STUDY OF POST-CRACKING TORSIONAL BEHAVIOUR OF HIGH-STRENGTH REINFORCED CONCRETE BEAMS WITH A FERROCEMENT WRAP

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Abstract

The post-cracking behaviour of structures subjected to torsion can be well predicted by Hsu's softened truss model. The softened truss model is applicable to structures having symmetry in their material properties on all four sides. Wrapping on three faces is a common phenomenon when the top face is provided with a flange or slab. Such a wrapping on three faces of a beam is referred to as a "U" wrap. "U" wraps are better wrapping strategies for distressed structures, as their top face is not accessible for many structures. The material property of an unwrapped face differs from the rest of wrapped faces. For the effective use of wrapping, the unwrapped face needs to be provided with a material having a higher resistance to tension and shear. For this, high-strength concrete in the core is a better option. Here, an attempt is made to predict the torsional capacity of "U" wrapped high-strength concrete beams having an asymmetry in the material using a softened truss model with suitable modifications of the material properties. Efficient algorithms are proposed for the solution of simultaneous equations. The predictions are found to be in good agreement with the experimental test results.

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Key words

- High-strength concrete,
- torsional strength,
- twist,
- toughness,
- stiffness.

1 INTRODUCTION

In the recent past, the world has witnessed many natural calamities. Distressed structures need to be retrofitted rather than demolished (Adrian and Riadh, 2007; Ghobarah and Ghorbel, 2002). Wrapping with composites is an effective way of retrofitting to address the torsional deficiency of distressed reinforced concrete (RC) structures and to change the failure mode from brittle to ductile. FRP wrapping is a better solution to protect distressed structures against shear, torsion or bending (Adrian and Riadh, 2006; Mehran and Harid, 2007). It is more costly. Ferrocement, a composite material which possesses a better distribution of strain and crack-arresting characteristics (ACI-549 R, 1997) can strengthen reinforced concrete structures effectively in a similar way to that of FRP. Ferrocement is a relatively

cheaper material compared to FRP; therefore, ferrocement "U" wraps can be a substitute for developing countries. Research has attained its pace for the quantification of a ferrocement composite as a wrapping against shear, bending and axial forces (Bansal et al., 2007). No such analytical model is available to quantify the torsional strength of ferrocement as a wrapping material. Again, for the effective use of wraps, the cracking should be initiated from the wrapping face. To avoid the initiation of cracking from an unwrapped face, the concrete must be of a higher strength. High-strength concrete (HSC) possesses some characteristics and properties that may differ from those of normal-strength concrete (Rasmussen, 1995). Distressed beams cannot be wrapped on all four sides, which are required to resist torsion due to the presence of flanges (slabs). "U" wraps are a solution for this problem (Saravanan and Belarbi, 2002). The torsional behaviour of

Fig. 1 Cracked beam section due to torsion.

“U” wrapped beams up to cracking is well covered by (Behera et al., 2008). Beyond cracking, the softened truss model by Hsu is a powerful tool for the analysis of the torsional behaviour of beams with symmetry in their material properties. As there is asymmetry in the material properties of ferrocement “U” wrapped HSC reinforced beams, a softened truss model cannot be directly applied for the post-cracking stage. So, here an attempt is made for the prediction of torsional strength in the post-cracking stage by applying Hsu’s softened truss model with suitable modifications in the material properties.

1.1 Post-Cracking Stage of “U” Wrapped RC High-Strength Beams

The torque-twist response of “U” wrapped reinforced concrete beams up to cracking is well predicted by the equations (Behera et al., 2008). After the cracking, the softened truss model by (Hsu, 1988) is applied for the prediction of torsional strength with suitable modifications in the material properties.

1.1.1 Formulation of Problem

After cracking, “U” wrapped beams can be analyzed as a thin tube (Hsu, 1988; Rao and Seshu, 2003; Rao and Seshu, 2005; Rao and Seshu, 2006). Diagonal cracks separate the concrete and ferrocement into a series of diagonal struts. The longitudinal and transverse reinforcements keep the cracked pieces of concrete and ferrocement together in their respective faces (Hsu, 1988; Leu and Lee, 2000). The concrete struts are subjected to compression, and the steel bars act as tension links in the concrete face, thus forming a truss action in the concrete face. The mortar on the other three faces acts as strut and wire meshes, with the steel reinforcement acting as a tension link in the ferrocement faces. The propagation and growth of cracks are diminished in the ferrocement faces in comparison to the unwrapped concrete face due to the presence of the ferrocement, as the ferrocement has a better crack-arresting ability. The beams with ferrocement “U” wraps subjected to torsion after cracking can be analyzed as hollow tubes of variable thicknesses on the periphery with an equal shear flow ‘q’ as shown in Fig.1 due to the asymmetry in the material. After cracking, the hollow tube thickness can be taken as T_{d1} on the three ferrocement-wrapped faces and T_{d4} on the top concrete face.

Fig. 2 Co-ordinate system.

Fig. 3 Applied stresses.

The theories available to date in the literature for the prediction of the torsional strength of beams are based on an equal thickness of a shear flow tube due to the homogeneity in the material. When a “U” wrap is present, the same theories cannot predict the torsional strength due to the asymmetry in the material. The thickness of the shear flow tube will vary according to the shear-resisting capacity of the materials present on the outer periphery of the beam. So, the shear flow thickness of the reinforced concrete portion will be different from that of the ferrocement faces. The shear flow is the same on all the faces, but the shear stress is different for different faces. So, the beam can be idealized as a space truss with co-ordinate systems as shown in (Figs. 2 and 3). The post-cracking behaviour of ferrocement “U” wraps can now be analyzed by the softened truss model with necessary modifications in the equations.

