DETERMINATION OF SUBGRADE REACTION COEFFICIENTS AND SPRING STIFFNESSES FOR A COMBINED PILE RAFT FOUNDATION (CPRF) BY MEANS OF A CLUSTER ANALYSIS

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Abstract

An analysis of combined piled raft foundations could be performed by means of a special geotechnical finite element software that utilizes 3D modeling and advanced constitutive models for the soil. However, a foundation is generally included in the structural models in a structural design program. The least advanced part of finite element software that has been developed for structural FEM modeling are the tools used for modeling behaviour. A method is required with which the input parameters of the structural design software that are used for modeling the soil structure interaction as well could be determined so that the results calculated by means of a geotechnical software that approximates realistic soil behaviour the best can be reproduced by structural design software as well. The procedure outlined in this paper by means of a cluster analysis provides a tool for a substantial and innovative improvement in subsoil modeling, to which not enough attention is generally paid in the structural FEM software, thereby resulting in a reduced amount of work associated with the task of inputting data.

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- Coefficient of subgrade reaction,
- Pile spring stiffness,
- Cluster analysis,
- FEM.

1 INTRODUCTION INTRODUCTION, ORIGIN OF THE PROBLEM

The MOL Campus Office Building is currently under construction in Budapest, Hungary. With its planned height of 143 m, it is regarded as the pioneer project of future high-rise buildings (Hudacsek et al., 2020). The designed program justifies the research aimed at the foundation of high-rise buildings. During the investigation of applicable foundation systems, it was demonstrated that it is reasonable to form the foundation of high-rise buildings with combined piled raft foundations (Kanizsár, 2017).

The favourable, but on the other hand, complex internal force distribution of the foundation system referred to as a combined pile raft foundation (CPRF) is difficult to follow with conventional design calculations. No standards are available for their dimensioning; only design principles that provide general guidelines exist (Katzenbach and Choudhury, 2013). The calculations can be carried out following the

principles shown in these guidelines by means of finite element modeling, provided that the results of an appropriate soil investigation program are available. The calculation of deformations that belong to the different load levels, for the determination of which special geotechnical FEM software that can work with advanced soil models are indispensable, is a part of the bearing capacity calculations.

The structural design practice evidently regards a CPRF structure as part of a superstructure model. It is a fundamental problem, however, that the subsoil modeling of structural design software rests on different theoretical basics than do geotechnical software. The displacements and stresses of a combined foundation structure modelled by means of the subgrade reaction coefficients and pile spring stiffnesses can only approximate the results which are calculated by the geotechnical software that work with realistic soil models if the distribution of subgrade reaction coefficients is simulated by selecting an appropriate distribution for them (Bak et al., 2010). As they are not constant, but change from point to point, i.e., to a smaller or greater amount, the subgrade reaction coefficients and spring stiffnesses of

structural design software should be determined in accordance with the above diverse values. This means a significant amount of data input even in the case of a medium-sized foundation slab, which is stressful for the practice. Due to the estimation of the coefficients of subgrade reaction and spring stiffness values by means of various approximative calculations and simplified presumptions and their excessively small number in relation to the variability of the subgrade reaction distribution, the results of the calculations obtained by means of structural design software differ from the results obtained with geotechnical software (Kanizsár, 2020).

A more precise method, which can be achieved by means of more advanced tools, is presented in the present paper. The development of software with similar purposes is still in progress, however, they are not published with a sufficient degree of scientific accuracy. Apart from the similarities of the fundamental concepts, however, an additional novelty of the method presented here is the application of cluster analysis, which is known from mathematical statistics, although not used in civil engineering practice; it provides an opportunity for a significant reduction of the data required for the description of spatial variations of the coefficients of subgrade reactions.

2 DESCRIPTION OF CLUSTER ANALYSIS

2.1 The principles of cluster analysis

The essence of statistical methods for data reduction is that a size reduction is performed on the data matrix, after which the statistical conclusions drawn from the reduced data set will still be valid for the original statistical multitude. In addition to numerous procedures (e.g., factor analysis, discrimination analysis), cluster analysis also belongs to this group.

Cluster analysis is the collective expression for dimension reduction procedures with the help of which the elements of data sets can be divided into groups, i.e., clusters on the basis of certain properties. The fundamental features of cluster analysis or clustering for short are cohesion and separation. Cohesion means data homogeneity within the individual clusters: the smaller the diversion of cluster elements from each other and the cluster average is, the stronger the cohesion is. In an analogous manner, the greater the difference between the individual cluster elements, i.e., the more the groups formed on the basis of their clustering aspects differ, the greater the separation. The joint assertion of the two principles results in a stable cluster structure; thus, certain mathematical and statistical properties of the significantly reduced data are equal to the identical characteristics that refer to the originally greater data set. The advantage of clustering lies precisely in the fact that we can work with much less data (thus saving time, work, and the capacity for calculations), so that the statistical properties of the smaller data set represent those of the original large data set.

2.2 Methods of cluster analysis

Numerous clustering procedures exist, which can be divided into two main groups: hierarchical and non-hierarchical algorithms. In hierarchical clustering, new clusters are obtained by the combination (agglomerative) or division (divisive) of the previously formed clusters. The non-hierarchical algorithm forms all the clusters in one single step. Since the latter clustering procedure might be applied more advantageously due to its lesser sensitivity to outlier values and high-

er speed, it has therefore been chosen for the computational method outlined. Among the non-hierarchical clustering methods considered feasible by the author, is the application of the most common, centroid-based, so-called K-means procedure, which was justified by its advantages (e.g., suitable, simple and fast for a large dimension data matrix) and its disadvantages, which are less relevant from the aspect of the current task (e.g., built-in metrics, the previously supplied cluster number). It must be stated prior to the description of the K-medium cluster analysis, that for measuring the distance within and between clusters, the general Minkowski metric is used:

$$X = (x_1, x_2, \dots, x_n) \text{ and } Y = (y_1, y_2, \dots, y_n) \in R^n \quad (1)$$

$$D(X, Y) = \left(\sum_{i=1}^n |x_i - y_i|^p \right)^{\frac{1}{p}} \quad (2)$$

, where $p \geq 1$: the Minkowski constant (in the case of $p = 2$, the Euclidean distance is obtained).

2.3 Formation of the cluster structure by means of the K-means method

The definition of the K-means clustering analysis is the following:

Given a $(x_1, x_2, \dots, x_n)$ data set in which all the variables are a d element vector, the aim of the K-means clustering is to divide the n number of data groups $C = (C_1, C_2, \dots, C_k)$, consisting of k ($\leq n$) numbers of clusters, so that the sum of the square differences of the elements within the cluster from the cluster average should be minimum. The target function to be minimized is:

$$\min \sum_{i=1}^k \sum_{x \in C_i} \|x - \mu_i\|^2 = \sum_{i=1}^k |C_i| \text{Var } C_i \quad (3)$$

, where μ_i is the average value of the C_i elements.

The simple algorithm consists of the alternating repetition of a further two steps of the initial step; the procedure lasts until the elements do not change clusters any more:

- K number of the initial cluster centre are presumed: $m_1^{(1)}, \dots, m_k^{(1)}$.
- All the elements are divided into the cluster that corresponds to the cluster centre that is closer on the basis of the Euclidean distance:

$$C_i^{(t)} = \{x_p : \|x_p - m_i^{(t)}\|^2 \leq \|x_p - m_j^{(t)}\|^2 \forall j, 1 \leq j \leq k\} \quad (4)$$

All x_p is put into one single cluster, even though it could be placed into two or more different groups. (t) is the actual number of steps of the algorithm.

- The cluster centres are recalculated:

$$m_i^{(t+1)} = \frac{1}{|C_i^{(t)}|} \sum_{x_j \in C_i^{(t)}} x_j \quad (5)$$

2.4 Measurement of the validity of the cluster structure and the definition of the reasonable cluster number

The reasonable number of clusters might be defined by the previously performed hierarchic clustering. In this method, however, a cluster

validity index, i.e., the so-called silhouette coefficient (also known as the silhouette index), was chosen for this purpose. Numerous metrics exist that might be used for regarding the “goodness” of the clustering, i.e., the strength of the cluster structure, thus the accuracy of the cluster number applied (Szüle, 2019). A reason for the use of the silhouette coefficient is the fact that, as opposed to other cluster validity indexes, it might also be used in the case of a non-hierarchical clustering procedure; it measures cohesion and separation in a simple manner and can be defined as the result of an algebraic expression, not in a graphic manner (such as the cluster elbow index). It can be calculated according to the following:

Indicate $a(i)$ the x_i average distance C_i of the element from other elements within the cluster:

$$a(i) = \frac{\sum_{j \in C_i, j \neq i} d(x_i - x_j)}{|C_i| - 1} \quad (6)$$

C' should refer to the set of clusters outside the C_p , the elements of which are indicated by K .

