Transport and Telecommunication, 2021, volume 22, no. 3, 301–311
Transport and Telecommunication Institute, Lomonosova 1, Riga, LV-1019, Latvia
DOI 10.2478/ttj-2021-0023

MODERN SEDIMENT MODEL OF TRAFFIC FLOW

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The work deals with the mathematical modeling of traffic phenomena. The submitted model is based on a prospective analogy of some described phenomena with particle sedimentation. Both the qualitative analysis of the model and the numerical experiment is carried out. Qualitative results of the research have been compared with the known data of supervision of the traffic on city highways. As a result, the main control parameters which can use for optimal traffic management are highlighted and justified.

Keywords: traffic flow, sediment model, traffic intensity, hysteresis, traffic jam, time delay

1. Introduction

Improvement of the reliability of transportations, as well as their security and quality with ensuring the environmental characteristics, are extremely relevant issues of today. The solution of these problems requires an increase in the cost of infrastructural improvements of the transport network for turning it into a flexible and well-controlled logistics system (Greenshields, 1934; Gasis *et al.*, 1959; Cremer and Ludwig, 1986). However, the investment risk is greatly increased if you do not take into account the laws of development of the transport network and balance of load on its sites. Ignoring these laws leads to the formation of traffic jams, overloading or underloading the individual lines and nodes in the network, as well as increasing the level of accidents and environmental damage. To identify effective strategies for traffic management in the city, the optimal solutions for designing the road network, and traffic management we should take into account a wide range of characteristics of the traffic flow and the influence of external and internal factors on the dynamic characteristics of the traffic flow. We have now various theories of traffic flows developed by researchers of different fields of knowledge - physicists, mathematicians, and specialists in operations research (Greenberg, 1959; Haight, 1963; Cremer and Ludwig, 1986).

Research experience on this topic is great. However, the overall level of the reached results in trying to their practical application is often not sufficient. In our opinion, this can be explained by the following factors. Traffic flow is unstable and diverse in its nature, and the objective information about it is the most complex and resource-intensive part of the control system.

The quality criteria for traffic control are contradictory. It is conditioned by the need to ensure the smooth flow while reducing the costs of movement by imposing restrictions on the speed and directions of travel. Enforcement of traffic management in practical implementation is always inaccurate, and, as a result, the road leads to unintended effects. Thus, the difficulty of formalizing the regularities of traffic flow has become a major cause of the backlog of research results from the requirements of practice. The first macroscopic model, in which the traffic regularities have been investigated from the standpoint of continuum mechanics, as proposed in 1955 by Lighthill and Whitham (Lighthill and Whitham, 1955; Nagel *et al.*, 2003). They showed that the methods for describing transport processes in continuous media can be applied to models of traffic. In the 60-70s of the 20th century, interest in the study of transport systems was renewed. This interest manifested itself in the funding of numerous contracts for the appropriate research topics with the participation of outstanding scientists in the field of mathematics,

physics, a theory of control, such as I. Prigogine (Prigogine, 1961; Prigogine and Herman, 1971), M. Athans, author of fundamental works on statistics L. Breiman and others.

In the late '80s and early '90s in the USA the study of transportation systems has been put to the rank of national security problems. The solution to this problem attracts the best "physical minds". Today there is an extensive literature on the study and modeling of vehicular traffic. Several academic journals are devoted exclusively to the dynamics of vehicular traffic.

Interesting models based on the mathematical tool of multiphase flows were proposed in works (Kerner, 2004; Hindawi, 2019; Zhiyuan *et al.*, 2019). The works (Hobbs, 1984; Antonella *et al.*, 2018; Pell *et al.*, 2017) focuses on the construction of models for narrower purposes of automated regulation and control of traffic flows in real conditions of urban transport.

There are two main approaches to modeling the traffic intensity: deterministic and probabilistic models. Deterministic models are created on the basis of the functional relationships between different indicators, such as speed and distance between vehicles in the stream. Traffic flow in stochastic models is considered as a random process. All models of traffic flow can be divided into three classes (Helbing *et al.*, 2001): model-analogs, models of "follow the leader", and probabilistic models. In the models-analogs, the motion of vehicles set is simulated by the help of mathematical tools of hydrodynamics. This class of models is called macroscopic.