1.1.2 Assumptions

- After cracking, the tensile force in the longitudinal and transverse directions is taken by the steel bars in the concrete core for the unwrapped portion. The same is resisted by the mortar, steel bars and GI wire mesh in the ferrocement shell. The tensile stress of the concrete is ignored, but the tensile strength of the mortar is taken into account.
- A perfect bond exists between the concrete and the ferrocement.
- The diagonal concrete on the top face and the mortar on the other three sides only take axial compression.

In Fig.1 the concrete top face is treated as a fourth face and has a different thickness than the other three wrapped faces. The ferrocement-wrapped faces are assumed to have an equal thickness due to the symmetry of the material.

Fig. 2 represents the coordinate system used in the analysis. There are two main coordinate directions: the l-t direction represents the longitudinal and transverse directions; whereas, the d-r direction represents the principal stress directions respectively. A torsional load induces a biaxial tension-compression state along the diagonal directions. It is assumed that the compressive stress σ_d of the biaxial state acts along direction ‘d’, whereas, the tensile stress σ_r acts in the ‘r’ direction as shown in (Fig. 3). A ferrocement composite is made up of a mortar matrix and mesh reinforcement, which is finely dispersed across the section. Therefore, the resistance provided by the ferrocement section can be idealized as the sum of the resistance provided by the mortar matrix and mesh reinforcement (Fig. 4).

1.1.3 Equilibrium Equations

Considering a small element in the concrete and ferrocement faces, the equilibrium condition results in the following equations:

$$q = \frac{T}{2A_0} \quad (1)$$

where q = shear flow, T = Torsional moment, A_0 = Area enclosed by the centre line of the cracked tube.

Fig. 4 Idealisation of the composite.

a) Longitudinal direction

$$\sigma_{d1} \cos^2 \alpha_1 + \sigma_{r1} \sin^2 \alpha_1 + \rho_{l1} F_{l1} = 0 \quad (2)$$

for the ferrocement face

$$\sigma_{d4} \cos^2 \alpha_4 + \rho_{l4} F_{l4} = 0 \quad (3)$$

for the concrete face where $\rho_{l1} = \frac{A_{l1}}{T_{d1}d}$ $A_{l1} = A_{lw1} + A_{ls1}$

$$\rho_{l4} = \frac{A_{l4}}{T_{d4}b}$$

$$b = B - T_{d1}$$

$$d = H - \frac{T_{d1}}{2} - \frac{T_{d4}}{2}$$

b) Transverse direction

$$\sigma_{d1} \sin^2 \alpha_1 + \sigma_{r1} \cos^2 \alpha_1 + \rho_{t1} F_{t1} = 0 \quad (4)$$

for faces 1, 2 and 3 (ferrocement faces)

$$\sigma_{d4} \sin^2 \alpha_4 + \rho_{t4} F_{t4} = 0 \quad (5)$$

for 4th face (concrete face)

$$\rho_{t1} = \frac{A_{t1}}{T_{d1}s_w}$$

$$\rho_{t4} = \frac{A_{t4}}{T_{d4}s_4}$$

where σ_{d1} and σ_{d4} are the principal compressive stresses in faces 1 and 4 respectively. σ_{r1} and σ_{r4} are the principal compressive stresses in faces 1 and 4 respectively. ρ_{l1} and ρ_{l4} are the reinforcement ratio in the longitudinal direction in faces 1 and 4 respectively. ρ_{t1} and ρ_{t4} are the reinforcement ratio in the transverse direction in faces 1 and 4 respectively. A_{l1} = Area of steel reinforcement along the longitudinal direction in the ferrocement face taking the steel bars and wire mesh. A_{l4} = Area of steel reinforcement along the longitudinal direction in the concrete face. F_{l1} and F_{l4} are the stresses in the longitudinal reinforcement in faces 1 and 4 respectively. F_{t1} and F_{t4} are the stresses in the transverse reinforcement in

faces 1 and 4 respectively.

 α_1, α_4 crack angle along the ferrocement face and concrete face respectively. $A_{l1} = A_{lw1} + A_{ls1}$ A_{l1} = Area of steel reinforcement along the transverse direction in the ferrocement face taking the steel bars and wire mesh. A_{l4} = Area of steel reinforcement along the transverse direction in the concrete face. A_{lw1} = Area of one wire in the mesh A_{ls1} = Equivalent Area of steel bar in the transverse direction of the ferrocement face calculated for a spacing of s_w (spacing of the wire in the wire mesh)

c) Shear direction

$$q = (\sigma_{d1} - \sigma_{r1}) \sin \alpha_1 \cos \alpha_1 \quad (6)$$

for 1st, 2nd and 3rd faces (Ferrocement)

$$q = (\sigma_{d4} - \sigma_{r4}) \sin \alpha_4 \cos \alpha_4 \quad (7)$$

for 4th face (Concrete face)

Generally σ_{r1} and σ_{r4} can be taken as zero, where σ_{r1} is the tensile stress along the 'r' direction for the ferrocement face, and σ_{r4} is the tensile stress of the concrete along the 'r' direction under the bi-axial compression-tension state. The actual calculation of σ_{r1} is given in Equation 32. Combining Equations 6 and 7 gives

$$(\sigma_{d1} - \sigma_{r1}) T_{d1} \sin 2\alpha_1 = \sigma_{d4} T_{d4} \sin 2\alpha_4 \quad (8)$$

1.1.4 Compatibility equations

In the softened truss model it is assumed that the section under consideration is completely cracked. Therefore, the Mohr's circle of strains cannot be applied due to its basic requirement of continuity of the material. However, it is assumed here that the Mohr circle of strains is also applicable in the cracked stage. These strains also include the crack width in the strain measurement. These strains are known as smeared strains.