For the $\forall K \in C'$ cluster

$$d(x_i, K) = \frac{\sum_{j \in K} d(x_i - x_j)}{|K|} \quad (7)$$

i.e., the average distance of the associated element x_i from all the elements of the other clusters $x_j \in K \in C'$ must be determined; then the minimum that must be indicated by the following $b(i)$

$$b(i) = \min_{K \in C'} d(x_i, K) \quad (8)$$

must be determined.

The $s(i)$ silhouette values might be defined with the above in the following manner (Rousseeuw, 1987):

$$s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}} \quad (9)$$

The cluster $K^* \in C'$, for which the distance $d(x_i, K^*)$ is only less in the case of the minimum $b(i)$, is called the neighbour of x_p , i.e., the second-best cluster which x_i could have belonged to.

After the clustering, the $s(i)$ values could be calculated for the cluster elements. The values range in the $[-1, 1]$ interval, and the following conclusions can be drawn:

- if $s(i) < 0$, then x_i is presumably in the wrong cluster; it is closer to K^* than C_p ,
- if $s(i) \approx 0$, but $s(i) > 0$, then x_i is situated between clusters K^* and C_p ,
- if $s(i) \approx 1$, then x_i is in the right cluster.

The SC (silhouette coefficient) characterizing the clustering is the average of the silhouette values:

$$SC = \frac{\sum_{i: x_i \in C} s(i)}{|C|} \quad (10)$$

, where $|C| = |C_i \cup C'| = k$. On the basis of the above:

- if $SC \in [0.71, 1.0]$, then the cluster structure is strong,
- if $SC \in [0.51, 0.7]$, then the cluster structure is appropriate,
- if $SC \in [0.26, 0.5]$, then the cluster structure is weak,
- if $SC < 0.25$, then the structure cannot be considered a true cluster structure.

3 REVIEW OF THE NEW CALCULATION METHOD

3.1 Principle of the method

The results related to the CPRF structure are obtained from the entire building model calculated in Plaxis 3D, which are regarded as

the reference values. In another FEM simulation performed by the structural design software, the coefficients of the subgrade reactions and spring stiffnesses of the identical structural model were calculated during a correction (approximation) process in a way that the settlements obtained for the CPRF structure should correspond to the reference values or should approximate the values within a previously set error bound. The primary aim with the structural design software is the calculation of the distribution of settlements obtained with the Plaxis-3D package with the prescribed degree of precision (under the distribution of the settlements, the deformation state intended is determined by the displacements and rotations belonging together and forming a common deformation system on the common plane of the foundation slab and the soil. The concept will be further specified in Chapter 5).

The distribution of settlements with the prescribed degree of precision could be obtained from the Plaxis 3D settlement values by the application of the subgrade and spring stiffnesses, which can be calculated from the kinematic load indirectly prescribed for the structure design model. These values that can be calculated directly from the Plaxis 3D model as input parameters that typically form a huge data set, the size of which could be significantly reduced at each correction step by the application of the cluster analysis. As a consequence, in the case of identical global and local structural stiffnesses of the two models and their corresponding external loads, the same displacements result in identical internal forces. On the basis of the above, the essence of the method can be summarized briefly in a way that the desired CPRF input parameters might be determined by means of the kinematic finetuning of the structural design model to the results of the Plaxis 3D reference model, while its input data set is reduced by cluster analysis to a manageable size for practical calculations.

3.2 The basic assumptions and requirements related to the application

3.2.1 Geometry

The superstructural parts of the FEM models created by the geotechnical and structural design models are identical in the geometrical sense. The identicalness of the applied beam and surface elements refers to the complete superstructural model. If certain parts of the superstructures are modelled by volume elements instead of beam and surface elements, then the required geometrical identity refers to the beam axes and central planes. The complete geometrical identity of the finite element meshes is not a condition.

3.2.2 Finite elements

The correspondence of the type of elements used by the two models is not identical, but the identicalness of the degree of freedom of their nodes is a requirement. Mindlin-type elements capable of capturing shear deformations were applied to the plates. Plaxis 3D provides for the usage of volume elements together with the so-called “dummy” plate and/or beam elements that meet the above requirements.

3.2.3 Loads and effects

An analysis of the CPRF structures was carried out for the vertical loads. The horizontal effects and subgrades and the moment vectors perpendicular to the plane of the plate were ignored due to their subordinate significance. Therefore, under the expression “stiffness of the subgrade reaction”, the vertical stiffness of the subgrade reaction is hereinafter intended. The rotation stiffness of the springs

representing the piles will only refer to the stiffness belonging to the rotations around the (horizontal) axes on the plane of the plate.

All the load combinations deriving from the load cases and those belonging to the service and ultimate limit states (SLS and ULS) theoretically require a separate analysis, as various subgrade reaction distributions and settlements are developing. In design practice from both the structural design side and the geotechnical aspect, it should be considered which load combinations require a separate analysis or which are critical. The settlement calculations are included in the analysis of the SLS limit state, which must be performed on the basis of a quasi-constant load combination. This provides mostly characteristic values regarding the displacements of the structure that can be measured in reality; thus, it is considered as the main combination in the analysis. This method can also be used for other load combinations in an unchanged form.

3.2.4 Mechanical and statical presumptions

The material models of the superstructures of both constitutive models are identical, i.e., a linearly elastic material model either for concrete or steel. It is presumed that in the case of identical settlement distributions gained from the two models, the force and displacement systems of the superstructural models are identical since the identical material equations can only connect the kinematically admissible identical displacement systems with identical, statistically admissible, force systems (Kaliszky et al., 1990).

3.2.5 Foundation structure

In the geotechnical software the piles of the CPRF structure are structures modelled by volume elements or by special beam elements embedded in the soil (*embedded beam*) in which normal forces and bending moments might operate. The connections between the piles and the raft are capable of transferring bending moments. The distribution of internal forces within the CPRF structure can be modelled by linear, vertically compressible, and spiral springs rotating around axes that are perpendicular to each other. The normal forces and moments that arise in the nodes of the pile caps and act upon the raft are simulated by the above-mentioned springs. The value of the variable vertical normal force is regarded as valid at the pile cap since this is the force that directly causes a load on the slab.

3.2.6 Soil

Besides the presumption of the soil models of the geotechnical software, no supplements should be added. The modelled ground profile might be built from an arbitrary number and type of soil layer with any spatial expansion. From the aspect of the soil structure interaction (*SSI*), i.e., the interaction between the soil and the structure, the relevant plane is the common part of the soil zone modelled by two different software. This is the plane defined by the surface elements in the structural design software modeling the foundation slab, which in the case of modeling the planar structure, means its midplane and also the contact surface of the raft and the soil. Although the piles in reality are structures embedded in the soil under the raft, in the structural design software, the springs modeling the piles are located in the above-mentioned plane where the force transfer is realized during the modeling.

3.3 Main steps of the method

The sequence of the steps of the method is the following (some concepts will be provided in the next section):

- creation of a reference model in Plaxis 3D with a common, so-called matching grid,
- creation of an equivalent model in the structural design software and also with the matching grid,
- determination of the reference values in Plaxis 3D,
- assumption of the subgrade and spring stiffness values,
- clustering of the subgrade and spring stiffness values,
- definition of the optimal cluster number by the silhouette coefficient method,
- the running of the structural design software, after providing the optimum cluster number subgrade and spring stiffnesses,
- evaluation of the results of the structural design software on the basis of the reference settlements,
- decision on ending or continuing the recursive approximation process on the basis of the error analysis,
- in the case of continuing the correction of the subgrade and spring stiffnesses on the basis of relative errors,
- increase in the index of the correction step by 1; repetition from the assumption of the initial values.