This paper presents and discusses new models developed by the authors in order to provide a reasonable explanation of the phenomena observed in various experiments and surveys of traffic, which, in our opinion, have not yet received a proper mathematical description. Based on the analysis of the patterns of traffic formation previously discovered and reliably confirmed by various researchers in this work, mathematical models are proposed, with the help of which it is possible to calculate the optimal traffic intensity and obtain estimates of the main control parameters for quality management. It is in this that the authors see the main contribution and novelty of the work.

At the same time, it should be noted that this work did not set the goal of complete quantitative verification of the introduced new control parameters in specific situations. Therefore, the development of practical calculation methods based on the proposed models will require further derivation of dependencies for control parameters, and then it will be possible to carry out a full check of the adequacy of the models. In the proposed form, the simulation results are conceptual in nature. Nevertheless, the authors suggest this work for consideration and discussion, since many researchers, if they are interested in the issues discussed in the article, will be able to offer a larger amount of data for model verification.

2. Materials and methods

The models of "follow the leader" are based on the essential assumption of a connection between the regularities of movement of the cars following and the head car. According to this approach regularities of motion of the set of vehicles as a whole are determined by the local parameters as reaction times of isolated drivers, in particular. This class of models is called microscopic. In stochastic models, the concerted motions of vehicles are considered as a result of the stochastic interaction of vehicles on the elements of the transport network. The specific traffic pattern is formed due to the hard nature of the traffic network and by the mass character of the movement of the dense transport stream.

This pattern is characterized by distinct queues and intervals of downloads of the road bands, etc. The appropriate regularities are essentially stochastic. Recently, in studies of traffic, the researchers began to apply interdisciplinary mathematical ideas, methods, and algorithms of nonlinear dynamics. Their usefulness is justified by the presence in the traffic flow of stable and unstable modes of motion, and the instabilities when changing traffic conditions, and nonlinear feedbacks.

In our opinion, the hydrodynamic models of traffic flow are sufficiently well-grounded. Indeed, traffic flow can be written as one-dimensional compressible fluid flow, assuming that the consumption is conserved and there is a to one relation between the speed and the density of traffic. Of course, such a relation can be considered as true in nature only approximately.

The first assumption is expressed by the continuity equation. The second assumption is a functional relation between speed and density to account for the reduction of vehicle speed with increasing flux density. This is an intuitively correct assumption but theoretically, it could lead to the negative of the density or velocity. The point is that the certain value of the traffic density may correspond to several values of the speed of vehicles.

Therefore, for the correctness of the second assumption, the average flow rate at any time ought to meet the equilibrium value for a given density of cars on the road. However, the equilibrium situation is a

purely theoretical assumption and can be observed in areas without crossing roads only. For these reasons, some researchers abandoned the continuous models, and some consider them to be too rough.

So, it makes sense to continue the search for new models, which would combine the advantages of the microscopic and macroscopic models, and have minimal risks to repeat the shortcomings of known approaches.

According to the authors, the new model presented in the next section has mentioned advantages.

3. Sediment model of vehicular traffic

3.1. Preliminary notes

As we already noted, there are two main different concepts to describing the traffic, namely: on the base of microscopic or macroscopic approaches. And there are several different types of mathematical models to describe the dynamic evolution of transport systems within the conceptual framework of the microscopic approach.

For example, individual vehicles are modeled explicitly as particles that move in the environment. Incidentally, under the probabilistic description of traffic flow, the methods developed by the kinetic theory of gases can be used. Under the deterministic description of the movements of individual vehicles, the simulation is carried out on the principles of the classical Newtonian dynamics.

Computer models are useful to get a better description of the various phenomena characteristic for the traffic and to understand the basic principles that govern the motion system. Since the large-scale experiments with real street traffic are not practicable, the use of simulation models in this area of research has no alternative. In this case, empirical data is collected, as a rule, by the passive observation, in contrast to the controlled active experiments used in other fields of study.

The use of models and methods of computer simulation helps us to find a strategy for optimizing the transport network and to get the correct methods for traffic control. Vehicular traffic in the modern metropolis has a very complex structure. Therefore, the many usual models which ignore the influence of traffic lights on the vehicles flow cannot describe adequately the dynamics of traffic density on the streets, and they cannot make a proper evaluation of the air pollution by exhaust gases, as the pollution and the number of emissions depend on the work regime of the engine.

3.2. Mathematical details

We submit now the model of vehicular traffic based on the analogy with the sedimentation process (Figure 1).

Figure 1. Scheme of traffic flow

Let $V(\rho)$ be the transport rate, depending on the density of vehicles on a road.