For the ferrocement face

$$\varepsilon_{l1} = \varepsilon_{d1} \cos^2 \alpha_1 + \varepsilon_{r1} \sin^2 \alpha_1 \quad (9)$$

$$\varepsilon_{t1} = \varepsilon_{d1} \sin^2 \alpha_1 + \varepsilon_{r1} \cos^2 \alpha_1 \quad (10)$$

$$\gamma_{lt1} = (\varepsilon_{d1} - \varepsilon_{r1}) \sin 2\alpha_1 \quad (11)$$

For the concrete face

$$\varepsilon_{l4} = \varepsilon_{d4} \cos^2 \alpha_4 + \varepsilon_{r4} \sin^2 \alpha_4 \quad (12)$$

$$\varepsilon_{t4} = \varepsilon_{d4} \sin^2 \alpha_4 + \varepsilon_{r4} \cos^2 \alpha_4 \quad (13)$$

$$\gamma_{lt4} = (\varepsilon_{d4} - \varepsilon_{r4}) \sin 2\alpha_4 \quad (14)$$

$\varepsilon_{l1}, \varepsilon_{l4}$ = average strain in the “l” direction in faces 1 and 4 respectively

$\varepsilon_{t1}, \varepsilon_{t4}$ = average strain in the “t” direction in faces 1 and 4 respectively

$\varepsilon_{d1}, \varepsilon_{d4}$ = average principal strain in the “d” direction in faces 1 and 4 respectively.

$\varepsilon_{r1}, \varepsilon_{r4}$ = average principal strain in the “r” direction in faces 1 and 4 respectively.

$\gamma_{lt1}, \gamma_{lt4}$ = average shear strain in the “l-t” direction in faces 1 and 4 respectively.

From Hsu's softened truss model, the curvature Y_1 of the diagonal ferrocement strut can be related to the twist angle θ as per the following equations.

$$\psi_1 = \theta \sin 2\alpha_1 \quad (15)$$

In the same way as for the concrete face, the curvature Ψ_4 can be written as

$$\psi_4 = \theta \sin 2\alpha_4 \quad (16)$$

From the bending of the concrete strut the curvature can be related to the maximum strain by

$$\psi_1 = \frac{\varepsilon_{ds1}}{T_{d1}} \quad (17)$$

$$\psi_4 = \frac{\varepsilon_{ds4}}{T_{d4}} \quad (18)$$

$\varepsilon_{ds1}, \varepsilon_{ds4}$ = maximum principal strain in the “d” direction in faces 1 and 4 respectively.

Because of the linear strain distribution, the maximum strains $\varepsilon_{ds1}, \varepsilon_{ds4}$ can be related to the average strains ε_{d1} and ε_{d4} by the following equations;

$$\varepsilon_{ds1} = 2\varepsilon_{d1} \quad (19)$$

$$\varepsilon_{ds4} = 2\varepsilon_{d4} \quad (20)$$

combining the equations from (15) to (20),

$$\theta = \frac{2\varepsilon_{d1}}{T_{d1} \sin 2\alpha_1} = \frac{2\varepsilon_{d4}}{T_{d4} \sin 2\alpha_4} \quad (21)$$

The twist can be related to the shear strain using Bredt's equation as presented by (Hsu, 1984):

$$\theta = \frac{2d+b}{2A_0} \gamma_{lt1} + \frac{b}{2A_0} \gamma_{lt4} = \frac{\varepsilon_{ds4}}{T_{d4} \sin 2\alpha_4} \quad (22)$$

1.1.5 Constitutive laws for the materials

The equilibrium and compatibility conditions of the softened truss model remain the same for all kinds of materials. However, the material law changes with the change in the constitutive material.

This is due to the variation in the stress-strain response of the materials under tension and compression.

a.) Longitudinal reinforcement

The stress in the longitudinal reinforcement can be related to their strains. The longitudinal strain or transverse strain in the reinforcement is the same as the strain of the ferrocement mortar or concrete surrounding the reinforcement in that direction.

$$F_{l1} = \begin{cases} E_s \varepsilon_{l1} & \text{if } \varepsilon_{l1} \leq \varepsilon_{ly1} \\ F_{ly1} & \text{if } \varepsilon_{l1} > \varepsilon_{ly1} \end{cases} \quad (23)$$

$$F_{l4} = \begin{cases} E_s \varepsilon_{l4} & \text{if } \varepsilon_{l4} \leq \varepsilon_{ly4} \\ F_{ly4} & \text{if } \varepsilon_{l4} > \varepsilon_{ly4} \end{cases} \quad (24)$$

b.) Transverse reinforcement

The strain in the transverse reinforcement is the same as the surface strain in the transverse direction on that face on the concrete or mortar, assuming a perfect bond between them.

$$F_{t1} = \begin{cases} E_s \varepsilon_{t1} & \text{if } \varepsilon_{t1} \leq \varepsilon_{ty1} \\ F_{ty1} & \text{if } \varepsilon_{t1} > \varepsilon_{ty1} \end{cases} \quad (25)$$

$$F_{t4} = \begin{cases} E_s \varepsilon_{t4} & \text{if } \varepsilon_{t4} \leq \varepsilon_{ty4} \\ F_{ty4} & \text{if } \varepsilon_{t4} > \varepsilon_{ty4} \end{cases} \quad (26)$$

c.) Compression of the ferrocement struts

For the ferrocement faces, the compressive stress-strain diagram for the bi-axial state of tension and compression is taken into account. The softening coefficient for the stress and strain of the ferrocement by (Mansur and Abdullah, 1998) is included for an evaluation of the same.

$$\sigma_{d1} = \frac{1}{\varepsilon_{ds1}} \int_0^{\varepsilon_{ds1}} F_{c1} \xi_{\sigma} \left[\frac{1.77 \frac{\varepsilon_{ds1}}{\varepsilon_f \xi_{\varepsilon}}}{1 - 0.23 \frac{\varepsilon_{ds1}}{\varepsilon_f \xi_{\varepsilon}} + \left(\frac{\varepsilon_{ds1}}{\varepsilon_f \xi_{\varepsilon}} \right)^2} \right] d\varepsilon_{ds1} \quad (27)$$

