EFFECTS OF SURFACE ROUGHNESS AND SUPPLY INERTIA ON STEADY PERFORMANCE OF HYDROSTATIC THRUST BEARINGS LUBRICATED WITH NON-NEWTONIAN FLUIDS

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Abstract: The objective of present theoretical analysis is to study the combined effects of surface roughness and fluid inertia (including the inertia of the fluid in the supply region) on the steady performance of stepped circular hydrostatic thrust bearings lubricated with non-Newtonian pseudoplastic fluids using Rabinowitsch stress-strain model. To account for the effects of surface roughness, the classical Christensen theory of rough surface has been taken. Analytic expressions for film pressure in bearing regions have been established for radial and circumferential patterns of roughness. Numerical results for film pressure, load carrying capacity and lubricant flow rate has been plotted and analysed. Due to surface roughness and fluid inertia in overall regions, significant improvement in performance properties have been observed.

KEYWORDS: Hydrostatic lubrication, pressurized bearings, thrust bearings, surface roughness, Rabinowitsch stress fluids.

1 Introduction

The performance of bearings depends upon different geometric, operating and lubrication conditions. The disk break, crank and camshaft, wheel bearings, disk drives, telescope bases, rocket launcher, axel soft, turbine generators and heavy machine bearings, everywhere there is need of bearings for smooth working of systems [1]. The radius of bearing, design of bearings, fluid supply designs are the examples of geometric conditions. The speed of bearings, vibrations control, fluid film force, are some examples of operating conditions [2]. Nature of the lubricants and additive such as viscosity, thermal stability, density, film rupture, are some examples of the lubrication conditions [3]. Thus, depending upon the physical conditions, the need of lubrication systems led to invention of different types of bearings such as hydrodynamic, hydrostatic, squeeze film, sliding, rolling, bearings. Hydrostatic thrust bearings constitute a major class of externally pressurized bearings, and in this class, circular step thrust bearings received of aided importance in industrial and engineering applications due to large load sustaining capacity, longer life, and its wide applications in high-speed rotating machines. Theoretical as well as industrial researchers have paid their attention towards improvement of stability and performance of these bearings under various operating and lubricating conditions [4]–[8]. Numerous experimental as well as theoretical investigations have been carried out to analyse the dependence of different performance properties of these bearings on fluid inertia, compressibility and nature of contacting surface and other operating conditions. For theoretical researchers, experimental assessment of hydrostatic thrust bearings by [7] and theory for surface roughness of hydrodynamic bearings by Christensen [9] have been of great importance. Researchers have analysed results for pressure and load capacity of hydrostatic thrust bearings considering different aspects such as inertia effects, thermal effects, viscosity variation, non-Newtonian lubricants, and surface roughness effects. Yadav and Kapur [10] considered thermo-viscous and piezo-viscous effects

on hydrostatic stepped thrust bearings lubricated with Newtonian lubricants. Kapur and Verma [11] discussed the combined effects of thermal variation and fluid inertia. Lin [12] considered the static and dynamic characteristics of hydrostatic thrust bearings lubricated with couple stress fluids. Singh, Gupta and Kapur [13]–[15] presented annular hydrostatic thrust bearings lubricated with Rabinowitsch type fluids, and analyzed inertia effects on the performance of hydrostatic bearings. Many other authors also presented similar work for different fluids [16]–[19]. Resultant impacts of inertia and surface roughness on hydrostatic bearings have been also analyzed by Lin [20] and, Walicka and Walicki [21]. Peterson, Finn and Dareing [22] discussed the non-Newtonian, temperature, and inertia effects on lubricant slurry in hydrostatic step thrust bearings.

Researchers also worked on stabilizing lubrication characteristics of fluids by addition of viscosity index improver, [23]–[25]. Spikes [24] emphasized that the use of additives reduces the sensitivity of the lubricant to the change in the shearing strain rate with temperature, which changes lubricant's behaviour from Newtonian to non-Newtonian. To study the lubrication regimes with such lubricants, many non-Newtonian models such as couple stress, power law, micropolar, Ellis and Casson models are of common interest [12], [19]. Amongst these models, Rabinowitsch fluid model [26] fits the viscosity data of real fluids over a wide range of shear rate, which was verified by Wada and Hayashi [23] through an experiment with real lubricants blended with industrial additives. The stress-strain relation of the Rabinowitsch fluid model is given as:

$$\bar{\tau}_{rz} + \kappa \bar{\tau}_{rz}^3 = \bar{\mu} \frac{\partial \bar{u}}{\partial \bar{z}} \quad (1)$$

where $\bar{\mu}$ is the *zero shear rate viscosity* and κ is a coefficient accounting for the non-Newtonian nature of fluids. This model holds for dilatant, Newtonian and pseudoplastic fluids for $\kappa < 0$, $\kappa = 0$ and $\kappa > 0$ respectively. Recently many researchers [4], [27]–[32] used this model to analyse characteristics of different bearings lubricated with non-Newtonian fluids. Considering the present discussion, it is worth considering fluid inertia on supply region as well as on recess and land regions along with surface roughness and non-Newtonian characteristics of fluids to have a precise insight of performance of externally pressurized thrust bearings. The author has observed that none of the researchers have analysed the combined effects of fluid inertia (with special consideration to fluid inertia on supply hole), surface roughness, and non-Newtonian lubricants on performance characteristics of hydrostatic thrust bearings using a realistic fluid model.

In this work, it is proposed to study the combined effects of surface roughness, fluid inertia (including inertia of fluid in the supply region) on the performance of stepped hydrostatic thrust bearings lubricated with non-Newtonian fluids using Rabinowitsch fluid model.

2 Constitutive Equations

A schematic configuration of a hydrostatic thrust bearing with rough surfaces is shown in Fig. 1. It is assumed that the lubricant in the system is incompressible non-Newtonian fluid, the body forces and body couples are absent, and the assumptions of hydrodynamic lubrication are applicable to lubricant film. Under these assumptions, the governing equations for one dimensional, axially symmetric fluid flow in hydrostatic circular thrust bearings in dimensionless form [4], together with constitutive equation (1) can be written as:

$$\tau_{rz} + \alpha \tau_{rz}^3 = \mu \frac{\partial u}{\partial z} \quad (2)$$

$$-\frac{20}{3}S \frac{v^2}{r} + \frac{\partial p}{\partial r} = \frac{\partial \tau_{rz}}{\partial z} \quad (3)$$

$$0 = \mu \frac{\partial^2 v}{\partial z^2} \quad (4)$$

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) = 0 \quad (5)$$

These equations satisfy the boundary conditions $u=0$ at $z=0, \beta\hbar$; $v=0$ at $z=0$ and, $v=r\omega$ at $z=\beta\hbar$. The lubricant film thickness is taken as $\hbar = \beta h + h_s$, where h is the nominal smooth part of the film thickness, β is step ratio, h_s is the part of film thickness due to asperities on the surface measured from smooth level. In case of radial roughness, $h_s = h_s(\theta, \xi)$, and in case of transverse or circumferential roughness, $h_s = h_s(r, \xi)$; where, ξ is a random variable to characterize some definite arrangement of the surface asperities.

Fig. 1 A Schematic Diagram of Step Thrust Bearing with Rough Surfaces

3 Solution of the governing equations

From Eq. (4) and the boundary conditions $v(0)=0$ and, $v(\beta\hbar)=r\omega$, the expression for the circumferential velocity is calculated as $v=rz/\hbar$. Further, a close form solution to Eq. (3) satisfying the equation of continuity (5), the constitutive equation (2) and relevant boundary conditions without inertia effect is obtained as:

$$u = \frac{\delta}{2\mu} \left[(z^2 - z\beta\hbar)I + \alpha \left(\frac{z^4}{2} - z^3\beta\hbar + \frac{3}{4}z^2\beta^2\hbar^2 - \frac{1}{4}z\beta^3\hbar^3 \right) I^3 \right] \quad (6)$$

where $\hbar^3 I = -\frac{6\mu Q}{r} + \frac{162}{5}\alpha \frac{\mu^3 Q^3}{r^3 \hbar^4}$ [4] and $\hbar = \beta h + h_s$.

Multiplying both sides of Eq. (3) by $2\pi r$ and integrating with respect to $z(0, \hbar)$, the pressure gradient is obtained [4] as:

$$\frac{\partial p}{\partial r} = -\frac{1}{6\mu Q} \left[r \left(\hbar^3 I^2 + \frac{3\alpha}{20} \hbar^5 I^4 \right) + \frac{3\Omega}{10} r^2 \left(\hbar^3 I + \alpha I^3 \frac{13\hbar^5}{84} \right) \right] \quad (7)$$

where, $Q = \frac{\mu_o \bar{Q}}{\pi R^3 p_o} = 2 \int_0^{\bar{h}} r u d\bar{z}$ is dimensionless form of volumetric flow rate $\bar{Q} = 2\pi \int_0^{\bar{h}} r u d\bar{z}$.