The three characteristic densities can be defined. They are: the optimal density ρ_{opt} under which the maximum allowable speed in urban traffic can be realized, the density of vehicles in traffic in the area nearby traffic lights ρ_{st} , when the red light is on, and the density in a jam ρ_j at which the motion will be stopped (Kerner *et al.*, 1996).

Since due to the influence of traffic lights the density of vehicles on a road depends on the coordinates and time, then from the balance of the number of cars it follows:

$$V \frac{\partial \rho}{\partial y} + \rho \frac{\partial V}{\partial y} = \frac{\partial \rho}{\partial t}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} = - \frac{\partial J}{\partial y}, \quad (2)$$

where J is the traffic flow.

We offer the following logistic equation to describe the dependence of traffic rate on the traffic density:

$$\frac{\partial V}{\partial \rho} = -kV(V - V_*). \quad (3)$$

Thus, we get:

$$V = \frac{CV_* \exp(-kV_*\rho)}{1 - C \exp(-kV_*\rho)}. \quad (4)$$

In formula (4) the velocity V_* is taken for the flux density ρ_{st} , and the constant C is found from the following expression

$$V(\rho_{opt}) = V_{opt}. \quad (5)$$

Approximately, we can put

$$C \approx \frac{V_{opt}}{V_{opt} + V_*}. \quad (6)$$

As a result, the dependence of the flow rate on the local density reads:

$$V = \frac{V_{opt} V_* \exp(-kV_*\rho)}{V_{opt}(1 - \exp(-kV_*\rho)) + V_*}. \quad (7)$$

Figure 2 depicts a typical plot of (7)

Figure 2. The characteristic dependence of the velocity on the density of traffic flow

Considering equations (1), (2), (7) jointly, we can describe the changes in the density of vehicles and traffic intensity over time, and link these changes with the influence of traffic lights, and find also the specific areas of high and low flux densities before and after the traffic light.

Substituting (7) in (1), we obtain:

$$V \frac{\partial \rho}{\partial y} + \rho \frac{\partial V}{\partial \rho} \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial t}. \quad (8)$$

From this, it follows

$$(V(\rho) + \rho V') \frac{\partial \rho}{\partial y} - \frac{\partial \rho}{\partial t} = 0. \quad (9)$$

$$V'(\rho) = \frac{kV_{opt}V_*^2(V_{opt} + V_*)\exp(-kV_*\rho)}{[(V_{opt} + V_*) - V_{opt}\exp(-kV_*\rho)]^2}. \quad (10)$$

The full integral of equation (9) can be found by using the Lagrange-Sharpe method. The scheme of this method in our case is as follows.

Let us consider the system of ordinary differential equations:

$$\frac{dy}{p} = -dt = \frac{d\rho}{Pp-q} = -\frac{dp}{Rp} = -\frac{dq}{Rq}, \quad (11)$$

where

$$P = V(\rho) + \rho V'(\rho),$$

$$R = (2V'(\rho) + \rho V''(\rho))p,$$

$$p = \frac{\partial \rho}{\partial y}, \quad q = \frac{\partial \rho}{\partial t}.$$

The first integral of the system (11) reads:

$$\Phi(\rho, p, q) = A = \text{const}. \quad (12)$$

So, we obtain a coupled system of equations (9) and (12):

$$\begin{cases} (V(\rho) + \rho V'(\rho))p - q = 0, \\ \Phi(\rho, p, q) = 0 \end{cases} \quad (13)$$

Resolving (13) regarding the partial derivatives we can receive the system of "uncoupled" equations:

$$\begin{cases} p = F_1(\rho, A) \\ q = F_2(\rho, A) \end{cases} \quad (14)$$

Integrals of the system (14) will describe the density waves propagating in the traffic flow.

From system (11) it follows:

$$\frac{dp}{dq} = \frac{p}{q}. \quad (15)$$

Therefore, solutions of the model obey the relation

$$\frac{\partial \rho}{\partial t} + c \frac{\partial \rho}{\partial y} = 0, \quad (16)$$

where c is the phase velocity of the density.

Thus, the model has a solution in the form of a solitary density wave of the form wave

$$\rho = \rho(\xi), \quad (17)$$

where $\xi = y - ct$ - the self-similar wave variable.

The conclusion about that there are density waves on urban motorways is entirely consistent with well-known observations and empirical data (Chowdhury *et al.*, 2000; Lubashevsky *et al.*, 2002).