Where ε_f = strain in the ferrocement at maximum stress

ξ_{σ} = Stress softening coefficient for the ferrocement face

ξ_{ε} = Strain softening coefficient for the ferrocement face

$$F_{c1} = f_{cm} (1 + 0.13 S_F) \quad (28)$$

F_{c1} = compressive stress of the ferrocement,

S_F = specific surface factor.

f_{cm} = ferrocement compressive strength of mortar

$$S_F = s \frac{f_y}{f_{cm}} \quad (29)$$

s = specific surface area per volume

$$\xi_{\sigma} = \frac{0.85}{\sqrt{1 + 7.5 \sqrt{\varepsilon_{r1}}}} \quad (30)$$

$$\xi_{\varepsilon} = \frac{0.85}{\sqrt{1 + 18 \sqrt{\varepsilon_{r1}}}} \quad (31)$$

d.) Tensile strength of mortar

$$\sigma_{r1} = \begin{cases} \varepsilon_{r1} E_m, & \varepsilon_{r1} \leq \varepsilon_{cr} \\ (1.06 - 500 \varepsilon_{r1}) f_{tm}, & \varepsilon_{r1} > \varepsilon_{cr} \end{cases} \quad (32)$$

e.) Compression of concrete strut

Taking the effect of tension softening in concrete, the compressive stress of concrete σ_{d4} can be calculated as per the equations given below:

$$\sigma_{d4} = k_4 \xi_4 f_{c4} \quad (33)$$

where ξ_4 = softening coefficient for the concrete face

K_4 = ratio of average stress to the peak stress in the concrete stress block

f_{c4} = cylindrical compressive strength of concrete (Hsu, 1988)

$$K_4 = \frac{\epsilon_{ds4}}{\epsilon_{04}} \left(1 - \frac{\epsilon_{ds4}}{3\epsilon_{04}} \right) \quad (34)$$

where ϵ_{04} = strain in concrete at the peak stress = -0.002

The softening coefficient is to be changed for high-strength concrete as given by (Zhang and Hsu, 1998)

$$\xi_{\sigma 4} = \xi_{\epsilon 4} = \xi_4 = \frac{5.8}{\sqrt{f_{c4}} \sqrt{1 + \frac{400\epsilon_{r4}}{\eta}}} \quad (35)$$

where η = ratio of the tensile force in the transverse steel to the tensile force in the longitudinal steel and, if greater than 1.0, it should be taken as a value of 1/ η .

f_m = tensile strength of mortar

The number of unknowns and the number of available equations are 22 as shown in Tab. 1.

2 ALGORITHM

- Take the surface compressive strains ϵ_{ds4} and ϵ_{d4} at the center-line of the shear flow.
- Assume an initial value for ξ_4 .

- Calculate the ratio of the average stress to the peak stress K_4 using Equation 34.
- if $\xi_4 > 1$, then make it $\xi_4 = 1$; calculate ϵ_{p4} using Equation 35.
- Calculate s_{d4} using Equation 33.
- Assume a suitable value for T_{d4} .
- Assuming $\epsilon_{t4} < \epsilon_{ty4}$, calculate α_4 using Equation 5, putting F_{t4} as per Equation 26.
- Calculate ϵ_{t4} using Equation 13.
- If $\epsilon_{t4} \geq \epsilon_{ty4}$, recalculate α_4 taking $f_{t4} = F_{ty4}$ in Equation 5 and ϵ_{t4} using Equation 13.
- Assume a suitable value for a_1 .
- Assume a suitable value for T_{d1} .
- Calculate ϵ_{ds1} using Equation 21.
- Calculate g_{lt4} , g_{lt1} using Equations 14 and 22 respectively, as b and d can be calculated.
- Calculate ϵ_{r1} from Equation 11.
- Calculate the stress and strain softening coefficient for the ferrocement face, i.e., x_σ , x_ϵ by using Equations 30 and 31 respectively.
- Calculate s_{d1} using Equation 27 by Simpson's rule and s_{r1} from Equation 32.
- Calculate ϵ_{d1} from Equation 19.
- Calculate ϵ_{t1} using Equation 10.
- If $\epsilon_{t1} < \epsilon_{ty1}$, $F_{t1} = E_s \epsilon_{t1}$; if $\epsilon_{t1} \geq \epsilon_{ty1}$, $F_{t1} = F_{ty1}$ as per Equation 25.
- Calculate ρ_{t1} using Equation 4 and the improved value of T_{d1} using formulae $\rho_{t1} = \frac{A_{t1}}{T_{d1}s_w}$.
- Check the assumed value of T_{d1} with the calculated value of T_{d1imp} , if the error $1 = \frac{T_{d1} - T_{d1imp}}{T_{d1}} \times 100$ more than 1, then improve the assumed value as below and start the calculation from step 11, $T_{d1} = \frac{(T_{d1imp} + T_{d1})}{2}$.

Tab. 1 Variables and equations used for the torsion solution.