The boundary conditions for eq. (7) are $p(r_o) = 1$ and $p(1) = 0$ together with the continuity of pressure at the step.

Probability density function of the stochastic film thickness is taken from Christensen's [9] theory of rough surfaces for hydrodynamic bearings:

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3; & -c < h_s < c \\ 0 & \text{; elsewhere} \end{cases} \quad (8)$$

where, c is the half range assumed by of the random film thickness variable ξ with the standard deviation σ . The function $f(h_s)$ terminates at $c = 3\sigma$. In the present analysis, c will be referred to as roughness parameter.

Taking expectancy function $E(\Theta)$ as:

$$E(\Theta) = \int_{-\infty}^{\infty} \Theta f(h_s) dh_s \quad (9)$$

stochastic form of eq. (7) can be written as:

$$\frac{\partial E(p)}{\partial r} = -\frac{1}{6\mu Q} E \left[r \left(\bar{h}^3 I^2 + \frac{3\alpha}{20} \bar{h}^5 I^4 \right) + \frac{3S}{10} r^2 \left(\bar{h}^3 I + \alpha I^3 \frac{13\bar{h}^5}{84} \right) \right] \quad (10)$$

or, in simplified notations:

$$\frac{\partial P}{\partial r} = -\frac{1}{6\mu Q} \left[r \left(H_3 F^2 + \frac{3\alpha}{20} H_5 F^4 \right) + \frac{3S}{10} r^2 \left(H_3 F + \frac{13}{84} \alpha H_5 F^3 \right) \right] \quad (11)$$

and $E(\bar{h}^3)F = -\frac{6\mu Q}{r} + \frac{162}{5} \alpha \frac{\mu^3 Q^3}{r^3 E(\bar{h}^4)}$; where, $F = E(I)$, $P = E(p)$, $H_n = E(\bar{h}^n)$ for longitudinal roughness and $H_n = 1/E(\bar{h}^{-n})$ circumferential roughness.

Integrating the equation (11) from 0 to r , general expression for the film pressure can be calculated as:

$$P(r) = -\frac{1}{6\mu Q} \left[\int_0^r r \left(H_3 F^2 + \frac{3\alpha}{20} H_5 F^4 \right) dr + \frac{3S}{10} \int_0^r r^2 \left(H_3 F + \frac{13}{84} \alpha H_5 F^3 \right) dr \right] + c_1 \quad (12)$$

where $c_1 = P(0)$ is the pressure at $r = 0$. Because the distribution of film pressure $P(r)$ is different in different regions and involves the volumetric flow rate Q , it is calculated systematically for recess and land regions with help of the pressure boundary conditions and the continuity of fluid flow at the step position. However, the expression (12) can serve without any modification if there are no steps, that is, the single region bearings.

4 Pressure Distribution

4.1 Pressure Distribution in the supply region

If the supply hole r_o is not sufficiently small or bearing is being operated at a high speed or step ratio β is large, the lubricant in the region $0 \leq r \leq r_o$ will be influenced by the rotational inertia due to which, film pressure will not be exactly equal to the supply pressure at r_o , but it will be equal to supply pressure at the point $r = 0$. Therefore, the condition for pressure at r_o needs to be derived. Now, without loss of generality, it can be assumed that at any instant of

time, the volume of lubricant (Q_s) contained in supply region is volume contained in the closed cylindrical surface of radius r and height βh . Therefore, Q_s will be of form $Q_s = \lambda r^2$ for some constant of proportionality λ , and the value of Q_o at r_o must be equal to the total volumetric flow rate Q . This gives $\lambda = \frac{Q}{r_o^2}$ and $Q_o = Q \frac{r^2}{r_o^2}$. Denoting the pressure in this region as P_o , Eq. (7) can be written as:

$$\frac{\partial P_o}{\partial r} = -\frac{1}{6\mu Q_o} \left[r \left(H_3 F^2 + \frac{3\alpha}{20} H_5 F^4 \right) + \frac{3S}{10} r^2 \left(H_3 F + \frac{13\alpha}{84} H_5 F^3 \right) \right] \quad (13)$$

which on integration with respect to $r(0, r)$ gives:

$$P_o(r) = 1 - \frac{r_o^2}{6\mu Q} \int_0^r \left[\frac{1}{r^2} \left(\beta^3 h^3 F^2 + \frac{3\alpha}{20} \beta^5 h^5 F^4 \right) + \frac{3S}{10} \left(\beta^3 h^3 F + \frac{13\alpha}{84} \beta^5 h^5 F^3 \right) \right] dr \quad (14)$$

$$\text{where } F = -\frac{6\mu Q}{rH_3} + \frac{162\alpha}{5} \frac{H_5}{H_3^4} \frac{\mu^3 Q^3}{r^3}.$$

