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REDUCING THE IMPACT OF A VERTICAL REFRACTION BY A TWO-REGIME MODEL

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ABSTRACT

This paper deals with the creation of a mathematical model to eliminate or reduce the impact of a vertical refraction from the results of repeated trigonometric measurements of elevations. A multiple regression model with air temperature and pressure as the explanatory variables is proposed and subsequently modified into the form of a two-regime model. The threshold parameter is given by the threshold function, which represents an approximation of the relationship between the atmospheric pressure and temperature. An investigation of the quality of the proposed models and the application of the selected model on the experimental measurement results is also performed.

KEY WORDS

- trigonometric measurement of elevations,
- vertical refraction,
- regression model,
- two-regime model,
- threshold parameter.

INTRODUCTION

For determining the spatial position of points by means of terrestrial surveying techniques, both vertical and horizontal measurements have to be performed. The trigonometric measurement of elevations is a well-known surveying technique. With its speed and range it is mainly used in those cases where the use of a levelling method is not possible or inefficient from the viewpoint of time and economics.

The lowest layers of the atmosphere where terrestrial surveying measurements are conducted represent a non-homogenous environment in the horizontal as well as in the vertical directions. The light rays which pass through the atmosphere are reflected, absorbed and scattered, and their path is curved. The curvature of light rays arriving from a source to a receiver, which is caused by an unstable density of air layers, is denoted as a refraction

(STN 73 0404-1). As a consequence, the observer sights the target in the direction of a tangent to the spatially curved path from the point of the observation. The trigonometric measurement of the elevations is mainly influenced by its vertical part, the so-called vertical refraction, which affects the measured zenith angles. The physical model of the vertical refraction is based on a knowledge of the wavelengths of light, the meteorological parameters of the atmosphere's ground layer along the whole sight at the moment of light ray transition, and furthermore, the sight's distance and slope. Because the determination of the meteorological parameters along the whole sight is not practicable, this has led many authors to the development and further improvement of the various approaches to eliminate the impact of the vertical refraction (e.g., Brunner, 1979; Ramsayer, 1979; Hradilek, 1984; Štempelová, 1993; Flach, 2000; Böckem, 2001; Binnenbruck et al., 2002; Ingensand, 2002; Weiss, 2002; Gašinec, Gašincová, 2009, etc.).

Tab. 1 Arrangement of the measurement results.

File	Elevation	Quantity	Arithmetic mean	Temperature	Humidity	Pressure
1	$h_{1,1}, h_{1,2}, \dots, h_{1,J_1}$	J_1	h_1	T_1	e_1	p_1
2	$h_{2,1}, h_{2,2}, \dots, h_{2,J_2}$	J_2	h_2	T_2	e_2	p_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$h_{m,1}, h_{m,2}, \dots, h_{m,J_m}$	J_m	h_m	T_m	e_m	p_m

2 EXPERIMENTAL MEASUREMENTS

For the purpose of researching the impact of vertical refraction, we carried out twelve experimental measurements from 2007 – 2009. These experiments were conducted on separate sights with lengths of approximately 120 to 320 meters, over different surfaces and in different seasons under different atmospheric conditions. In order to measure the zenith angles and slope distances, we utilized modern total stations (Leica, Trimble). The zenith angles were measured in ten ranks; the standard deviation of the measured zenith angle in two faces did not exceed 0.44 mgon. The meteorological parameters – temperature, relative humidity and air pressure – were measured at the station point simultaneously with the zenith angles with a precise GFTB 100 hygro-/thermo-/barometer. All the data was measured in one or two hour lags, and we captured from seven to thirteen series per day. Each elevation was also determined by means of a precise levelling method.

In pursuance of the measured zenith angles, slope distances, heights of the instrument and targets, we calculated the elevation between the station and the point observed. We did not take into account the impact of the deflection of vertical and the vertical refraction correction. We considered the Earth's curvature correction and the physical correction for the slope distances according to the relationships specified by the manufacturer of the total station; we also applied an addition constant of an “instrument – reflecting prism” measuring system. Furthermore, we recalculated the Celsius temperature [°C] to the thermodynamic temperature [K] and the values of the relative humidity to the partial pressure of the water vapour. The arrangement of the measurement results from one day is shown in table 1.