VARIABLES and EQUATIONS used for Solution										
Stress or Force	Equations	Strain deformations	Equations	Material	Equations	Geometry	Equations	Equilibrium	Compatibility	Material
s_{d4}	33	ϵ_{t4}	12	K_4	34	α_4	5	2	9	23
s_{d1}	27	ϵ_{t4}	13	ξ_4	3	a_1	2	3	10	24
s_{r1}	32	ϵ_{r4}	35			T_{d1}	4	4	11	25
F_{t1}	23	g_{lt4}	14			T_{d4}	8	5	12	26
F_{t4}	24	ϵ_{t1}	9					8	13	27
F_{t1}	25	ϵ_{t1}	10						14	32
		ϵ_{r1}	11						21	33
		g_{lt1}	22						22	34
		ϵ_{d1}	21							35
Total = 7 + 9 + 2 + 4 = 22								Total = 5 + 8 + 9 = 22		

- xxii. Assuming $e_{11} < e_{ly1}$, calculate the improved value of a_1 using Equation 2, putting the value of F_{11} from Equation 23.
- xxiii. Calculate e_{11} using Equation 9.
- xxiv. Check if $e_{11} \geq e_{ly1}$, recalculate a_1 using Equation 2 putting $F_{11} = F_{ly1}$ and check e_{11} by Equation 9 respectively.
- xxv. Check the assumed value of a_1 and calculate the value of a_1 .
If $\text{error2} = \frac{\alpha_1 - \alpha_{1imp}}{\alpha_1} \times 100$ more than unity, improve the assumed value of $\alpha_1 = (\alpha_1 + \alpha_{1imp})/2$ and start the calculations again from step 10.
- xxvi. Calculate T_{d4} using Equation 8.
- xxvii. Check the assumed value of T_{d4} and calculate the value of T_{d4} .
If $\text{error3} = \frac{T_{d4} - T_{d4imp}}{T_{d4}} \times 100$ is greater than unity, improve the assumed value of $T_{d4} = \frac{(T_{d4imp} + T_{d4})}{2}$ and start the calculations again from step 6.
- xxviii. Calculate e_{14} by using Equation 12.
- xxix. Calculate σ_{d4} from Equation 3 taking F_{14} from Equation 24.
- xxx. Calculate ξ_{4imp} from Equation 33.
- xxxi. If $\xi_{4imp} > 1.0$, then make $\xi_{4imp} = 1.0$.
- xxxii. If $\text{error4} = \frac{\xi_4 - \xi_{4imp}}{\xi_4} \times 100$ is more than 1, start from step 2 by making $\xi_4 = \frac{(\xi_{4imp} + \xi_4)}{2}$.

- xxxiii. Calculate q , γ_{11} and γ_{14} by using Equations 7, 11 and 14 respectively.
- xxxiv. Calculate the torque T using Equation 1 and the corresponding value θ by Equation 21 in the post-cracking range.
- xxxv. Choose another value of ε_{ds4} . Find out the torque and twist. Repeat from step 1 till either of the matrix strains ε_{ds4} reaches the failure strain or the steel strains reach the yield strain.

3 VALIDATION OF THE PROPOSED MODEL

For the validation of the proposed model, beams were cast and tested. Experimental and analytical torque twist diagrams were presented to verify the accuracy of the model. To study the effect of the arrangement of the reinforcement on the torque twist response, six possible cases of the arrangement of the reinforcement were considered, keeping the number of mesh layers as four, the ferrocement matrix strength of 55 MPa, the core concrete strength of 60 MPa and the aspect ratio of 2 constant. The reinforcement can be arranged in six ways (Rao and Seshu, 2003), i.e. either a) in a longitudinal direction, b) a transverse direction, c) under-reinforced in both the longitudinal and transverse directions, d) longitudinally over reinforced and transversely under reinforced, e) longitudinally under reinforced and transversely over reinforced and, f) completely over-reinforced in both directions.

Tab. 2 Details of Beams Tested, Mesh wire diameter = 0.72mm.

Series	Designation	Dimension (mm)	Compressive strength		Reinforcement Details					
			Ferrocement matrix (MPa)	Concrete (MPa)	Core Reinforced Concrete				Outer Wrap	
					Longitudinal Steel		Transverse steel		No. of mesh layers (yield strength of wire mesh 250 MPa) Wire diameter = 0.72 mm dia = 0.72 mm	
					Diameter, No. of bars	Yield Strength (MPa)	Diameter, Spacing	Yield Strength (MPa)		
1	Plain Beams	BH	125 x 250	55					0	
2		B4H	125 x 250	55	60				4	
3	Only Longitudinal	L4H	125 x 250	55	60	12 mm, 6 nos.	440		4	
4	Only Transverse	T4H	125 x 250	55	60			10mm @ 70 mm c/c	445	4
5	Torsionally under reinforced in Longitudinal and transverse direction	U4H	125 x 250	55	60	6 mm, 6 nos.	350	6mm @ 70 mm c/c	350	4
6	Torsionally Over reinforced in Longitudinal direction and Under Reinforced in transverse direction	Lo4H	125 x 250	55	60	12 mm, 6 nos.	440	6mm @ 70 mm c/c	350	4
7	Torsionally under reinforced in Longitudinal direction and over reinforced in transverse direction	To4H	125 x 250	55	60	6 mm, 6 nos.	350	10mm @ 70 mm c/c	445	4
8	Torsionally over reinforced in Longitudinal and transverse directions	Co4H	125 x 250	55	60	12 mm, 6 nos.	440	10mm @ 70 mm c/c	445	4

The RC beam details are given in Table 2. In members under torsional loads, diagonal cracks are formed in the shape of a helix. To allow two complete spirals in the central test region of the test beam, a length of 1500 mm was kept for observation of the crack pattern. In order to force the failure in the central portion of the beam, the end zones of a length of 250 mm were heavily reinforced. Thus the total length of the beam was fixed as 2000 mm for the aspect ratio 2.0 for the beam size (125 mm X 250 mm). A control specimen, BH, of plain concrete without any reinforcement in its core was cast. Another control specimen, B4H, of the same size without any reinforcement in the core was cast with 25 mm thick ferrocement “U” wraps containing four mesh layers.