The stochastic film thickness H_3 and H_5 for radial roughness are:

$$H_3 = h^3 \beta^3 + \frac{c^2 h \beta}{3}; (0 \leq r \leq r_o),$$

$$H_5 = h^5 \beta^5 + \frac{5c^4 h \beta}{33} + \frac{10c^2 h^3 \beta^3}{9}; (0 \leq r \leq r_o)$$

whereas H_3 and H_5 for circumferential roughness are:

$$H_3 = \frac{32c^7/35}{3(6c^2 h^2 \beta^2 - c^4 - 5h^4 \beta^4) \log \frac{(h\beta + c)}{(h\beta - c)} + 30ch^3 \beta^3 - 26c^3 h \beta}$$

$$H_5 = \frac{(32/35)(c^9 - c^7 h^2 \beta^2)}{3(c^4 - 6c^2 h^2 \beta^2 + 5h^4 \beta^4) \log \frac{(h\beta + c)}{(h\beta - c)} - 30ch^3 \beta^3 + 26c^3 h \beta}$$

such that in case of circumferential roughness patterns, $H_3 = h^3 \beta^3$, $H_5 = h^5 \beta^5$ as $c \rightarrow 0$.

Now, it is easy to substitute F , H_3 and H_5 and find the integral on right hand side of Eq.(14), but the final expression is not presented in this analysis due to its long length.

Further, integrating Eq. (11) with respect r under the conditions of continuity of pressure at r_o, r_1 and $P(1) = 0$, pressure P_R for recess ($r_o \leq r \leq r_1$) and P_L for land ($r_1 \leq r \leq 1$) regions, can be obtained as follows.

4.1 Pressure Distribution in the Recess region

$$P_R(r) = P_o(r_o) - \frac{1}{6\mu Q} \int_{r_o}^r \left[r \left(H_3 F^2 + \frac{3\alpha}{20} H_5 F^4 \right) + \frac{3S}{10} r^2 \left(H_3 F + \frac{13\alpha}{84} H_5 F^3 \right) \right] dr \quad (15)$$

$$\text{where } F = -\frac{6\mu Q}{rH_3} + \frac{162\alpha}{5} \frac{H_5}{H_3^4} \frac{\mu^3 Q^3}{r^3}.$$

The expression for H_3 and H_5 for radial roughness are:

$$H_3 = h^3 \beta^3 + \frac{c^2 h \beta}{3}; (r_o \leq r \leq r_1)$$

$$H_5 = h^5 \beta^5 + \frac{5c^4 h \beta}{33} + \frac{10c^2 h^3 \beta^3}{9}; (r_o \leq r \leq r_1)$$

whereas H_3 and H_5 for circumferential roughness are:

$$H_3 = \frac{32c^7}{35} \frac{1}{3(6c^2 h^2 \beta^2 - c^4 - 5h^4 \beta^4) \log \frac{(h\beta + c)}{(h\beta - c)} + 30ch^3 \beta^3 - 26c^3 h \beta}$$

$$H_5 = \frac{c^9 - c^7 h^2 \beta^2}{35/32} \frac{1}{3(c^4 - 6c^2 h^2 \beta^2 + 5h^4 \beta^4) \log \frac{(h\beta + c)}{(h\beta - c)} - 30ch^3 \beta^3 + 26c^3 h \beta}$$

such that in case of circumferential roughness patterns, $H_3 = h^3 \beta^3$, $H_5 = h^5 \beta^5$ as $c \rightarrow 0$.