3 STATISTICAL ANALYSIS OF THE MEASUREMENT RESULTS

In the proposed statistical analysis of the measured elevations ordered into the files, we focused on the testing of the basic assumptions. The results of Grubb's test for outliers, Shapiro –

Wilk's test designed for the verification of a normal distribution, and Bartlett's test for testing the homogeneity of the empirical dispersions among the files, confirmed the suitability of the measurement results for further analyses and processing. The analysis confirmed the theoretical assumption that the results of the trigonometric measurement of elevations are affected by systematic errors. The impact of the systematic errors among the files of the elevations was verified by means of the ANOVA tool, and its existence was documented by the high values of the test statistics, which contradict the null hypothesis (Lipták, Sokol, 2009). We estimated the correlation characteristics in the subsequent correlation analysis. From the results achieved, we concluded that between the elevations and the conjunctive influence of the meteorological parameters, a statistically significant multiple correlation exists. Furthermore, the air temperature showed a statistically significant negative correlation; the air humidity characterised by the partial pressure of the water vapour was statistically insignificant in the context of the air temperature and influence of the pressure. On the other hand, in pursuance of the relations among the meteorological parameters, a correlation arises with the air pressure.

4 MATHEMATICAL MODEL OF THE VERTICAL REFRACTION

The aim of the proposed mathematical model is to express the systematic impact among the files of the elevations by means of the meteorological parameters. The creation of the model was based on determining the variables and equations, calculating the model's parameters, verifying the proposed model and applying it. From the point of view of the mathematical statistics, the classical approach for solving this task is to build a regression model.

The properties of the environment in the atmosphere's ground layer are most influenced by the changes in the air temperature and pressure. The regression model, which expresses the linear dependency of changes in elevations on changes in the air temperature and pressure, can be written in the form:

$$h_i' = \beta_0 + \beta_1 \cdot T_i' + \beta_2 \cdot p_i', \quad (1)$$

for $i = 1, 2, \dots, m$, where β_j are the unknown parameters, and h_i' , T_i' , p_i' are the values of the elevation, air temperature and pressure reduced by their arithmetic mean.

Theoretical knowledge and the results of the correlation analysis show that the atmospheric pressure is related to the air temperature. We proposed using this dependence of the variables to split the atmospheric pressure domain into two parts with potentially different rules – regimes for the interpreted variable. Then the determining equation of the model could be formulated in the following form:

$$h_i' = \beta_0 + \beta_1 \cdot T_i' + \beta_2 \cdot p_i' + \beta_3 \cdot \min\{p_i', p(T_i')\} + \beta_4 \cdot \max\{p_i', p(T_i')\} \quad (2)$$

or after specification into the regimes, as follows:

$$h_i' = \begin{cases} \beta_0 + \beta_1 \cdot T_i' + (\beta_2 + \beta_3) \cdot p_i' + \beta_4 \cdot p(T_i'), & p_i' \leq p(T_i') \\ \beta_0 + \beta_1 \cdot T_i' + (\beta_2 + \beta_4) \cdot p_i' + \beta_3 \cdot p(T_i'), & p_i' > p(T_i') \end{cases} \quad (3)$$

where min and max are the aggregation functions, which assign the smallest and biggest values from the pair of values p_i' and $p(T_i')$ (Figure 1), and $p(T_i')$ is the threshold parameter defined by the threshold function later in equation (4). The threshold parameter splits the atmospheric pressure domain and is incorporated as an explanatory variable to increase the fitting potential of the model. The threshold function represents an approximation of the relationship between the atmospheric pressure and temperature. The approximation of the functional relationship can be solved by different types of functions. Among the most commonly used

belong polynomial functions. The polynomial of the L^{th} degree expressing the relation between the atmospheric pressure and temperature can be written as follows:

$$p(T_i') = \gamma_0 + \gamma_1 \cdot T_i' + \gamma_2 \cdot (T_i')^2 + \dots + \gamma_L \cdot (T_i')^L = \sum_{l=0}^L \gamma_l \cdot (T_i')^l, \quad (4)$$

where $l = 0, 1, \dots, L$ and γ_l are the coefficients of the polynomial. Because the appropriate degree of the polynomial for this task is not known, we will start with the polynomial of the zero degree (constant), through the first degree (straight-line) and second degree (quadratic parabola) to the third degree (cubic parabola).