3.1.1 Material and Material Properties

- a) **Cement:** Ordinary 53 grade Portland cement conforming to Indian Standard (IS: 12269, 1987) was used throughout the experimental program. The standard consistency was 28%, whereas the initial and final setting time was 95 min. and 210 min. respectively. The specific gravity of the cement was 3.14, and its compressive strength after 28 days was 57 MPa.
- b) **Coarse Aggregate:** Crushed hard granite stone of a maximum size of 20 mm was used for the concrete. The bulk density of the aggregate was 1695 kg/m³, and the specific gravity was found to be 2.65.
- c) **Fine Aggregate:** The International Ferrocement Information Centre (IFIC) model code (2001) suggests that the size of the sand particle in a ferrocement matrix should not be more than one half of the size of the opening of the mesh. The mesh opening was 6.35 mm. River sand that passed through a 1.18 mm sieve was used in the ferrocement matrix's preparation. The specific gravity of the sand was 2.65. The fine aggregate used in this investigation for the concrete was river sand conforming to Zone-II of (IS: 383, 1970). The fineness modulus was 2.81.
- d) **Wire meshes:** Square woven wire meshes made of galvanized steel were used for the “U” wraps. The diameter of the wire was 0.72 mm, and the yield strength was 250 N/mm²; the centre-to-centre spacing of the wires was 6.35 mm.
- e) **Reinforcement:** 6, 10 and 12 mm diameter bars were used in the entire experimental study. The yield stresses of these bars were 350 N/mm², 440 N/mm² and 440 N/mm² respectively.

3.1.2 Casting of the Specimen

- a) **Preparation of the mesh layer:** The required length of the mesh layer was cut from a 1200 mm-wide mesh roll up to a length of 1980 mm and bent to the desired shape. All the layers required for the beam were prepared in the same manner and arranged one above the other. The layers were separated by smaller diameter spacer bars.
- b) **Preparation of the End Cage:** To avoid failure in the end regions, the ends were heavily reinforced, but care was taken to avoid congestion of the reinforcement.
- c) **Casting:** The “U” shaped mesh layers were properly put in the mould. The sides of the mesh layers were separated by spacer bars to provide proper gaps. The ferrocement matrix, which was properly mixed in the mixture machine, was put in the mould up to a depth of approximately 25 mm. A closed wooden box of the size of the concrete core was put in the mould, and the ferrocement matrix was poured from the top to fill up the portions of the wrappings. Within the elapsed initial setting time of the ferrocement matrix, the hollow box section was

carefully removed. The end cage reinforcements and the reinforcement in the core portion were put in their proper place. The filled concrete was vibrated. After 24 hours, the mould was carefully removed along with its companion and put in the curing tank.

3.1.3 Testing

The beams were tested on a torsion test rig available in the laboratory. The marked beams were mounted on the wing table of a Tinius-Olsen testing machine. Rollers in the lateral direction at the reaction end were provided to allow the beam to slide freely along the longitudinal direction to avoid any axial restraint. The loading end was supported on a cylindrical roller placed in the longitudinal direction of the beams to allow a twist under pure torsion. This set up is shown in (Fig. 5). Specially made steel trusses were used as lever arms and placed at both supports to apply transverse loads, which in turn produced torsion on the member. The load was gradually applied through a mechanical jack. The dial gauge readings attached to the twist meters were recorded for each increment of load. The load at the first crack was noted, and then the cracks formed for each increment of the load were marked along with the crack marker. The peak load was recorded.

3.2 Discussion of the Torsional Behaviour of the HSC Beams

The torsional behaviour of the high-strength concrete beams with a core and without a core reinforcement were studied separately.

3.2.1 Torsional Behaviour of Unreinforced HSC beams

The first crack for the unreinforced high-strength concrete beam (BH) appeared on the longer face; on a further attempt to load, the failure of the beam resulted in cracks connecting each face at 45° as spirals, the same as noticed by (Hsu, 1984). This is due to the fact that the shear stress induced is more than the shear strength of the material provided on that face. The failure of the BH beam was catastrophic, as it suddenly failed with the appearance of a crack on the longer face and was connected spirally. The crack on the B4H beam wrapped with a ferrocement “U” wrap appeared on the unwrapped concrete's shorter face. The beams that failed with a single potential crack appeared on the top of the un-wrapped concrete face in a similar manner as observed in normal strength concrete beams (Behera *et al.*, 2010). The failure plane changed from the longer face to the shorter un-wrapped face in the case of B4H due to the higher shear resistance provided by the ferrocement shell on the longer face. The ultimate failure of these two beams was found to be due to the formation of a single potential crack with an inclination of approximately 45° to the longitudinal axis. The BH beam failed with a torque of 4.612 kNm, while the B4H beam failed with a torque of 6.50 kNm. The experimental torque-twist diagram of the ferrocement “U” wrapped plain high-strength beam plotted is in (Fig. 6). The torque varies linearly with the twist, and the linearity ends once the torque is more than the elastic torque. This is because of the formation of micro-cracking in the ferrocement portion. The micro cracking region ends when the visible crack appears on the surface.

Ultimate Torque and Crack Angle

The ultimate torque and twist were found to be 4.61 kNm and 0.0028 rad/m for the BH beam respectively. The same as calculated by

Tab. 3 Experimental and theoretical values of the torsional parameters.

Beam designation	Torsional stiffness (kN-m ²)			Torque (kN-m)		Toughness (kN-m/m)	Twist (rad/m)	
	Initial	Cracking	Ultimate	Cracking	Ultimate		Ultimate	Breaking
BH (Expt)	2108			4.612				0.0028
B4H (Expt.)	1750	1145		6.500		0.0219		0.00546
L4H (Expt)	1686	1167	869.3982	6.79	6.71	0.038	0.005818	0.00781
L4H (Analytical)	The model cannot predict the torsional capacity as the beam lacks one transverse reinforcement							
T4H (Expt)	1686	1153	915.2778	6.59	6.59	0.03317	0.00569	0.0072
T4H (Analytical)	The model cannot predict the torsional capacity as the beam lacks one longitudinal reinforcement							
U4H(Expt)	1634	1188	38	6.4248	7.68	1.1441	0.1305	0.1945
U4H (Analytical)	1626	1188	38	6.52	7.729	1.1550	0.13736	0.197
Lo4H(Expt)	1721	1239	68	6.675	7.87	0.4258	0.056	0.0875
Lo4H (Analytical)	1627	1250	66	6.52	7.8288	0.4185	0.0055	0.0873
To4H(Expt)	1636	1225	72	6.618	8.66	0.7840	0.0921	0.1124
To4H (Analytical)	1627	1226	72.75	6.52	8.58	0.7821	0.0921	0.1116
Co4H(Expt)	1662	1242	130	6.7163	12.91	0.8121	0.0754	0.092
Co4H (Analytical)	1627	1289	129	6.52	12.98	0.8016	0.074	0.0929