4 DESCRIPTION OF THE BASE ELEMENTS OF THE METHODS

4.1 Reference model of Plaxis 3D

4.1.1 Modeling principles

The complex model of the structure examined must be created in the Plaxis 3D software, which is suitable for the spatial finite element analysis of geotechnical tasks. “Complex” in our case means that the modeling of the soil environment and the superstructure is performed with the application of the actual geometry, loads, and realistic soil models. The building is not divided into various parts, either in the geometric or the statics sense (e.g., the division in the top plane of the foundation slab), but the results related to the structural element to be examined are gained from the complex model of the entire structure. The advantage of this procedure as opposed to separate modeling is that by ignoring the division into parts, the uncertainties arising during the load transfer might be eliminated (e.g., managing the action and reaction forces by means of necessary averaging, the reduction of extreme values, smoothening of the loads, etc.); the actual stiffness of the building could be taken into account in all phases of the analysis, and the desired results can be obtained with identical superstructural models. Thus, the procedure is in accordance with the presumption that has been formulated by many authors (Lee and Brown, 1972; Brown and Yu, 1986; Zhang and Small, 1994; Poulos, 2016): the correlation between the stiffening effect derived from the superstructure and the relative settlements is substantial. In practice it can only be realized in a way that the geotechnical software, similarly to Plaxis 3D, is suitable for the appropriate modeling of the superstructure.

4.1.2 Modeling of the structural elements

The modeling of the CPRF structure in the Plaxis 3D software can be performed by the combination of the plate and beam elements embedded in the soil specifically developed for pile modeling. Besides, there is also an opportunity for the modeling of the structure by means of the actual volume elements either in the case of the pile or the plate. This option, besides certain restrictions, enables a more realistic modeling of the driven piles or might also serve as an analysis of the spatial stress conditions of the structural connections. One limitation in the modeling could be the construction technology in-

fluencing the load-bearing capacity or the large number of structural elements. For obtaining the internal forces, however, with the use of the volume elements, it is necessary to apply the apparent so-called “dummy plate” elements that are defined in the middle plane of the foundation slab and the “dummy beam” element created in the pile axes. The interaction between the raft and soil and the potential relative displacement could be ensured by the application of the interface elements.

4.1.3 Structural connections

The structural connection between the pile and the foundation slab is assumed to be moment bearing. This naturally depends on the reinforcement and the structural layout; however, for the aim of the general task definition besides the normal forces acting upon the piles, the bending moments that are transferred from the raft are also examined. In the case of the zero values of the rotation pile stiffnesses, a special case of the CPRF is achieved; the piles are loaded only by normal forces. The definition of the subgrade stiffnesses of the foundation slab without piles (raft foundation (*RF*)) could be regarded as another special case. The method could also be used for combined foundation structures in which there is no structural connection between the raft and the deep foundation structures (non-connected pile raft foundation (*NCPRF*)). What is more, the applicability also expands to *rigid inclusion* foundation systems that are completed with a load transfer layer between the raft and the piles (Kanizsár, 2019).

4.1.4 Concept of matching grid – stress definition points and nodal settlements

The Plaxis 3D model and the nodal geometry of the CPRF models of the structural design software could be tuned so that the results of the individual nodes can directly be compared and that the output of the Plaxis 3D might be used as a direct input in the structural design software. Naturally, in the identical layout the finite element meshes are not required to be perfectly identical. However, with pre-defined coordinates, a “mesh in the mesh” point network could be created with the coordinates of whose points correspond to each other and also as part of the finite element meshes of each type of software. Prior to providing its details, the concept of a “domain” must be introduced. It can be created in the structural design software by the unification of more surface elements with identical features, but a domain specifically consisting of a single element could also be generated.

The geometry of a mesh consisting of m number of nodes might be defined by two-dimensional raft elements in Plaxis 3D by the $\underline{P}$ vector, which, when distributed to m blocks, contains the nodal coordinates for each block. The set containing all the elements of $\underline{P}$ is indicated by A . Similarly, in the structural design software, $\underline{Q}$ is the vector of the mesh created by node n , and B is the set including the elements of $\underline{Q}$.

$$\underline{P} = \begin{bmatrix} \underline{p}_1 \\ \vdots \\ \underline{p}_m \end{bmatrix} \text{ and } A = \{ \underline{p}_i \mid i = 1 \dots m, m \in N \} \quad (11)$$

$$\underline{Q} = \begin{bmatrix} \underline{q}_1 \\ \vdots \\ \underline{q}_n \end{bmatrix} \text{ and } B = \{ \underline{q}_j \mid j = 1 \dots n, n \in N \} \quad (12)$$

The k number of blocks of vector $\underline{H}$, including the central nodes of $\underline{P}$ and $\underline{Q}$ with identical coordinates, are formed from the elements of set C , which are defined as the intersection of A and B :

$$C = A \cap B = \{ \underline{h}_l \mid l = 1 \dots k, k \in N \} \Rightarrow \underline{H} = \begin{bmatrix} \underline{h}_1 \\ \vdots \\ \underline{h}_k \end{bmatrix}, \quad m > k < n. \quad (13)$$

The mesh that was defined by the elements of vector $\underline{H}$ by means of secondary finitization will be called the “matching” grid, thus distinguishing it from the actual finite element mesh of each type of software. As a definition of the nodal coordinates of the matching grid, it must be taken into account that in the structural design software, the value of the average subgrade stiffness for a domain must be defined for its centroid. The subgrade stiffness that can be calculated from the nodal stress and the settlement can only be obtained from the results of the point of the same coordinate in Plaxis 3D. Therefore, in all the points where we wish to provide a subgrade stiffness value related to the domain in the structural design software, we also need to have a Plaxis 3D node. The distance between the nodes in the matching grid, i.e., the selection of the mesh density is the result of a subjective decision. However, similar to the general layout principles of the geometry of the finite element meshes, certain aspects must naturally be taken into account. The layout expansion of the CPRF structure, the raft thickness, the distance between single piles, the pile axis distances within the pile groups, and the positions of the columns and walls should also be considered. It is generally true that an increase in the density of the matching grid enables a more precise matching between the deformation and load results of the two kinds of software, especially in the environment of the more intensely altering function values.

4.2 Model of the structural design software

4.2.1 Geometric alignment of the FE-mesh, domains

The modeling of the raft is performed by surface elements that obtain a Winkler-type flexible subgrade soil side support. It is feasible to divide the finitized range into domains formed by the merging of finite elements with the same characteristics. According to the definition of the geometry of the domains, it must be kept in mind that the coefficient of the subgrade reaction of the surface elements in the merged ranges are identical. This average value might be provided in the centroid node of the domain, and this will be valid for each surface element of the range.

The matching grid used by the structural design software must include nodes, the coordinates of which are identical with the connection points common to the piles defined in the Plaxis 3D software. Vector $\underline{H}$ can be divided in two different blocks. One block should be indicated by $\underline{H}_L$, which includes the same number of nodes as the k_L number of domain centroids. It is possible to provide the average coefficient of the subgrade reaction related to the domains in the nodes included in vector $\underline{H}_L$. The other block, i.e., $\underline{H}_C$ includes the coordinates of the k_C number of nodes modelled by the springs:

$$\underline{H} = \begin{bmatrix} \underline{H}_L \\ \underline{H}_C \end{bmatrix} \text{ and } k = k_L + k_C \quad (14)$$

, where

- k is the number of blocks of vector $\underline{H}$, i.e., the number of nodes of the matching grid,
- k_L is the number of domains of the foundation slab,
- k_C is the number of piles.

The coordinate vectors $\underline{X}$ and $\underline{Y}$ which include the x and y coordinates of the nodes of vector $\underline{H}$, must be defined. Both can be divided into two blocks on the basis of the raft and pile nodes:

$$\underline{X} = \begin{bmatrix} \underline{X}_L \\ \underline{X}_C \end{bmatrix} \quad \text{and} \quad \underline{Y} = \begin{bmatrix} \underline{Y}_L \\ \underline{Y}_C \end{bmatrix} \quad (15)$$

It cannot be expected, particularly in the case of very large layout rafts, that the subgrade stiffness values are individually determined on the basis of the 3D results. Therefore, finding the optimum reduced number of the unified subgrade reaction zones is one of the crucial steps of the method. In the later section of the review, it will be discussed in which way the optimum number $k_{opt} \ll k$ of different coefficients of the subgrade reaction could be assumed so that the settlement distribution obtained in the structural design software would essentially correspond with the settlement distribution achieved in Plaxis 3D. The concept of settlement distribution that was generally defined in Section 3 must be supplemented with information so that it can be regarded as valid on the set of points of the matching grid.