The determining equations (1) and (2) of both models can be written in a matrix form:

$$y = f(\beta) \quad (5)$$

or after the introduction of the approximate values of the unknown parameters:

$$y = f(\beta_0) + A \cdot \Delta\beta \quad (6)$$

where β is a k – dimensional vector of the unknown parameters; β_0 is a k – dimensional vector of the approximate values of β ; $A = \partial f(\beta) / \partial \beta^T|_{\beta=\beta_0}$ is an $m \times k$ – dimensional matrix of the first derivatives of a vector function $f(\beta)$ by β , quantified for the approximate values, and $\Delta\beta$ is a k – dimensional vector of the increments. The difference between the regression and the two-regime model is in the dimension of the vector β and the resulting changes in the dimensions of the other vectors and matrix A . Moreover in the case of a two-regime model, when the atmospheric pressure is approximated by the polynomial of the zero or first degree, the variables T' , p' , $\min\{p', p(T')\}$ and $\max\{p', p(T')\}$

Fig. 1 Distribution of the values of the variables using the min and max aggregation functions.

are linearly dependent. In this case it is not necessary to consider parameter β_4 .

The task is to find the appropriate values of the unknown vector parameter β by means of the realization $\mathbf{w}$ of the observation vector ξ . Because β is calculated in the form $\hat{\beta} = \beta_0 + \Delta\hat{\beta}$, the subject of the calculation is the vector of increments $\Delta\hat{\beta}$ in pursuance of the realization $\mathbf{x}$ of a random vector η , $\eta = \xi - f(\beta_0)$. This task is usually solved by the least squares method, i.e., by minimizing a quadratic form (Kubáčková, 1990):

$$Q = (\mathbf{x} - \mathbf{A} \cdot \Delta\hat{\beta})^T \cdot \Sigma^{-1} \cdot (\mathbf{x} - \mathbf{A} \cdot \Delta\hat{\beta}) = \min, \quad (7)$$

whence we obtain:

$$\Delta\hat{\beta} = (\mathbf{A}^T \cdot \Sigma^{-1} \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \Sigma^{-1} \cdot \mathbf{x}, \quad (8)$$

where Σ is an $m \times m$ – dimensional covariance matrix of the measurements. The solution is conditional to the existence of all the necessary inversions, i.e., under the assumption $\text{Rank}(\mathbf{A}) = k \leq m$, $\text{Rank}(\Sigma) = m$. The least squares method is also applied to obtain the estimates $\hat{p}(T_i)$ of the threshold parameter $p(T_i)$.

An important part of the model's creation is its verification; in this case we will focus on an assessment of the model's quality. The selection of a characteristic, an assessment of the quality of the proposed models, and a comparison of models with different degrees of the polynomial is the subject of the following part. The final stage of the model's creation is its application. According to the given aim, the proposed, quantified and verified model will be applied in order to reduce the impact of the vertical refraction in the following form:

$$h_i^{corr} = h_i - \begin{cases} \hat{\beta}_0 + \hat{\beta}_1 \cdot T_i + \hat{\beta}_2 \cdot p_i, & \text{regression model,} \\ \hat{\beta}_0 + \hat{\beta}_1 \cdot T_i + \hat{\beta}_2 \cdot p_i + \hat{\beta}_3 \cdot \min\{p_i, \hat{p}(T_i)\}, & L = 0, 1, \\ \hat{\beta}_0 + \hat{\beta}_1 \cdot T_i + \hat{\beta}_2 \cdot p_i + \hat{\beta}_3 \cdot \min\{p_i, \hat{p}(T_i)\} + \hat{\beta}_4 \cdot \max\{p_i, \hat{p}(T_i)\}, & L = 2, 3, \end{cases} \quad (9)$$

where h^{corr} is the elevation corrected for the impact of the vertical refraction.

5 COMPARISON AND APPLICATION OF THE MATHEMATICAL MODELS

To assess the quality of the proposed models we chose the residual sum of the squares:

$$SS_u = \mathbf{u}^T \cdot \mathbf{u}, \quad (10)$$

Tab. 2 A comparison of the residual sums of the squares.

Number of experiment	Regression model [mm ²]	Two-regime model			
		L = 0 [mm ²]	L = 1 [mm ²]	L = 2 [mm ²]	L = 3 [mm ²]
E1	0.62	0.53	0.53	0.14	0.14
E2	0.83	0.37	0.75	0.64	0.60
E3	0.96	0.79	0.71	0.67	0.81
E4	0.71	0.62	0.57	0.50	0.63
E5	3.24	1.29	1.07	0.13	0.62
E6	4.09	1.82	1.97	1.38	0.45
E7	6.75	5.82	4.67	1.96	4.16
E8	3.50	3.31	3.47	1.30	1.52
E9	1.24	1.15	1.24	0.61	0.71
E10	10.60	10.32	8.53	3.41	7.37
E11	6.32	3.90	4.44	1.82	2.35
E12	6.06	3.60	3.85	1.36	2.43

Tab. 3 Comparison of elevations – E7 (date 2008-09-19, slope distance 203.831 m).