the skew bending theory was found be 4.34 kNm and 0.003468 rad/m. When a similar beam was provided with ferrocement “U” wraps with four layers of mesh and even with a ferrocement matrix of a lower strength (55 MPa) than that of the core concrete, the torsional strength was found to be 6.50 kNm versus the theoretical value of 6.52 kNm.

This shows that beams with “U” wraps have more strength than that of plain beams and that their strength cannot be estimated by the skew bending theory. The enhancement may be due to the pre-stressing effect of the “U” wrap (Panchacharam and Belarbi, 2002). The twist at the ultimate torque of the B4H beam was predicted by the theoretical model as 0.0056 rad/m versus the experimental values of 0.0055 rad/m. The theoretical crack angle for the concrete and ferrocement face should be 45°. The reason is due to the equal volume fraction of the reinforcement in both directions. The experimental crack angles for the wrapped ferrocement face and concrete were 46.17° and 45° respectively. The initial stiffness of the B4H was found to be 1626 kNm². The twist up to the elastic torque depends on torsional rigidity and is independent of the face (wrapped or un-

wrapped) of the cracking, but at the ultimate torque, the twist is dependent on the face of the initiation of the formation of the crack, as the cracks on the wrapped faces allow the beams to rotate more, thereby providing better ductility. The toughness of the B4H beam was found to be 0.0219 kNm/m.

3.2.2 Torsional Behaviour of Ferrocement “U” Wrapped High-Strength RC Beams

The reinforcement is activated after cracking. So, the torque-twist response of a RC beam beyond cracking is influenced by the reinforcement present in the beam. The post-cracking torque-twist response of a ferrocement “U” wrapped beam is characterized by the reinforcement present in the core concrete and the mesh layers in the ferrocement shell. After the cracking, torsional resistance is offered by the longitudinal reinforcement, the transverse reinforcement, the ferrocement on three faces, and the concrete present between the

Fig. 5 Torsion test set up. **Fig. 6** Torque twist response of BH & B4H. **Fig. 7** Torque twist response of L4H.

Fig. 8 Torque twist response of T4H.

Fig. 9 Torque twist response of U4H.

Fig. 10 Torque twist response of Lo4H.

diagonal struts. As the first two categories lack one of the resisting components, they can be analyzed as plain beams with “U” wraps.

Beams with only Longitudinal Reinforcement

The L4H beam was cast with six 12- mm diameter bars as longitudinal reinforcements and without any transverse reinforcement with four mesh layers in the ferrocement shell.

- a) **Torque:** The cracking and ultimate torque of the L4H beam was found to be 6.79 kNm. The increase in the torque of the L4H beam over its plain “U” wrap B4H beam is 4.46%. The torque-twist response as shown in (Fig.7) shows that only the behaviour of the longitudinally reinforced beam was similar to the plain wrapped beam. The first crack was initiated on the unwrapped face of the beam. The L4H beam was a replica of the B4H, having only longitudinal reinforcement in the core area. The cause of the initiation of the first crack in the unwrapped face was due to the induced shear stress, which is more than the shear capacity of the material in the core area.
- b) **Twist:** There was an increase in the twist at the ultimate torque with respect to its plain B4H beam. The twist at the ultimate torque was found to be 0.0078 rad/m. The increase in the twist over the plain “U” wrapped B4H beam was found to be 39.28%.

c) **Stiffness:** The initial stiffness of a beam is independent of its reinforcement. The initial stiffness of the L4H beam was found to be 1686 kNm².

d) **Toughness:** Toughness is measured as the area under the torque twist curve. The increase in the twist enhances the toughness of the beam over the plain “U” wrapped beam. The toughness of the beam was found to be 0.038 kNm/m with an increase of 73.5% over the plain “U” wrap B4H beam.

Beams with only Transverse Reinforcement

To investigate the effect of only transverse reinforcement, the T4H beam was cast and tested. T4H was cast with stirrups of 10 mm diameter bars at a spacing of 70 mm c/c without any longitudinal reinforcement in the test region. The first crack appeared on the unwrapped face of the beam. There was no such improvement in the cracking torque of T4H over the plain B4H beam as shown in (Fig. 8).

- a) **Torque:** The cracking and ultimate torque of the beam was found to be 6.59 kNm. The increase in the cracking torque over the B4H beam was only 1.38%.
- b) **Twist:** After the formation of the first crack, the beam was unable to resist a further load, but twisted continuously under the constant ultimate loading. The twist at the cracking of the

Fig. 11 Torque twist response of To4H.

Fig. 12 Torque twist response of Co4H.

Fig. 13 Crack pattern of Co4H.

Fig. 14 Percentage increase in torque w.r.t. plain beam.

Fig. 15 Increase twist w.r.t. plain beam.

Fig. 16 Experimental torque twist responses of different states of torsion.

beam was found to be 0.0056 rad/m and the twist at the ultimate torque of 0.0072 rad/m. The increase in the ultimate twist over the cracking twist was found to be 28.57%.

c) **Stiffness:** The initial stiffness of the T4H beam was found to be 1686 kNm² over the predicted value of 1626 kNm².

d) **Toughness:** The toughness of the beam was found to increase 51.14% over the plain beam.

A single type of reinforcement could not improve the torsional strength of the ferrocement “U” wrapped beams, but was capable of increasing their toughness.