4.2 Pressure Distribution in the Land region

$$P_L(r) = \frac{1}{6\mu Q} \int_r^1 \left[r \left(H_3 F^2 + \frac{3\alpha}{20} H_5 F^4 \right) + \frac{3S}{10} r^2 \left(H_3 F + \frac{13\alpha}{84} H_5 F^3 \right) \right] dr \quad (16)$$

$$\text{where } F = -\frac{6\mu Q}{rH_3} + \frac{162\alpha}{5} \frac{H_5}{H_3^4} \frac{\mu^3 Q^3}{r^3}.$$

The stochastic film thickness H_3 and H_5 for radial roughness are:

$$H_3 = h^3 + \frac{c^2 h}{3}; (r_o \leq r \leq r_1)$$

$$H_5 = h^5 + \frac{5c^4 h}{33} + \frac{10c^2 h^3}{9}; (r_o \leq r \leq r_1)$$

whereas H_3 and H_5 for circumferential roughness are:

$$H_3 = \frac{32c^7}{35} \frac{1}{3(6c^2 h^2 - c^4 - 5h^4) \log \frac{(h+c)}{(h-c)} + 30ch^3 - 26c^3 h}$$

$$H_5 = \frac{32c^7 (c^2 - h^2)}{35} \frac{1}{3(c^4 - 6c^2 h^2 + 5h^4) \log \frac{(h+c)}{(h-c)} - 30ch^3 + 26c^3 h}$$

such that in case of circumferential roughness patterns, $H_3 = h^3$, $H_5 = h^5$ as $c \rightarrow 0$.

5 Load Capacity and Flow Rate

The dimensionless load carrying capacity of bearing can be calculated as:

$$W = \int_0^1 rP(r)dr = 2 \int_0^{r_o} rP_o(r)dr + 2 \int_{r_o}^{r_1} rP_R(r)dr + 2 \int_{r_1}^1 rP_L(r)dr \quad (17)$$

and, volumetric flow rate (Q) is calculated numerically from the non-linear function in terms of Q derived by satisfying the continuity of pressure at radial positions r_0 and r_1 , that is, $P_0(r_0) = P_R(r_1)$ and $P_R(r_1) = P_L(r_1)$.

Fig. 2 Variation of film pressure (P) with radius (r) for $S = 2$ and $b = 2.18$. (a) Radial roughness (b) Circumferential roughness

5 Results and Discussion

To analyse the effect of surface roughness and lubricant inertia (including inertia in supply region) on the steady state performance characteristics of externally pressurized thrust bearings lubricated with non-Newtonian (pseudoplastic) lubricants, numerical results for pressure and load capacity have been obtained for the different values of roughness parameter c ($0 \leq c \leq 0.005$). For the practical justification, results have been discussed for the experimental value for coefficient of pseudoplasticity, $\kappa = 5.65 \times 10^{-6} \text{ m}^4 / \text{N}^2$ (\$0.3\%\$ additive) [23]. The values of operating parameters : film thickness ratio $\beta = 2.18, 5$; parameter of centrifugal inertia $S = 0, 0.5, 1.0, 2.0$; supply radius $r_o = 0.05$, and step radius $r_1 = 0.4$ have been taken from experimental results [7]. Further, the theoretical results of pressure and load capacity for Newtonian and non-Newtonian lubricants have been compared with the experimental results [7] and theoretical results [4]. In present analysis, calculated values of pressure have been presented for larger values of step ratio $\beta = 2.18, 5$ and higher values of inertia $S = 2$ for two reasons; first, to avoid the messy graphical presentation and second, earlier analyses have shown deviation from experimental data for larger values of S and β only.

Fig. 2 shows the variation of film pressure along the radial direction for $\beta = 2.18$ and, different values of roughness parameter (c), coefficient of pseudoplasticity (κ) and inertia parameter (S). It is readily observed that calculated values of pressure for Newtonian lubricants and smooth surfaces are sufficiently close to the experimental results [7] in comparison to earlier theoretical results [4]. This significant enhancement in the theoretical

results is due to the consideration of lubricant inertia on supply region. Therefore, the accountability of inertia on supply region is established. Further, it can be observed that at each radial point, pressure for pseudoplastic lubricants is lower than that for Newtonian lubricants and closer to the experimental results. Influence of surface roughness on pressure is dependent on the type of roughness patterns. Radial roughness causes a pressure drop at every point, Fig. 2(a); while circumferential (azimuthal) roughness causes an increase in film pressure, Fig. 2(b). A small value of roughness parameter ($c = 0.0005$) causes a significant pressure-drop in case of radial patterns and, a significant pressure-rise in case of azimuthal patterns. The influence of roughness is also not much dependent type of lubricants. It is, further, observed that the calculated pressure with radial roughness patterns and pseudoplastic lubricants is much closer to the experimental results, Fig.2(a).

Fig. 3 Variation of film pressure (P) with radius (r) for $S = 0.5, 1$ and $b = 5$.