4.2.2 CPRF modeling for vertical loads

In modeling software using subgrade stiffnesses, the support conditions of vertically loaded CPRF structures could generally be characterized by four different input parameters, i.e., by the coefficient of subgrade reaction K_z of the raft foundation, vertical spring stiffness R_z of the piles, and the rotational stiffness of the springs R_{xx} and R_{yy} representing the pile caps. The latter two rotation stiffnesses are neglected by the design practice in many cases; this simplifies the task, although it is a fundamental mistake. According to the Mindlin Plate Theory, besides the displacement function, displacements $\underline{u}$ must be described by the rotation functions that cannot directly be derived from it (Bojtár and Gáspár, 1992).

$$\underline{u} = \begin{bmatrix} w(x, y) \\ \varphi_x(x, y) \\ \varphi_y(x, y) \end{bmatrix} \quad (16)$$

At the pile-raft connection point, the raft rotations should be interpreted together with the rotations of the pile caps, which depend on the bending stiffness, length, and soil as the subgrade medium. The latter determine the rotation stiffness, which, in the case of a group of large diameter piles, has a significant impact even on the rotations of a thick foundation slab.

4.2.3 Subgrade and spring stiffnesses

By applying the displacement method known from statics, the finite element method provides nodal displacements as direct results. Vector $\underline{s}$ includes the settlements of the nodal points of vector $\underline{H}$. So that the further amounts can be defined, $\underline{s}$, similarly to $\underline{H}$, is divided into two blocks for the sake of differentiation between the centroids of the domains and the nodal points of the pile springs:

$$\underline{s} = \begin{bmatrix} \underline{s}_L \\ \underline{s}_C \end{bmatrix} \quad (17)$$

According to the definitions of the subgrade and spring stiffnesses, they are the ratios of the forces, specific forces, moments, specific moments to displacements, and rotations:

$$\underline{K}_{z,i} = \frac{\sigma_{z,i}}{\underline{s}_{L,i}}; \underline{R}_{z,j} = \frac{F_{z,j}}{\underline{s}_{C,j}}; \underline{R}_{xx,j} = \frac{M_{x,j}}{\varphi_{xx,j}}; \underline{R}_{yy,j} = \frac{M_{y,j}}{\varphi_{yy,j}} \quad (18)$$

- $\underline{K}_{z,i}$: i^{th} element of the vector of the subgrade stiffnesses,
- $\sigma_{z,i}$: i^{th} element of the effective vertical soil stress vector,
- $\underline{s}_{L,i}$: i^{th} element of the raft settlement vector block,
- $\underline{R}_{z,j}$: j^{th} element of the vertical spring stiffness vector,
- $F_{z,j}$: j^{th} element of the vertical pile force vector,

- $\underline{s}_{C,j}$: j^{th} element of the pile settlement vector block,
- $\underline{R}_{xx,j}$ and $\underline{R}_{yy,j}$: j^{th} elements of the pile cap rotation spring stiffness,
- $\underline{M}_{x,j}$ and $\underline{M}_{y,j}$: j^{th} elements of the moment vectors causing the pile cap rotations,
- $\varphi_{xx,j}$ and $\varphi_{yy,j}$: j^{th} elements of the pile cap rotations,
- $i=1 \dots k_L$ and $j=1 \dots k_C$.

Section 5 presents a detailed description of the steps of the method, furthermore, an explanation can be found about which model results the vector elements should be calculated by at each step of the approximation.

4.3 Definition of the Reference Values

In Plaxis 3D the results obtained for the points of the matching grid from the calculations performed on the reference model are regarded as reference values. Among the above primary reference values are the ones on the basis of which the settlement distributions of the model of the structural design software might be tuned to that of the reference model and on the basis of which the subgrade and spring stiffnesses are corrected at each correction step. As secondary reference values, the values were interpreted that are not required in each step of the correction procedure and can typically be used as error indicators or fixed threshold values.

Primary reference values:

- $\underline{s}_{ref}$: settlements,
- $\underline{M}_{x,ref}$ and $\underline{M}_{y,ref}$: moments causing the pile cap rotations.

Secondary reference values:

- $\sigma_{z,ref}$: vertical effective soil stresses,
- $F_{z,ref}$: vertical pile force,
- $\underline{m}_{x,ref}$ and $\underline{m}_{y,ref}$: raft moments,
- $\underline{N}_{column,ref}$: normal forces of the columns standing on the foundation slab,
- $\underline{M}_{z,column,ref}$ and $\underline{M}_{y,column,ref}$: moments of the columns standing on the foundation slab.

4.4 Implementation of the Cluster Analysis

4.4.1 Aim of the application

The matching grid of large layout dimension CPRF structures includes too many nodes, so that the tuning of the structural design model can be performed by handling them individually. As an example, the matching grid of a CPRF plate of 100x50 m layout dimensions with a 2x2 metre division includes more than 1300 points, for which, in the case of an 8x8 metre column raster, the number of piles/pile groups under the columns extends the data set to approximately 1400 elements. It becomes evident that the determination of the subgrade and spring stiffness values at so many points is very labour intensive and time consuming. What is more, it must be performed on more than one occasion. In the model tuning (correction) procedure of the method, the cluster analysis described in general in Section 2 serves as a significant reduction of the above-mentioned amount of work. The aim is to set up a description for the subgrade stiffness distribution with a $k_{opt} \ll k$ size, i.e., a strong (SC ≈ 0.7) cluster structure, which is capable of reproducing the settlement distributions produced by Plaxis 3D with the predefined degree of accuracy despite the significant reduction of the data. The smallest number that is suitable for the formation of this cluster structure will be referred to as the optimum cluster number. When it is applied, the error convergence of the correction steps is of an acceptable speed. It typically and asymptotically proceeds towards zero and not towards any other error threshold. Clustering might be used for

the spring stiffnesses of the piles; however, due to their much smaller number related to the surface elements, the advantages of the method might primarily be perceived in the case of the subgrade stiffnesses:

$$k_{opt,L} \ll k_L; \quad k_{opt,C} \leq k_C; \quad k_{opt} \ll k$$

$$\text{and } |\underline{s}_{ref,i} - \underline{s}_i| \rightarrow 0, \quad i = 1 \dots k \quad (19)$$

, where

- $k_{opt,L}$: optimum number of subgrade clusters,
- $k_{opt,C}$: optimum number of pile spring clusters,
- $k_{opt} = k_{opt,L} + k_{opt,C}$: joint optimum number of subgrade clusters and pile spring clusters.

4.4.2 Presentation of the clustering performance

The size of the initial database corresponds with the number of nodes (k_L) of block $\underline{H}_L$ of the matching grid, as this amount of subgrade stiffness can be calculated for the domains from the stress and settlement values of the nodes. The analysis is performed with three variables, i.e., at all points apart from the initial coefficient of the subgrade reaction value, an x and y coordinate also appears as a variable that is interpreted in the local coordinate system assigned to the matching grid. At the formation of the cluster structure, not only is the value of the subgrade stiffnesses exclusively essential, but the layout position of the points is also important, since the subgrade stiffness of an individual point becomes important in accordance with its immediate environment. As an example, in the analysis of the stress peaks at the edges and corners of the raft, it is unfortunate if the subgrade stiffness values of the points that are located far from the phenomenon, and thus are independent, are included in the clusters for this area. This could distort the approximation of the stress function, which is very sensitive in this location with high gradient changes.

Vector $\underline{K}_z$ includes the ratio of the vertical soil stresses and the settlements at the nodes of the matching grid calculated by Plaxis 3D, while $\underline{X}_L$ is the vector of coordinate x , and $\underline{Y}_L$ is coordinate y . Although the dimensions of the three variables do not correspond, and in most cases, standardization is required prior to the clustering, in this case it can be disregarded. Since two variables out of three form a geometrical coordinate pair, the third coordinate, i.e., the subgrade stiffness, can be interpreted as the third, i.e., the coordinate z ; thus, we have a spatial vector at all the points. Spatial Euclidean distances might be calculated between the individual points.

The clustering should be completed in accordance with the above-mentioned three variables (K_z, x, y), for cases of the various cluster numbers, out of which the optimal cluster number $k_{opt,L}$ is se-

lected by means of the silhouette coefficients. As a rule of thumb, the following formula can be applied:

$$k_{max} = \sqrt{n/2} \quad (20)$$

which in the case of a set of data consisting of n elements, provides the maximum number of clusters (Kovács, 2014). On the basis of the experience gained from the clustering that was performed, the meaning of the formula was modified in a way so that it would provide the number around which the optimum cluster number $k_{opt,L}$ would probably be found. A slightly higher or lower cluster number than the value that can be calculated from the formula could also be optimal. It is advisable to perform clustering with a selected value upwards and downwards from the value that can be calculated from the formula with a chosen Δk value. There is no exact formula for the value of Δk (typically 1-2), as the expression $\sqrt{n/2}$ itself is not exact either. If, upon an evaluation of the silhouette coefficients, it becomes evident that Δk was chosen for a small value, then the width of the examination range might be increased.