General Behaviour of RC High-Strength Beams

All the beams were replicas of B4H with different amounts of reinforcement in the core concrete. As the initiation of the crack for the B4H beam was on an unwrapped face, as in all the reinforced concrete beams, the first crack was found on an unwrapped concrete face. Small segmental cracks were developed with subsequent torsional loading on the wrapped and unwrapped faces. The formation of cracks ceased before the ultimate loading and widening of the cracks took place on the unwrapped face, while there was the formation of still new cracks on the wrapped faces. The cracks on the different faces connected before failure. Beyond the ultimate torque, all the beams showed a drooping portion. The four beams provided with both longitudinal and transverse reinforcements showed a considerable amount of increase in their torsional strength beyond the cracking torque. The ultimate torque was found to be increasing in an order from the under reinforced, longitudinally over reinforced, transversely over reinforced to completely over reinforced categories. The experimental and theoretical torque twist response of the under reinforced, longitudinally over reinforced, transversely over reinforced and completely over reinforced beams are presented in (Figs. 9, 10, 11 and 12) respectively. The torque-twist diagram was linear up to the elastic torque after which there was deviation in the linearity. The change of linearity can be seen from the cracking as during the micro-cracking, any deviation is not prominent.

Crack Pattern, Torque, Twist and Stiffness

The first crack appeared on the unwrapped face in each beam. The cracks were formed in the unwrapped concrete face as well as the wrapped ferrocement faces. The crack angles were found to vary from 40.96° to 47.54° for the concrete face experimentally against the predicted values of 36.31° to 48.75° for the reinforced beams from the under reinforced state to the completely over reinforced state. The variation of the crack angle was found experimentally from 41.63° to 56.78° for the ferrocement faces against the predicted values of 43.90°

to 60.03°. The crack pattern for the Co4H beam was shown in Fig. 13.

The cracking torque was found to increase with the increase in the amount of reinforcement, particularly with the longitudinal reinforcement. At the cracking, the twist was found to be increasing at a constant cracking torque. Thereafter, the torque and twist were increasing up to the ultimate torque. With the increase in the torque beyond the cracking, the cracks were formed in the ferrocement jacketed longer faces. The experimental initial stiffness, secant stiffness at the cracking torque and the secant stiffness at the ultimate torque values, the cracking and the ultimate torques are reported in Table 3. The same values using the analytical model are presented in the same Table 3. The initial stiffness of these beams was found to be invariable, but the secant stiffness at the cracking and the secant stiffness at the ultimate torque were found to vary with the state of torsion. There was a maximum decrease in the post-cracking torsional stiffness for the under-reinforced beams provided with a fewer number of mesh layers on the ferrocement face. The ultimate torque calculated by using the proposed model was compared with respect to the “U” wrapped plain beams with the same number of mesh layers. The ultimate torque was found to increase from the under reinforced to the longitudinally over reinforced, transversely over reinforced, and completely over reinforced beams. There was a significant improvement in the ultimate torque with the variations in the state of torsion. The ultimate cracking torques of the U4H, Lo4H, To4H and Co4H were experimentally found to be 6.42 kNm, 6.675 kNm, 6.618 kNm and 6.71 kNm against the predicted values of 6.52 kNm. The theory underestimates the cracking torque by 2.28%, 1.36% and 2.83% for the Lo4H, To4H and Co4H beams respectively, while it overestimates by 1.5% for the U4H. The ultimate torque of these beams were experimentally found to be 7.68 kNm, 7.87 kNm, 8.66 kNm and 12.91 kNm for the U4H, Lo4H, To4H and Co4H beams respectively. The predicted values of the ultimate torques were found to vary within 1% of the experimental values. The cracking torque of the B4H, L4H, T4H, U4H, Lo4H, To4H and Co4H was increased by 40.93%, 47.22%, 42.89%, 39.30%, 44.73%, 43.50% and 45.62% with respect to the plain BH beam respectively, whereas the ultimate torque of these beams was increased to 40.94%, 47.22%, 42.89%, 66.52%, 70.64%, 87.77% and 179.92% respectively as shown in (Fig. 14). The twist at the ultimate torque of these U4H, Lo4H, To4H and Co4H beams was found to vary by 0.1305 rad/m, 0.056 rad/m, 0.0921 rad/m and 0.0754 rad/m respectively. The predicted values were found to be more than 4.98% for the U4H and less than 1.78%, 0.6% and 2.4% for the Lo4H, To4H and Co4H respectively. The twist at the breaking was reported in (Tab. 3),

and the ratio of the twist of a beam with respect to the plain BH beam is presented in (Fig. 15).

The toughness of the beams was found to be the maximum for the under-reinforced beams as it allows the beams to twist more due to the less torsional stiffness. The To4H beam has more toughness in comparison to the Lo4H beam. When the number of layers is changed in the ferrocement wrap, it is found that toughness is increasing in the analytical model, but this increase is marginal. The variations of the torque were more prominent in the various states of torsion. The variations in the torque for the various states of torsion are presented in (Fig. 16).

4 CONCLUSIONS

- RC beams strengthened with ferrocement “U” wraps exhibit a significant increase in cracking torque, ultimate torque and ultimate twist when tested under pure torsion.
- The torsional strength of under-reinforced, partially over-reinforced and completely over-reinforced sections can be well predicted by this developed model.
- The equations developed are not only able to predict the torsional strength of “U” wrap beams, but can also predict the twists and strains in steel and concrete throughout their entire loading history.
- The developed model can well predict the torsional strength of “U” wrapped beams strengthened with any material other than ferrocement with suitable modifications in the material properties.
- The proposed equations well predict the ultimate torsional moments very closely to the experimental results.
- The experimental results reveal that the torque twist response of a ferrocement “U” wrap beam is more influenced by the state of torsion than the amount of ferrocement reinforcement.

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