(a) Radial roughness (b) Circumferential roughness

Variations of pressure with respect to radius have been also plotted for $\beta = 5$ in Fig. 3. To compare the results to the experimental values, $S = 0.05, 1$ have taken. It is easy to observe that calculated values of pressure for Newtonian lubricant and smooth surfaces in the present analysis are very closed to experimental values of [7] even in the vicinity of step position. Pressure calculated for pseudoplastic lubricant ($\kappa = 5.65 \times 10^{-6}$) with small radial roughness ($c = 0.0005$) fit to the experimental values more accurately, Fig.3(a). Variation of pressure with pseudoplasticity and surface roughness is similar in nature to that discussed for $\beta = 2.18$. In comparison to smooth surfaces, pressure is lower for radial roughness, while higher for azimuthal roughness patterns. Pressure for pseudoplastic lubricant is lower to that for Newtonian lubricants.

In Fig. 4, dimensionless load capacity with respect to the step position r_1 are plotted for different values of roughness parameter (c), coefficient of pseudoplasticity (κ) and inertia parameter (S). A first observation to the results for Newtonian lubricants and smooth surfaces ($\kappa = 0, c = 0$) shows that load capacity obtained in the present analysis fit to the experimental data more precisely than that obtained in [4] for each value of $S = 0, 1, 2$, and the

enhancement of results are due to the consideration of inertia on supply region. The load capacity for rough surface with longitudinal pattern ($c = 0.0005$) is less than that for smooth surfaces in both the cases of Newtonian and non-Newtonian lubricants; and the trend of variations sustain for each value of S . The decreased value of load capacity for radial roughness is due to the fall in film pressure. In case of azimuthal roughness patterns, load capacity is comparatively higher because the pressure for azimuthal roughness is higher than that for smooth surfaces. Because both the radial roughness and pseudoplastic lubricants decrease the film pressure, the best approximation to the experimental data is obtained for radial roughness ($c = 0.0005$) and pseudoplastic lubricants ($\kappa = 5.65 \times 10^{-6}$).

Furthermore, it can be clearly observed from the figures of pressure and load capacity that the influences of pseudoplasticity and roughness are more significant for higher values of inertia parameter S .

Fig. 4 Variation of load capacity (W) with step parameter (r_1) for $S = 0.5, 1$. (a) Radial roughness (b) Circumferential roughness

To have a close observation of influence of surface roughness on load capacity, Fig. 5 has been plotted, which depicts the variation of load carrying capacity with roughness parameter c for inertia parameter $S = 0.5, 1, 2$ and a typical value of step parameter $r_1 = 0.4$. Results are presented for Newtonian and pseudoplastic lubricants ($\kappa = 0, 5.65 \times 10^{-6}$) considering the cases of radial and circumferential roughness patterns. It is easy to observe that load carrying capacity decreases with increase of longitudinal roughness for each value of S , while the load capacity increases with the increase of circumferential roughness. It is, further, analysed that influence of surface roughness on load capacity is more significant for larger values of inertia parameter S .

CONCLUSION

Combined effects of surface roughness, non-Newtonian pseudoplastic lubricants and lubricant inertia on the steady performance of hydrostatic thrust bearings, considering the lubricant inertia on supply region and neglecting cavitation effects, have been presented using Rabinowitsch fluid model for non-Newtonian nature of the fluid and Christensen's theory for surface roughness. It is concluded that the inertia of lubricant on supply region has significant

influence on film pressure and load capacity, specially, in case of higher values of inertia parameter S and larger step ratio β . In comparison with smooth surfaces, dimensionless film pressure and load capacity is lower for longitudinal roughness and higher for circular roughness patterns in case of Newtonian as well as non-Newtonian lubricants. With the increase in the value of roughness parameter c , load carrying capacity decreases for radial roughness and increases for circumferential roughness. In comparison with Newtonian lubricants, dimensionless pressure and load is lower for pseudoplastic lubricants. Non-Newtonian lubricants influence the film pressure and, hence, load capacity regardless of surface roughness. Effects of surface roughness and non-Newtonian lubricants become more significant for larger values of inertia parameter S , that is, for larger bearing radius and higher operating speed.

The present analysis is expected to aid in better bearing designs because the theoretical results are closer to experimental data and physical situation of the problem.

Fig. 5 Variation of load capacity (W) with roughness parameter (c) for different values of inertia parameter S and coefficient of pseudoplasticity k .

(• - Radial roughness; o - Circumferential roughness)

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