After the clustering has been completed by $\sqrt{n/2} \pm \Delta k$ pieces of cluster, the silhouette coefficients should be determined for each cluster number, which could be illustrated in relation to the cluster numbers. Since the value of the silhouette coefficient characterizes the strength of the cluster structure, the environment of the local maximum near the k_{max} value should be examined. Fig. 1 represents such a curve. It is evident that the value of the silhouette coefficient reaches its maximum at $k = 2$. Nevertheless, this value was indicated for the sake of explanation and completeness; practically, it is irrelevant. In a raft design task (and the same is valid for large element statistical tasks), two clusters are definitely not sufficient for a description of the distribution of subgrade stiffness values even if the value of the silhouette coefficient is otherwise high in the absolute sense.

Fig. 1 Silhouette coefficients in relation to cluster numbers

Silhouette Statistics				
Cluster	Case Count	Statistics		
		Mean	Minimum	Maximum
1	8,000	,565	,040	,778
2	1,000	1,000	1,000	1,000
3	20,000	,626	,096	,777
4	1,000	1,000	1,000	1,000
5	18,000	,799	,527	,874
6	4,000	,798	,629	,873
7	3,000	,747	,678	,815
8	3,000	,633	,418	,753
9	52,000	,653	,015	,789
Total	110,000	,679	,015	1,000

Dissimilarity measure = Euclid

Fig. 2 Table of silhouette values and frequency histogram from the SPSS software

It is appropriate to examine the curve following the lowering section after the initial absolute maximum section assumed at $k = 2$, around the cluster number; $\sqrt{n/2}$, the value of $k_{opt,L}$, is expected on this section of the curve. In Figure 1, which was presented as an example illustrating the silhouette coefficients, it is evident in the case of the point indicated for cluster number 9, that the value of SC is approximately 0.7, which indicates a strong cluster structure. The same becomes clear from Figure 2, which derives from the results of the SPSS statistical software package used for the clustering. The improvement of the subgrade stiffnesses, and as a result, the rate of approximation of the settlement values to the reference values, is the largest in cases of the optimum number of clusters. If we differ from the cluster number $k_{opt,L}$ in any direction by a small amount, then, as a result, the convergence speed of the errors that can be calculated is smaller in order to achieve the correspondent settlement distributions, which requires more steps than in the case of $k_{opt,L}$. In cases of greater discrepancies, the average relative errors derivable from the reference values of each correction step cannot be reduced under a certain level, which results from the application of a cluster structure diverse from the optimum one.

The following observations can be made regarding the model tasks:

- the statistically optimum value of the silhouette coefficient in the case of a raft problem does not necessarily correspond with the optimum cluster number, which is due to the fact that the local stress peaks of the raft cannot be traced with a number smaller than a certain number of clusters,
- with identical SC values, it is worth choosing the larger cluster number,
- a cluster number smaller than the $\sqrt{n/2}$ value usually proves to be insufficiently low,
- the $SC \approx 0.7$ value results in an appropriate cluster structure,
- following the reduction of a cluster number compared to the previous correction step, the average error of the correction step generally increases in relation to the previous one, while the convergence slows down,
- if the value of SC in the case of a smaller cluster number significantly exceeds the value of 0.7 (e.g., around 0.8 - 0.9), but in the case of greater cluster numbers, it also approximates it (even from downwards), it is worth choosing a smaller cluster number related to the SC value.

After the definition of $k_{opt,L}$ during the clustering, the $k_{opt,L}$ cluster centre, i.e., the values of the subgrade stiffness parameters are determined and assigned to the elements of the data set, i.e., the domains. This is referred to as the classification into subgrade stiffness clusters, by the end of which the subgrade stiffness is available for all the domain centres.

A reduction might be obtained, depending on the size of the set of data regarding the types of the subgrade stiffness. Due to the $\sqrt{n/2}$ rule of thumb, the increase in the number of elements of the data set means an exponentially increasing saving of work. A data set consisting of a few hundred elements might be approximated by 3-10% of the cluster, which at the magnitude of thousands, approximates to 1%. This means that in case of the subgrade stiffnesses determined in just a few hundred points in the data supplied, at least 90% of the work and time savings can be realized. Naturally, it is reduced by the time and work load spent on clustering, but the balance (particularly in the case of a large database) is absolutely positive.

The clustering procedure outlined for raft foundations is also applicable for single spring stiffnesses individually if it is justified by the number of piles. In the case of a small number of piles, the clustering work might exceed the work spent on providing individual spring stiffnesses that are different in cases of various piles. In the latter case

it is not worth clustering the pile spring stiffnesses; it should be considered earlier. Due to the high calculation demands for clustering, the use of a statistical target software is recommended (e.g., SPSS, XLSTAT etc.) with which the task can be completed simply and fast.

It must be emphasized that the silhouette coefficient used for the selection of the reasonable cluster number is not exclusive, but is an alternative element of the method. Further procedures might also be applied for the selection of the optimum cluster structure since numerous further indexes exist for the measurement of validity.

4.5 Parameter Correction

In the steps of the correction procedure, the initial values of the subgrade and pile stiffnesses can be modified on the basis of the results of the runs performed with these input parameters. The rate of correction of the subgrade and vertical spring stiffness is determined by the diversions of the settlement results obtained that are related to the reference settlements. For the correction of the rotational stiffnesses, the pile cap bending moments and their reference values are used, for which there is a modeling reason that will not be outlined here. The description specified by the vector is the following:

At each correction step the calculations performed with the input parameter $\underline{K}_z$ of the structural design software provide an output value $\underline{s}_L$. With the difference of the latter to $\underline{s}_{ref}$, the value of $\underline{K}_z$ is modified. In the next step with the modified $\underline{K}_z$ input, a newer $\underline{s}_L$ can be calculated, by which the difference to $\underline{s}_{ref}$ could be determined again. As a result of the repeated process, we can reach the value of $\underline{K}_z$, with which the difference of $\underline{s}_L$ to $\underline{s}_{ref}$ is within a set error bound from the calculations.

This principle might also be applied to the correction of the stiffnesses, $\underline{R}_x$, $\underline{R}_{xx}$ and $\underline{R}_{yy}$, considering the fact that the corrections of the latter two quantities are calculated on the basis of the differences of the moments $\underline{M}_{x,spring}$ and $\underline{M}_{y,spring}$ and their reference values $\underline{M}_{x,ref}$ and $\underline{M}_{y,ref}$. The corrections of four different stiffness input parameters are performed simultaneously in the same correction steps during the correction procedure.

5 DETAILED DESCRIPTION OF THE CALCULATION PHASES

5.1 Initial Values of the Subgrade and Spring Stiffnesses

The determination of the final stiffness values in most cases is a multi-step correction process. In the initial step the subgrade and spring stiffness vectors, $\underline{K}_z^{(N)}$, $\underline{R}_z^{(N)}$, $\underline{R}_{xx}^{(N)}$ and $\underline{R}_{yy}^{(N)}$ are produced $\underline{X}_L$ along with the $\underline{Y}_L$ coordinate vectors. In case we also wish to cluster the spring stiffness values, the corresponding $\underline{X}_C$ and $\underline{Y}_C$ coordinate vectors are also required. In the first correction step the initial values can be calculated from the results of the reference Plaxis 3D (stresses and settlements); in the further steps they are identical with the corrected stiffness values of the previous step:

- in the case of $N = I$:

$$\underline{K}_{z,i}^{(N)} = \underline{\sigma}_{z,i} / \underline{s}_{L,i}, \quad \underline{R}_{z,j}^{(N)} = \underline{F}_{z,j} / \underline{s}_{C,j} \quad (21)$$

calculating from the reference values of the Plaxis 3D model, $\underline{R}_{xx}^{(N)}$ and $\underline{R}_{yy}^{(N)}$ are arbitrary initial values;

- in the case of $N > I$:

$$\underline{K}_z^{(N)} = \underline{K}_{z,corr}^{(N-1)}; \quad \underline{R}_z^{(N)} = \underline{R}_{z,corr}^{(N-1)}; \quad \underline{R}_{xx}^{(N)} = \underline{R}_{xx,corr}^{(N-1)}; \quad \underline{R}_{yy}^{(N)} = \underline{R}_{yy,corr}^{(N-1)} \quad (22)$$

where the $N \geq 1$ value in brackets indicates the actual number of the correction step, the corrected stiffnesses indicated by the index “corr” should be calculated in accordance with Section 5.6.

