{\beta}_s z} - \frac{\bar{\beta}_i K_i}{\bar{\beta}_s}. \quad (A12)$$

Insert (A11) and (A12) in (27)

$$K_i = \tilde{d}_i K - \tilde{d}_s, \quad (A13)$$

where

$$\tilde{d}_i(z, Z) \equiv \left((1 + d_i) - \frac{(1 + d_s) \bar{\beta}_i}{\bar{\beta}_s} \right)^{-1}, \quad \tilde{d}_s(z, Z) \equiv \frac{(1 + d_s) \tilde{d}_i N}{\bar{\beta}_s z}.$$

From (A11)-(A13) we solve

$$K_b = \tilde{d}_{1i} K - \tilde{d}_{1s}, \quad (A14)$$

where

$$\tilde{d}_{1i} \equiv \left(d_i - \frac{\bar{\beta}_i d_s}{\bar{\beta}_s} \right) \tilde{d}_i, \quad \tilde{d}_{1s} \equiv \left(d_i - \frac{\bar{\beta}_i d_s}{\bar{\beta}_s} \right) \tilde{d}_s - \frac{d_s N}{\bar{\beta}_s z}.$$

From (A10) and (A14) we have

$$D(z, Z) = \frac{\tilde{d}_{1i}}{d_0} K - \frac{\tilde{d}_{1s}}{d_0}. \quad (A15)$$

From (22)

$$m(z, Z) = \bar{m} = \frac{\gamma D}{\bar{N}}. \quad (A16)$$

From (A9) and (A1) we have

$$\frac{\xi \bar{N} \bar{y}}{\bar{p}} = \frac{w \bar{\beta}_s z K_s}{\beta_s p}. \quad (A17)$$

Insert (A12) and (A13) in (A17)

$$\frac{\bar{y}}{\bar{p}} = u_0 - u_1 K, \quad (A18)$$

where

$$u_0(z, Z) \equiv \frac{w z}{\xi \bar{N} \beta_s p} \left(\frac{N}{z} + \bar{\beta}_i \tilde{d}_s \right), \quad u_1(z, Z) \equiv \frac{\bar{\beta}_i w z \tilde{d}_i}{\xi \bar{N} \beta_s p}.$$

Insert the definitions of $\bar{y}$ and $\bar{p}$ in (A18)

$$\mu = (u_0 - u_1 \bar{k} \bar{N}) \left(1 - \frac{a r \bar{k}}{m} \right) \frac{p}{m} - \frac{\bar{a} + r_b d - \pi d + w}{m}. \quad (A19)$$

It is straightforward to confirm that all the variables can be expressed as functions of z , Z , and $\bar{k}$ by the following procedure: r , w , and R by (A2) $\rightarrow p$ by (A3) $\rightarrow K = \bar{k} \bar{N}$ by (A3) $\rightarrow d_0$ by (A5) $\rightarrow \gamma_E = \varphi_0 \Lambda^{-1/2} \rightarrow \gamma$ by (A7) $\rightarrow r_b$ by (A6) $\rightarrow \pi$ by (A8) $\rightarrow K_b$ by (A14) $\rightarrow D$ by (A15) $\rightarrow d = D/\bar{N} \rightarrow K_i$ by (A13) $\rightarrow K_s$ by (A12) $\rightarrow N_i$ and N_s by (A1) $\rightarrow N_i$ and N_s by (A2) $\rightarrow X = X_i + X_s$ by (A2) $\rightarrow m$ by (A17) $\rightarrow m = \bar{m} \rightarrow \mu$ by (A19) $\rightarrow \tau = \mu m \rightarrow \bar{a} = \bar{k} + d \rightarrow \bar{p}$ and $\bar{y}$ by the definitions $\rightarrow c$ and s by (A19) $\rightarrow F_j$ by the definitions $\rightarrow Y$ by the definition. From this procedure (20), (23) and (26) we have

$$\dot{\bar{a}} = s - \bar{a}, \quad (A20)$$

$$\dot{Y} = \Psi_Y(z, Z, \bar{k}) \equiv (\mu - \pi) \frac{Y}{\mu_0}, \quad (A21)$$

$$\dot{\bar{k}} = \Psi_{\bar{k}}(z, Z, \bar{k}) \equiv \frac{F_i}{\bar{N}} - \delta_k \bar{k}. \quad (A22)$$