From (9), (10), (11) the equation for calculating the phase velocity at the front of the traffic wave follows immediately:

$$-c = V(\rho) + \rho V'(\rho). \quad (18)$$

And, as a result, we obtain

$$c = -V \left[1 - \rho k V \frac{V_{opt} + V_*}{V_{opt} \exp(-k V_* \rho)} \right]. \quad (19)$$

When specifying a certain fixed phase velocity, we obtain from (18, 19), the expression for the velocity of vehicles, depending on the density of vehicular traffic at the front of the density wave

$$V = \frac{1 + \sqrt{1 + 4ckA}}{2\rho kA}, \quad (20)$$

where

$$A = \frac{V_{opt} + V_*}{V_{opt} \exp(-k V_* \rho)}. \quad (21)$$

Let us consider now the dependence of the total vehicles flow on the road on the density.

From the general expression for the velocity, we obtain

$$J = \frac{\rho V_{opt} V_* \exp(-kV_* \rho)}{V_{opt}(1 - \exp(-kV_* \rho)) + V_*}. \quad (22)$$

Figure 3 depicts the typical plots of the total vehicular traffic, calculated from equation (22) at different values V_* . Figure 4 depicts the experimental data corresponding to observations made on Canadian highways (Lubashevsky *et al.*, 2002; Kerner, 2004).

Figure 3. Typical plots of the total traffic flow J (1/(ms)) on the density of traffic ρ (1/m)

Figure 4. Experimental data on the flow (1/h) and the relative traffic density (%) on Canadian highways (Lubashevsky *et al.*, 2002)

It is easy to be convinced that the submitted model gives a true qualitative description of the noted phenomenon. Of course, to fully check the adequacy of the model, it is necessary to have a more representative sample of data.

3.3. Assessment of the critical density of vehicular traffic and pre-jam situations

In this section, the evaluation for the value of the critical density, at which there is a maximum of the vehicular traffic intensity on straight stretches of road without intersections and crossings will be obtained.

This critical value should satisfy:

$$(V_{opt} + V_*)(1 - kV_* \rho_{kp}) = V_{opt} \exp(-kV_* \rho_{kp}). \quad (23)$$

Figure 5 is an illustration of relation (23) that can be interpreted as a transcendental equation for the calculation of the critical density, after which the rapid decline in traffic intensity may be observed.

Since the reduction of the total flux begins in the area of the strong dependence of the motion velocity on the flux density, we can propose the approximate explicit formula for the calculation of the critical density.

To do this, we expand the exponent in (23) into series, and after saving of two terms of the expansion we obtain the following expression:

$$\rho_{kp} \approx \frac{V_* + \sqrt{V_*^2 + 2V_{opt}V_*}}{2V_{opt}V_*}. \quad (24)$$

Figure 5. The method of calculating the critical density according to the sediment model (24)

The last formula is easily converted to the form:

$$\rho_{kp} \approx \frac{1 + \sqrt{1 + 2s}}{2V_{opt}}, \quad (25)$$

where $s = \frac{V_{opt}}{V_*}$ is the ratio of the optimum speed of the roadway to the speed of braking.

Figure 6 depicts the results of measurements of traffic flow under decreasing and increasing the relative traffic density. The characteristic hysteresis is noticeable, i.e., reducing the traffic density does not lead to a significant increase in the traffic flow (Chowdhury *et al.*, 2000; Treiterer *et al.*, 1974).

Figure 6. Transport hysteresis according to (Chowdhury *et al.*, 2000)

This phenomenon is also easily explained by the sediment model. Indeed, with decrease flow in the flux density on the descending branch of the dependence of flow on the density $J(\rho)$ the wave front with the corresponding phase velocity (3.22) is formed. And the speed at the wave front (20), under the great density, is almost inversely proportional to it.

Hence for the total flux under the "reverse course" of the density we obtain:

$$J = \frac{1 + \sqrt{1 + 4ckA}}{2kA}, \quad (26)$$

where

$$A = \frac{V_{opt} + V_*}{V_{opt} \exp(-kV_*\rho)}.$$

When $\rho \rightarrow 0$ we get

$$J_0 \approx \frac{1 + \sqrt{1 + 4ck\left(1 + \frac{1}{s}\right)}}{2k\left(1 + \frac{1}{s}\right)}. \quad (27)$$

Figure 7 is the illustration of this phenomenon, which according to the sedimentation model can be interpreted as instability of the descending branch of the dependence $J(\rho)$

Figure 7. Interpreting the transport hysteresis according to the sediment model

Other well-known characteristic traffic problems are associated with the appearance of "traffic jams". It is known that phenomena of the spontaneous formation, propagation, and "resolution" of traffic jams on the stretch of road are often observed for no explicit reason.