The vectors, including the initial values of the rotational stiffnesses, cannot be produced by calculating $\underline{R}_{xx}^{(1)}, \underline{R}_{yy}^{(1)}$, since the reference values related to the pile cap rotation are not available. However, the stiffnesses might be assumed by an arbitrary initial value, since in the course of the correction procedure, they will be modified until the calculated moments become identical to those from the reference model. In a manner analogue with the Newton-Raphson numerical method, which is also used by finite element software, if the initial value falls too far from the target value, then the convergence speed is low (there is no convergence in extreme cases). Thus, it is worth starting from a realistic range, e.g., by means of estimation based on an approximation formula (Gazetas, 1984).

5.2 $\underline{K}_z^{(N)}, \underline{R}_z^{(N)}, \underline{R}_{xx}^{(N)}, \underline{R}_{yy}^{(N)}$ size reduction of stiffness vectors by means of a cluster analysis

$\underline{K}_z^{(N)}, \underline{R}_z^{(N)}, \underline{R}_{xx}^{(N)}, \underline{R}_{yy}^{(N)}$, the previously outlined clustering procedure, is performed on the elements of the vectors. The optimal cluster number is selected on the basis of the silhouette coefficients that are determined for more cluster structures. The cluster centres belonging to the above will be the elements of the vectors, including the clustered stiffnesses with which the running of the model of the structural design software will be run:

$$\underline{K}_z^{(N)} \rightarrow \underline{K}_{z,cl}^{(N)}; \underline{R}_z^{(N)} \rightarrow \underline{R}_{z,cl}^{(N)}; \underline{R}_{xx}^{(N)} \rightarrow \underline{R}_{xx,cl}^{(N)}; \underline{R}_{yy}^{(N)} \rightarrow \underline{R}_{yy,cl}^{(N)} \quad (23)$$

If there are few piles, no clustering is performed for the spring stiffnesses; then the calculations proceed with the initial values:

$$\underline{R}_{z,cl}^{(N)} = \underline{R}_z^{(N)}; \underline{R}_{xx,cl}^{(N)} = \underline{R}_{xx}^{(N)}; \underline{R}_{yy,cl}^{(N)} = \underline{R}_{yy}^{(N)} \quad (24)$$

5.3 Running the model with the clustered stiffnesses

The calculations are performed in the structural design software using the $\underline{K}_{z,cl}^{(N)}, \underline{R}_{z,cl}^{(N)}, \underline{R}_{xx,cl}^{(N)}, \underline{R}_{yy,cl}^{(N)}$ deflectional and rotational stiffnesses; as a result, the settlements of the nodal points of the matching grid are produced and included in vector $\underline{s}^{(N)}$. For the error evaluation as the next step, further results of the $\underline{M}_{x,spring}^{(N)}$ and $\underline{M}_{y,spring}^{(N)}$ moments arising in the pile springs are also required beside the settlements.

5.4 Error Analysis of the Running Results

From the results of the structural design software, the relative errors can be calculated if the settlements obtained and the values of the moments arising in the pile springs are compared to the primary reference values.

With the knowledge of $\underline{s}^{(N)}$, $\underline{M}_{x,spring}^{(N)}$ and $\underline{M}_{y,spring}^{(N)}$, the relative errors related to the primary reference values are determined for the nodal points of the matching grid, expressed in %:

- for the settlements of the points of the raft ($\underline{H}_L$) and piles ($\underline{H}_C$):

$$\begin{aligned} \underline{\delta s}_{L,i}^{(N)} &= \frac{\underline{s}_{L,i}^{(N)} - \underline{s}_{ref,L,i}}{\underline{s}_{ref,L,i}} \cdot 100, \\ \underline{\delta s}_{C,j}^{(N)} &= \frac{\underline{s}_{C,j}^{(N)} - \underline{s}_{ref,C,j}}{\underline{s}_{ref,C,j}} \cdot 100 \end{aligned} \quad (25)$$

- for the moments in the pile springs:

$$\begin{aligned} \underline{\delta M}_{x,j}^{(N)} &= \frac{\underline{M}_{x,spring,j}^{(N)} - \underline{M}_{x,ref,j}^{(N)}}{\underline{M}_{x,ref,j}^{(N)}} \cdot 100, \\ \underline{\delta M}_{y,j}^{(N)} &= \frac{\underline{M}_{y,spring,j}^{(N)} - \underline{M}_{y,ref,j}^{(N)}}{\underline{M}_{y,ref,j}^{(N)}} \cdot 100 \end{aligned} \quad (26)$$

For the measurement of the approximation to the reference value, the Laplace average error could be determined at each correction phase by the following general formula:

$$\vartheta R = \frac{\sum_{\kappa=1}^{n_R} |\delta R_{\kappa}|}{n_R} \quad (27)$$

where

- R = any quantifier of the results, e.g., raft settlement (s_L), pile cap moment (M_x), etc.,
- δR_{κ} = the relative error of the R result quantity in the κ^{th} point,
- n_R = the total number of relative errors of the R results quantity (k_L or k_C),
- $\kappa = 1 \dots n_R$

As an example, for the k_L number of raft point settlements, it could be defined as follows:

$$\vartheta s_L = \frac{\sum_{\kappa=1}^{k_L} |\delta s_{L,\kappa}|}{k_L} \quad (28)$$

The average relative errors following the N^{th} correction step are obtained by calculating the scalar product of the error vectors and the summing vectors; subsequently, they are multiplied by the scalars $1/k_L$ or $1/k_C$:

$$\vartheta s_L^{(N)} = \underline{\delta s}_L^{(N)} \cdot \underline{1} \cdot \frac{1}{k_L}, \quad \vartheta s_C^{(N)} = \underline{\delta s}_C^{(N)} \cdot \underline{1} \cdot \frac{1}{k_C} \quad (29)$$

$$\vartheta M_x^{(N)} = \underline{\delta M}_x^{(N)} \cdot \underline{1} \cdot \frac{1}{k_C}, \quad \vartheta M_y^{(N)} = \underline{\delta M}_y^{(N)} \cdot \underline{1} \cdot \frac{1}{k_C} \quad (30)$$

In the correction steps performed on the basis of the primary reference values, it is appropriate to determine the error reductions of the raft bending moments in at least one direction as in this way, as secondary reference values, these might be compared with the criteria outlined in Section 7. In accordance with the above:

$$\underline{\delta m}_{x,i}^{(N)} = \frac{\underline{m}_{x,i}^{(N)} - \underline{m}_{x,ref,i}^{(N)}}{\underline{m}_{x,ref,i}^{(N)}} \cdot 100, \quad (31)$$

$$\underline{\delta m}_{y,i}^{(N)} = \frac{\underline{m}_{y,i}^{(N)} - \underline{m}_{y,ref,i}^{(N)}}{\underline{m}_{y,ref,i}^{(N)}} \cdot 100$$

$$\vartheta m_x^{(N)} = \underline{\delta m}_x^{(N)} \cdot \underline{1} \cdot \frac{1}{k_L}, \quad \vartheta m_y^{(N)} = \underline{\delta m}_y^{(N)} \cdot \underline{1} \cdot \frac{1}{k_L} \quad (32)$$

5.5 Decision making on the basis of the error analysis

The change in each correction step of the average relative errors is presented in a coordinate system. If the optimum cluster structure was used in every step, the curve asymptotically approximates the zero value of the relative errors. The decision as to whether to follow the procedure with the correction of the subgrade stiffnesses or to

terminate, is subjective. By the observation of the shape of the curve, it is appropriate to finish the procedure if no substantial improvement can be expected from the new iterations. The termination might be justified by the following:

- the rate of error reduction is a fraction of the improvements that were achieved in the previous steps (the rate of convergence drastically reduces),
- the amount of error reduction in the last two subsequent steps has a negative impact (with the cluster structure applied last, no error reduction was not achieved),
- the previously determined absolute error value was achieved, which could be a part of the typical or a certain specific settlement value (average, maximum, minimum etc.) in percentages (e.g., the error corresponding to 1% of the 20 mm average settlement: $\vartheta s_L^{(N)} < 0.2$ mm).