From (A22) and $\bar{a} = \bar{k} + d$, we rewrite (A20) as follows

$$\dot{d} = \Psi_d(z, Z, \bar{k}) \equiv s - \bar{a} - \Psi_{\bar{k}}. \quad (A23)$$

By the above procedure we already have

$$Y = \Phi_Y(z, Z, \bar{k}), d \equiv \Phi_d(z, Z, \bar{k}). \quad (A24)$$

Here we don't give explicit expressions as they are tedious. Taking derivatives of (A24) with respect to time yields

$$\begin{aligned} \frac{\partial \Phi_Y}{\partial z} \dot{z} + \frac{\partial \Phi_Y}{\partial Z} \dot{Z} &= \Psi_Y - \Psi_{\bar{k}} \frac{\partial \Phi_Y}{\partial \bar{k}}, \\ \frac{\partial \Phi_d}{\partial z} \dot{z} + \frac{\partial \Phi_d}{\partial Z} \dot{Z} &= \Psi_d - \Psi_{\bar{k}} \frac{\partial \Phi_d}{\partial \bar{k}}. \end{aligned} \quad (A25)$$

This is a linear equations system with $\dot{z}$ and $\dot{Z}$ as the variables. Solve (A25)

$$\dot{z} = \Psi_z(z, Z, \bar{k}), \dot{Z} = \Psi_Z(z, Z, \bar{k}). \quad (\text{A26})$$

The differential system (A22) and (A26) has 3 variables, z, Z and $\bar{k}$ and consists of 3 differential equations. In summary, we proved the lemma.

6. References

- Abel, A., Bernanke, B.S., and Croushore, D. (2007) *Macroeconomics*. New Jersey: Prentice Hall.
- Barro, R.J. and X. Sala-i-Martin (1995) *Economic Growth*. New York: McGraw-Hill, Inc.
- Bhattacharya, J. and Guzman, M.G., Huybens, E., and Smith, B.D. (1997) Monetary, Fiscal, and Reserve Requirement Policy in a Simple Monetary Growth Model. *International Economic Review* 38 (2), 321-50
- Bencivenga, V.R. and Smith, B.D. (1991) Financial Intermediation and Endogenous Growth. *Review of Economic Studies* 58, 195–209.
- Burmeister, E. and Dobell, A.R. (1970) *Mathematical Theories of Economic Growth*. London: Collier Macmillan Publishers.
- Chang, C., Liu, Z., Spiegel, M.M., and Zhang, J.Y. (2019) Reserve Requirements and Optimal Chinese Stabilization Policy. *Journal of Monetary Economics* 103(May), 33-51.
- Chang, J.J., Chang, W.Y., Lai, C.C., and Wang, P. (2007) Equilibrium Dynamics in an Endogenous Growth Model of Money and Banking. *Journal of Money, Credit and Banking* 39 (7), 1683-710.
- Chang, W.Y., and Lai, C.C. (2000) Anticipated Inflation in a Monetary Economy with Endogenous Growth. *Economica* 67, 399–417.
- Chari, V.V. and Phelan, C. (2014) On the Social Usefulness of Fractional reserve Banking. *Journal of Monetary Economics* 65(July), 1-13.
- Chowdhury, I. and Schabert, A. (2008) Federal Reserve Policy viewed Through a Money Supply Lens. *Journal of Monetary Economics* 55(May), 825-34.
- Chu, A. and Cozzi, G. (2014) R&D and Economic Growth in a Cash-in-Advance. *International Economic Review* 55, 507-24.
- Chu, A., Ning, L., and Zhu, D. (2014) Human Capital and Innovation in a Monetary Schumpeterian Growth Model. *Macroeconomic Dynamics* 23(5), 1875-94.
- Dotsey, M. and Sarte, P.D.G. (2000) Inflation Uncertainty and Growth in a Cash-in-Advance Economy. *Journal of Monetary Economics* 45, 631–55.
- Ennis, H.M. (2018) A Simple General Equilibrium Model of Large Excess Reserves. *Journal of Monetary Economics* 98(October), 50-65.
- Espinosa-Vega, M. and Yip, C.K. (1999) Fiscal and Monetary Policy Interactions in an Endogenous Growth Model with Financial Intermediaries. *International Economic Review* 40, 595–615.
- Friedman, M. (1969) The Optimum Quantity of Money, in *The Optimum Quality of Money and Other Essays*, edited by Friedman, M. Chicago: Aldine.
- Gennaioli, N., Martin, A., and Rossi, S. (2018) Banks, Government Bonds, and Default: What Do the Data Say? *Journal of Monetary Economics* 112(June), 113-28.
- Gomme, P. (1993) Money and Growth Revisited. *Journal of Monetary Economics* 32, 51–77.
- Greenwood, J. and Jovanovic, B. (1990) Financial Development, Growth, and the Distribution of Income. *Journal of Political Economy* 98, 1076–107.
- Galati, G. and Moessler, R. (2013) Macro Prudential Policy – A Literature Review. *Journal of Economic Survey*, 27, 846–878.