Time of observation	h [m]	h ^{corr} [m]	h _{PL} [m]	h _{PL} – h [mm]	h _{PL} – h ^{corr} [mm]
7 ⁰⁰	0.0766	0.0747	0.0749	-1.7	0.2
8 ⁰⁰	0.0755	0.0745		-0.6	0.4
9 ⁰⁰	0.0747	0.0739		0.2	1.0
10 ⁰⁰	0.0747	0.0749		0.2	0.0
11 ⁰⁰	0.0740	0.0743		0.9	0.6
12 ⁰⁰	0.0737	0.0746		1.2	0.3
13 ⁰⁰	0.0718	0.0740		3.1	0.9
14 ⁰⁰	0.0718	0.0746		3.1	0.3
15 ⁰⁰	0.0746	0.0748		0.3	0.1
16 ⁰⁰	0.0738	0.0737		1.1	1.2
17 ⁰⁰	0.0751	0.0751		-0.2	-0.2
18 ⁰⁰	0.0756	0.0743		-0.7	0.6
19 ⁰⁰	0.0758	0.0743		-0.9	0.6

where $\mathbf{u}$ is an m – dimensional vector of the residuals which arise when comparing the empirical data and the data achieved by the mathematical model. The residual sum of the squares expresses that part of the variability of the interpreted variable which is not explained by the model; therefore, the lower value of this criterion indicates a better built model. A comparison of the residual sums of the squares between the regression model and the two-regime

Tab. 4 Comparison of elevations – E10 (date 2009-03-31, slope distance 318.963 m).

Time of observation	h [m]	h^{corr} [m]	h_{PL} [m]	$h_{PL} - h$ [mm]	$h_{PL} - h^{corr}$ [mm]
7 ¹⁵	0.3707	0.3640	0.3637	-7.0	-0.3
8 ¹⁵	0.3690	0.3652		-5.3	-1.5
9 ¹⁵	0.3669	0.3640		-3.2	-0.3
10 ¹⁵	0.3645	0.3641		-0.8	-0.4
11 ¹⁵	0.3632	0.3644		0.5	-0.7
12 ¹⁵	0.3626	0.3646		1.1	-0.9
13 ¹⁵	0.3623	0.3647		1.4	-1.0
14 ¹⁵	0.3625	0.3650		1.2	-1.3
15 ¹⁵	0.3600	0.3630		3.7	0.7
16 ¹⁵	0.3609	0.3644		2.8	-0.7
17 ¹⁵	0.3627	0.3645		1.0	-0.8
18 ¹⁵	0.3651	0.3647		-1.4	-1.0
19 ¹⁵	0.3663	0.3642		-2.6	-0.5

model with the polynomials of degrees $L = 0, 1, \dots, 3$ is given in table 2.

The above-mentioned comparison of the residual sums of the squares documents the increasing of the quality of the two-regime model versus the regression model. With the exception of the E2 and E6 experiments, the residual sum of the squares acquires minimum values when using the polynomial of the second degree. Based on the selected criterion to assess the quality, we can denote the two-regime model with the polynomial of the second degree as the most optimal of all the tested models, which can then be used in order to reduce the impact of the vertical refraction. The method of applying the selected model is defined by equation (9). The results of the two experiments are given in tables 3 and 4, which also

include a comparison with the elevation determined by the precise levelling (h_{PL}).

CONCLUSION

Despite the progress and continuous modernization of measuring devices, the crucial factor in using the trigonometric measurement of elevations is the impact of the vertical refraction. In this paper we focused on the elimination of the vertical refraction from the results of repeated trigonometric measurements by means of a mathematical model. Besides the classic regression model, we proposed a two-regime model based on aggregation functions. In this model, the threshold function is an approximation of the relationship between the air pressure and temperature. The given comparison of the residual sums of the squares shows that the two-regime model contributes at a higher rate to the explanation of the variability of the modelled elevations and that in any case it does not provide a lower degree of quality than the regression model. On the other hand, the disadvantage of the model is the requirement to calculate the estimates of the threshold parameter and also more unknown parameters, which leads to the need to increase the number of measurements. A comparison of the match between the corrected elevations and the elevations determined by precise levelling shows that a consideration of the meteorological parameters significantly contributes to the reduction of the impact of the vertical refraction, but does not lead to its complete exclusion. This fact is, on the one hand, determined by the quality of the model used; on the other hand, it is a consequence of the fluctuations in the systematic influence, which assumes a variable character and may not converge to the zero mean value.

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