No everlasting, fixed criteria totally excluding subjectivity can be set. Due to the approximative character of the method, one cannot rule out the necessity of special consideration. In the case the result of the decision making is the termination of the approximation procedure, the subgrade and spring stiffnesses are the clustered values achieved in the last step. In the case of continuation, these values are further corrected in accordance with the next step.

5.6 Correction of the subgrade and spring stiffnesses

The subgrade and spring stiffnesses can be corrected in the N^{th} correction step with the knowledge of the relative errors calculated in that step. The corrected values are the following:

$$\begin{aligned} \underline{K}_{z,corr,i}^{(N)} &= \underline{K}_{z,i}^{(N)} \cdot \left(1 + \frac{\delta s_{L,i}^{(N)}}{100} \right) \\ \underline{R}_{z,corr,j}^{(N)} &= \underline{R}_{z,j}^{(N)} \cdot \left(1 + \frac{\delta s_{C,j}^{(N)}}{100} \right) \end{aligned} \quad (33)$$

$$\begin{aligned} \underline{R}_{xx,corr,j}^{(N)} &= \underline{R}_{xx,j}^{(N)} \cdot \left(1 - \frac{\delta M_{x,j}^{(N)}}{100} \right) \\ \underline{R}_{yy,corr,j}^{(N)} &= \underline{R}_{yy,j}^{(N)} \cdot \left(1 - \frac{\delta M_{y,j}^{(N)}}{100} \right) \end{aligned} \quad (34)$$

In step $(N+1)$ the initial values are the corrected values of step N .

$$\begin{aligned} \underline{K}_z^{(N+1)} &= \underline{K}_{z,corr}^{(N)} ; \quad \underline{R}_z^{(N+1)} = \underline{R}_{z,corr}^{(N)} ; \\ \underline{R}_{xx}^{(N+1)} &= \underline{R}_{xx,corr}^{(N)} ; \quad \underline{R}_{yy}^{(N+1)} = \underline{R}_{yy,corr}^{(N)} \end{aligned} \quad (35)$$

5.7 Recursion

With the subgrade and spring stiffnesses corrected on the basis of the error analysis in the new correction step, i.e., $(N:=N+1)$, by repeating the previous series of steps, the relative errors determined from the comparison with the primary reference values can be reduced further. The final results are provided by the termination of the recursive process by the decision-making procedure in the last step.

6 DECISION CRITERIA

The tuning of the structural design model takes place at the points of the matching grid; thus, with the amounts that primarily depend on the displacement of the associated node of this mesh, in the case of

identical correction steps, a smaller relative error might be calculated. Quantities such as this are the settlement, the normal force of the columns, and the pile force. The amounts that are tied not to a point, but to a surface in the case of an identical number of correction steps, show greater relative errors. The reason for this is that a match between the settlements calculated with the two different models is not expected at all the surface element points but only at the alignment nodes. These amounts are the raft moments and the pile and pillar moments that are closely related to the first. The convergence rate of the error reduction related to the individual quantities differ, and, as was outlined in Section 5.5, an absolute criterion cannot be assigned to a solution that might be regarded as the final result. It becomes evident from the test runs that the displacement type amounts can be rendered quasi error-free with a small number of steps to a level that is required by engineering precision (settlements accurate by tenths of mm). Before the correction procedure is terminated, it is also advisable to analyze the matching of the bending moments since they usually show a greater error.

It is known that finite element software produce a relative error from the vector of the unbalanced loads related to the load vector and that they compare it to a threshold value. A similar principle might be applied in this procedure. For the raft moments a threshold value might be prescribed by considering the aspect of the structural design and demands, which might be the selected value of the Laplace error of the moments, e.g., $\varepsilon \rightarrow \vartheta m_v = 5\%$. There is an opportunity for the

application of an absolute value E error barrier:

- a value related to a special moment, e.g., a certain percentage of the maximum, minimum, average, etc., moment, e.g., $E \rightarrow 0.05 m_{max}$
- the value selected on the basis of the moment reinforcement: e.g., a certain amount of the value of the ultimate moment related to the minimum reinforcement in tension or of the value of the ultimate bending moment with the base reinforcement mesh to be applied, e.g., $E \rightarrow 0.01 m_R$

It sometimes occurs that in the course of correction steps completed with a matching grid of a certain density, it is not possible to go below the desired error value even if the number of correction steps is increased. In this case rendering the matching grid denser globally or around the critical places locally might lead to an improvement.

7 APPLICATION LIMITS, RELEVANCE

The premises related to the application of the method were outlined in Chapter 3.1. The lack of meeting the uniformity requirement might be an applicability barrier, which primarily refers to the modelability of the superstructure in Plaxis 3D. If the two superstructural models differ from each other as far as stiffness is concerned, the basic principle of the method becomes irrelevant. The buildings constructed as monolithic reinforced concrete structures in site formwork can generally be appropriately modelled in Plaxis 3D. An issue might rise when the features of the structural design software would be required for the modeling of the special structural elements and when connections should be required for the model to be constructed in Plaxis 3D. A structure like this could be a composite ortotropic slab structure with eccentric connection elements. In several cases when a substituting slab of equivalent stiffness is implemented in Plaxis 3D, difficulties like these might be solved in a way that the basic principles and assumptions of the method should not be violated. These modeling considerations must in all cases be made individually for the task provided.

As the structural design software is concerned, an application barrier is meant by supplying input parameters enabling the CPRF modeling. In the case the software enables providing the parameters outlined

in 4.2.2, there are no theoretical obstacles to the CPRF modeling. The above is met in the case of the Axis VM software, which is typically applied in Hungarian practice; however, the method is suitable if any other structural design software is used that operates in a similar manner. There are no limits regarding soil types or soil layers, since it is not relevant from the aspect of the method in which soil environment the settlements determined by the Plaxis 3D modeling were formed. With mathematical or statistical tools, only the modelled results might be planted into the model environment of the structural design software.

8 EXAMPLE OF APPLICATION

8.1 Preparation of a numerical model experiment

For the practical application and presentation of the results available, a test model was created. Of the structure that is axially symmetric in both layout directions and whose layout dimensions are $\sim 35 \times 40$ m, only one quarter of the structure was included in the software. The modelled structural part is of a 15-level reinforced concrete column building, the bottom four basement levels of which are located in a construction pit with an earth-retaining diaphragm wall structure. The thickness of the foundation slab of the CPRF is 0.90 m; under the (4 x) 4 pcs of internal columns, $\varnothing 0.80$ m diameter-drilled, 10.0 m long reinforced concrete piles are located. The building is constructed of plane slabs that are directly supported by columns. Although regarding its architectural and structural dimensions and column distribution, the model building might be regarded as an actual building, due to the research nature of the task, reinforced concrete columns were used instead of reinforced concrete basement walls for the sake of better transparency of the force distribution. This is significant for a comparison of the model results; however, it does not affect the theoretical and practical applicability of the method. The parameters for the characterization of the subsoil were chosen in a way that the displacement-reducing effect of the piles on the raft settlements would become evident. The primary and secondary reference values were calculated

in Plaxis 3D, using the HSS soil model. The matching grid ensuring the alignment link between the models includes $k_L = 110$ pcs of raft domain points; the further $k_C = 4$ points are the connection points for the piles under the 4 pcs internal columns in each structural quarter.

8.2 Analysis of calculations performed by means of the finite element software

The finite element analysis of the complex test model was performed in Plaxis 3D; the primary and secondary reference values were created as a result. Subsequently, the correction of the subgrade and spring stiffness values in 6 correction steps was completed, which meant 6 calculations in the structural design software to approximate the reference values. On the basis of the error convergence graphs, the sixth correction step as the last step was accepted and regarded with the associated results and the final values of the corrected stiffnesses as the final results.