- Haslag, J. and Young, E. (1998) Money Creation, Reserve Requirements and Seigniorage. *Review of Economic Dynamics* 1, 677–98.
- Jha, S. K., Wang, P., and Yip, C.K. (2002) Dynamics in a Transactions-Based Monetary Growth Model. *Journal of Economic Dynamics and Control* 26, 611–35.
- Jones, L. E. and Manuelli, R.E. (1995) Growth and the Effects of Inflation. *Journal of Economic Dynamics and Control* 19, 1405–28.
- Levine, R. (2002) Bank-Based or Market-Based Financial Systems: Which is Better? *Journal of Financial Intermediation* 4, 398–428.
- Magill, M., Quinzii, M., and Rochet, J.C. (2019) The Safe Asset, Banking Equilibrium, and Optimal Central Bank Monetary, Prudential and Balance-Sheet Policies. *Journal of Monetary Economics* 112(June), 113-28.
- Miles, D. and Scott, A. (2005) *Macroeconomics – Understanding the Wealth of Nations*. Chichester: John Wiley & Sons, Ltd.
- Oikawa, K. and Ueda, K. (2018) The Optimal Inflation Rate Under Schumpeterian Growth. *Journal of Monetary Economics* 100(December), 114-25.
- Patinkin, D. (1965) *Money, Interest, and Prices: An Integration of Monetary and Value Theory*, 2nd edition. New York: Harper & Row.
- Robinson, J. (1969) *The Accumulation of Capital*, 3rd ed. London: Macmillan.
- Sidrauski M (1967) Rational Choice and Patterns of Growth in a Monetary Economy. *American Economic Review* 57, 534-44.
- Schumpeter, J.A. (1939) *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*. New York: McGraw-Hill.
- Solow, R. (1956) A Contribution to the Theory of Growth. *Quarterly Journal of Economics* 70, 65-94.
- Tobin, J. (1965) Money and Economic Growth. *Econometrica* 33, 671-84.
- Tobin, J. (1969) A General Equilibrium Approach To Monetary Theory. *Journal of Money, Credit and Banking* 1(1), 15-29.
- Wang, P. and Yip, C.K. (1992) Examining the Long-Run Effect on Economic Growth. *Journal of Macroeconomics* 14, 359-69.
- Uzawa, H. (1961) On a Two-Sector Model of Economic Growth. *Review of Economic Studies* 29, 47-70.
- van der Ploeg, F. and Alogoskoufis, G.(1994) Money and Endogenous Growth. *Journal of Money, Credit, and Banking* 26, 771–91.
- Zhang, W.B. (1993) Woman's Labor Participation and Economic Growth - Creativity, Knowledge Utilization and Family Preference. *Economics Letters* 42, 105-10.
- Zhang, W.B. (2009) *Monetary Growth Theory: Money, Interest, Prices, Capital, Knowledge, and Economic Structure over Time and Space*. London: Routledge.
- Zhang, W.B. (2012) Economic Growth with Heterogeneous Households, Gender Division of Labor, and Elastic Labor Supply. *Journal of Income Distribution* 21, 15-37.
- Zhang, W.B. (2013) A Synthesis of the Uzawa-Lucas Model with the Walrasian-General-Equilibrium and Neoclassical-Growth Theories. *Economic Annals* 58, 8-38.