The next section is devoted to a novel model for describing this phenomenon.

3.4. Phenomenon of jam

Figure 8 shows a plot of the trajectories of individual vehicles on a multi-lane highway. These data were obtained with the help of aerial photography.

Figure 8. Aerial view of the trajectories of individual vehicles on multilane highway (Lubashevsky *et al.*, 2002)

In our opinion, none of the known models can lay claim to an adequate description of the formation of traffic jams. In (Zawar *et al.*, 2019), it was proposed to take into account while building the macroscopic model the certain local parameters, describing the driver's reaction to the emerging transport incentives.

A similar approach is also proposed in the paper (Zawar *et al.*, 2019). At the same time, models with technology-appropriate assumptions to simulate different vehicle types with distinct communication capabilities were considered, and the practical recommendations were given.

Note that such ideas were early submitted in our presentation on the WCTSR'13 conference (Brener *et al.*, 2013).

Here we wish to submit some ideas and simple calculations, which in certain cases may be the basis for models describing the phenomenon of jams.

Suppose that a set of vehicles with the traffic density ρ moves along a single-lane highway with a given width h and with a length L of the unimpeded flow of traffic (Figure 9).

Figure 9. To deriving condition (35)

The residence time of vehicles with average speed V at the given site is

$$T = \frac{L}{V}. \quad (28)$$

The number of cars arriving per the time unit to this site is:

$$q = \rho h. \quad (29)$$

Thus, the following number of cars is on the highway simultaneously.

$$N = \frac{\rho h L}{V}. \quad (30)$$

Let us estimate the area, which is occupied by every car on the road as

$$f = \Psi(r_1 + r), \quad (31)$$

where

r_1 is the characteristic size of a single-vehicle;

r is the characteristic size of the car maneuver;

Ψ is the factor of the form of the car.

From relations (30) and (31) it follows

$$\frac{\rho h L}{V} = \frac{h L}{\Psi (r + r_1)^2}, \text{ or} \quad (32)$$

$$r_1 = \sqrt{\frac{V}{\Psi \rho}} - r.$$

Our main hypothesis is that the characteristic radius of the maneuver cannot be less than a certain value that is determined by the time of driver's response to disturbances which can arise during movement. In accordance with this hypothesis the size of maneuver at a given speed should satisfy

$$r_1 = \sqrt{\frac{V}{\Psi \rho}} - r \geq VT. \quad (33)$$

$$\rho > \rho_{kp} = \frac{1}{4\Psi r T}.$$

Hence, we obtain

$$\frac{V}{\Psi \rho} \geq (r + VT)^2. \quad (34)$$

From the existence of solutions of inequality (34) regarding the velocity, we obtain

$$\rho \leq \frac{1}{4\Psi r T}. \quad (35)$$

The motion becomes impossible under the condition.

$$\rho > \rho_{kp} = \frac{1}{4\Psi r T}. \quad (36)$$

From this, it follows that the critical speed corresponding to the "pre-jam" state is defined by the relation

$$V_{kp} = \frac{r}{T}. \quad (37)$$

In other words, a jam occurs when the vehicle speed corresponding to a given density becomes equal to the speed of maneuver at the length, which is equal to the characteristic size of the car.

4. Conclusions

The novel "sediment" model of vehicular traffic has been submitted. It is shown that the use of the logistic equation to describe the dependence of the density of traffic flow on its intensity allows us to give an interpretation of the known experimental data of vehicle traffic on the highway. It is also shown that the developed model explains the known phenomenon of the hysteresis of traffic intensity depending on the density of traffic flow. The new model for prediction of the effects of jams on the roads has been presented too. The model is based on the idea of evaluation of the characteristic maneuver time while driving the vehicle in the stream.

It is important to note that the submitted models provide a substantiated and convincing explanation of the hysteresis phenomenon and a deeper mathematical description of the phenomenon of traffic jams for the first time from one conceptual point of view. The work showed that the conclusions drawn from the analysis of these models are qualitatively fully consistent with long-term observations of various researchers.

At the same time, the submitted models introduce a number of new control parameters, the verification of which should be the subject of the next studies using more extensive experimental data.

The offered model and appropriate equations can be used at designing highways, reconstruction of roads and highways, for exact regulations of traffic lights, and also for working out actions for preventing the jams on roads which are one of the principal causes of air pollutions and discomfort of inhabitants of cities.

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