8.3 The test model in Plaxis 3D and in the structural design software

Figure 3 shows the complex model created in Plaxis 3D and its match constructed in a structural design software. The test results to be presented derive from the runs of the subsequent test model:

8.4 Assessment and analysis of the results

8.4.1 The base results

In the evaluation of the results, the so-called base results were defined for the consideration of the practicality and efficiency of the method. These do not constitute part of the method and are only required for the sake of the presentation of the accuracy obtainable by means of the method. By the term “base results”, the values calculat-

Fig. 3 Finite element test model in Plaxis 3D (a) and in the structural design software (b)

ed by the structural design software without clustering and performed with $\underline{K}^{(1)}$, $\underline{R}^{(1)}$, $\underline{R}_{xx}^{(1)}$, $\underline{R}_{yy}^{(1)}$, the subgrade and spring stiffness values were referred to. These stiffnesses were calculated from the Plaxis 3D settlement and stress results as unique values for each domain centre (these were the initial stiffness values in the first correction step indicated in the upper index). As an example, the base settlements $\underline{s}^{(B)}$ are the settlement values derived from the above-mentioned run, the vector of which is indicated by the upper index (B) for the sake of differentiation. The base results could be compared to the reference values similarly with the results of the correction steps. Thus, the relative errors of the base results could be calculated in a way similar to the previously-described manner. As an example, the relative errors of the base settlements at point (H_L) of the raft and point (H_C) of the piles:

$$\underline{\delta s}_{L,i}^{(B)} = \frac{\underline{s}_{L,i}^{(B)} - \underline{s}_{ref-L,i}}{\underline{s}_{ref-L,i}} \cdot 100, \quad \underline{\delta s}_{C,j}^{(B)} = \frac{\underline{s}_{C,j}^{(B)} - \underline{s}_{ref-C,j}}{\underline{s}_{ref-C,j}} \cdot 100 \quad (36)$$

The Laplace average relative error might also be expressed as:

$$\vartheta s_L^{(B)} = \underline{\delta s}_L^{(B)} \cdot \underline{1} \cdot \frac{1}{k_L}, \quad \vartheta s_C^{(B)} = \underline{\delta s}_C^{(B)} \cdot \underline{1} \cdot \frac{1}{k_C} \quad (37)$$

The base results could be considered as the most accurate of all the calculations performed with the subgrade stiffnesses not using the method suggested. These are referred to as settlements “indirectly calculated from the Plaxis results” by Móczár et al. (2014).

The base results are produced in relation to the test model by providing subgrade and spring stiffnesses determined individually for all 110 domain central points and 4 pile central points (but evidently without clustering). As it becomes evident from Sections 8.4.2 - 8.4.6 of the numerical outline of the results, the accuracy that might be achieved by the method developed can exceed the accuracy level of the base results by one magnitude, whereas a value one magnitude smaller subgrade stiffness value is sufficient for the simulation of the support relations of the CPRF structure. The latter difference in the case of a larger subgrade stiffness database might be two or three magnitudes.

8.4.2 Clustering results

The subgrade stiffnesses were clustered in all 6 correction steps performed by the structural design software, but the spring stiffness values were not clustered due to the small number of piles. From the diagrams of the silhouette coefficients, the values of the optimum cluster number included in Tab. 1 resulted in each step.

Tab. 1 Optimum cluster numbers in each correction step

step	1	2	3	4	5	6
$k_{opt,L}$	10	9	9	8	8	8

8.4.3 Vertical displacements, pile cap rotations

Instead of the 110 different subgrade stiffnesses applying 8 clusters in the sixth correction step, for the average value of the errors related to the reference settlements ranging between 34 and 61 mm, $\vartheta s_L^{(6)} = 0.17\%$ was achieved at the raft support points and also $\vartheta s_C^{(6)} = 0.17\%$ at the pile spring points (Fig. 4). The absolute differences of the settlement values varied between 0.0 and 0.3 mm. For the base results of the structural design software, an average relative error could be calculated that is approximately one magnitude greater compared to the previous results, i.e., $\vartheta s_L^{(B)} = 1.49\%$ or $\vartheta s_C^{(B)} = 1.47\%$.

In the case of the rotation spring stiffnesses, one could refer to the reference moments. The average values of the relative errors were achieved in the sixth improvement phase, i.e., $\vartheta M_x^{(6)} = 0.46\%$ and $\vartheta M_y^{(6)} = 0.44\%$ (Fig. 5). Since the initial rotation spring stiffness values in the 1st correction step were not calculated from Plaxis 3D but are arbitrary values, the average relative errors of the base results cannot be interpreted for $\vartheta M_x^{(B)}$ and $\vartheta M_y^{(B)}$.

8.4.4 Column forces and moments

The examination of the normal forces and the moments of the columns loading the foundation slab revealed similar tendencies to the previous ones. The numerical values of the differences were the following: For the normal forces ranging between ~2.000 and 10.000 kN, the average relative error to the reference values calculated in the last correction step was smaller than that from the base results, i.e., $\vartheta P_N^{(6)} = 1.66\% < \vartheta P_N^{(B)} = 1.77\%$. For the moments at the bases of the columns in one direction, the base results were approximated by a difference of 1.5%, i.e., $\vartheta P_{MZ}^{(6)} = 4.12\% > \vartheta P_{MZ}^{(B)} = 2.64\%$, whereas in the case of the other direction moments, an improvement could be achieved compared to the base results, i.e., $\vartheta P_{MY}^{(6)} = 5.97\% < \vartheta P_{MY}^{(B)} = 8.93\%$.

8.4.5 Raft moments

Examining the largest negative moments under the columns in the range of ~2300-2500 kNm/m, it can be stated that the base re-

Fig. 4 Laplace average errors of raft (a) and pile (b) settlements

Fig. 5 Laplace average errors of pile cap moments - Direction X (a) and Y (b)

sults demonstrate a 7-8% diversion in both directions. After the termination of the correction steps, moments identical to the base results were revealed at each column. The base results of moment m_x were approximated with a difference of 1.1%: $\vartheta m_x^{(6)} = 8.9\% > \vartheta m_x^{(B)} = 7.8\%$; for the moment m_y , results more accurate than the base results were achieved, i.e., $\vartheta m_y^{(6)} = 7.6\% < \vartheta m_y^{(B)} = 8.1\%$. A more significant diversion from the base results can be observed at one examined point in each moment direction, the amount of which resulted in 25% and 22%. The fact must be taken into account, i.e., that at these points, the moments are the minimum, numerically 51 kNm/m and 69 kNm/m. For these low values a relatively high 22-25% relative error means minor differences expressed in absolute terms, i.e., 13-15 kNm/m. Related to the values of the maximum moments, these are smaller values by two magnitudes.

8.4.6 Pile forces and moments

It is not necessary to define the separate error values for these quantities, as due to the linear relationship between the spring stiffnesses and settlements, the same values might be calculated as in the case of the vertical displacement and the rotation of the pile caps.

9 CONCLUSIONS

The outlined method provided that the presumptions were met and that within the application limits, it is suitable for the modeling of CPRF structures by means of finite element structural design software, the subgrade and spring stiffness values might be provided with higher precision. The Plaxis 3D geotechnical software, which has advanced soil models, enables spatial modeling. Besides tasks relating to soil mechanics, it is also suitable for the modeling of, e.g., monolithic reinforced concrete framed buildings, while in most cases, no restrictions should be made regarding the material and layout of the superstructure. Due to the above, the results related to the CPRF structure of the building provided in the software as a complete model should be regarded as of reference value as far as the finite element analysis of the equivalent model performed by means of the structural design model is concerned.

By the application of a cluster analysis method, the number of different value subgrade and, if required, spring stiffness values might radically be reduced, which in the case of a medium-sized foundation slab, can result in extremely great savings in time and work. Meanwhile, the correction procedure performed by means of subgrade re-

action coefficient cluster structures chosen for correction steps on the basis of cluster validity index values reaches, and in terms of settlements and column forces, even exceeds the accuracy of the procedure regarded as the best of the other available methods. As far as settlements are concerned, this might mean results more precise by even one magnitude.

By means of this method the rotation stiffnesses of the pile springs might be determined in spite of the fact that the initial values are selected in an arbitrary manner. This is particularly useful as this modeling parameter is much more difficult to estimate than the vertical spring stiffness of the piles. It is possible to complete the modeling of the piles included in the Plaxis 3D reference model on the basis of the results gained from the back-analysis of the pile load test; thus realistic modeling might be realized at the most advanced level, even in everyday design practice.

It becomes evident from the step-by-step approximation of the settlement distributions in Plaxis 3D that the raft subgrade stiffness serving as alignment parameters, the vertical spring stiffness of the piles, and the rotation stiffnesses modeling two-directional moments are amounts that depend on each other; their separate definition is a fundamental mistake. As the displacements of the raft apart from the displacement function $w(x,y)$ are defined by the rotation functions $\varphi_x(x,y)$ and $\varphi_y(x,y)$, the desired distribution of the settlements can be obtained by the simultaneous tuning of these stiffness values in one model.

The method eliminates the uncertainties and diversions deriving from the application of subgrade and spring stiffness values estimated by means of dated approximative formulae both in relation to the settlements and the internal forces.

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